Price Postponement in a Decentralized Newsvendor Model under Uncertain Supply and Demand

Xiao-qin ZHANG\textsuperscript{1,a,*} and Yan-yi XU\textsuperscript{1}

\textsuperscript{1}School of Business, East China University of Science and Technology, 130 Meilong Road, Shanghai, P.R. China
\textsuperscript{a}Email: zhangxiaqin0827@163.com
\textsuperscript{*Corresponding author

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Abstract. With the rise of global procurement, economic globalization, on the one hand, promotes the development of supply chain; on the other hand it also increases the risk of supply chain, which makes the supply increasingly uncertain. According to the newsvendor model, we analyze the effect of price postponement in a decentralized newsvendor model with additive and price-dependent demand, wherein the supplier sets the wholesale price, and the retailer determines the order quantity and retail price. Such postponement strategies can be used by the retailer by delaying his retail price until after supply and demand uncertainties are observed. This research demonstrates the existence and uniqueness of the optimal order quantity of the retailer. We further shows that retailer adopts the price postponement strategy can reduce market uncertainty risk for retailers under the price dependent stochastic demand and uncertain supply by numerical simulation. At the same time, with the increase of the supplier’s production cost, the selling price (or expected price) will increase, order quantity will decrease, and the expected profit of the supplier and retailer will decrease under pricing postponement model.

Introduction

Over the past two decades, with globalization and faster product development, an unprecedented number and variety of products are competing in markets ranging from apparel and toys to power tools and computers. With the rise of global procurement, economic globalization, on the one hand, promotes the development of supply chain; on the other hand it also increases the risk of supply chain, which makes the supply increasingly uncertain. The uncertainty of supply has become one of the main issues of global supply chain management.

With the increasing globalization of economy, the loyalty of consumers to a single product is declining, and the demand is becoming more and more personalized and diversified. This trend brings benefit to consumers but intensifies the demand uncertainty in the market risk, which has brought great difficulties to the production enterprises and sales enterprises in a supply chain system.

Despite the benefits to consumers, this phenomenon is making it more difficult for suppliers and retailers to predict which of their products will sell and to plan production, ordering, and pricing decisions accordingly. To be able to make supply meet demand in an uncertain world, supply chain members have invested considerable resources to control supply and demand variability and reduce risk due to their uncertainty. Postponing pricing has emerged as a strategic mechanism to manage some of the risks associated with uncertain supply and demand. In academy,
the problem of delay pricing and supply uncertainty has been widely studied, but most of them are independent. We combine price postponement and supply uncertainty together and build the supply and demand uncertainty newsvendor model to study the optimal price and order quantity.

**Literature Review**

We mainly study the effect of price postponement in a decentralized newsvendor model with additive and price-dependent demand under uncertain supply and demand. The research on postponement policy has 3 aspects: production, pricing and ordering. The concept of postponement was first proposed by Aldeson [1], he examined that to delay the difference in the form and characteristics of the product in the production line as far as possible, can greatly reduce the cost and promote production efficiency. Bucklin [2] further analyzed the importance of postponement on cost reduction from production. Since 1980s, many international famous enterprises with production strategies achieved success with delaying production, which further attracted attention of scholars. However, the research on pricing postponement has just started, and the existing literature is not so much. In reality, the retailers tend to choose price postponement to reduce the risk from the market uncertainty. Van Mieghem and Dada [3] suggested that the bargaining practices in a car dealership can be viewed as an example for price postponement. Specifically, price postponement is implemented if a car dealership allows for some bargaining and haggling about the final price.

In this paper, we assume supply and demand are both uncertain. Recently, the existing work on supply uncertainty can be divided into three categories: (i) the random-yield model, which models the uncertainty by assuming that the supply level is a random function of the input level (e.g. Babich, Ritchken, Burnetas [4]; Deo, Corbett [5]; Federgruen, Yang [6]; Gao, Li, Shou [7]; Kazaz [8]; Parlar, Wang [9]; Swaminathan, Shanthikumar [10]; Wang, Gilland, Tomlin [11]; Fang & Shou [12]); (ii) the stochastic lead-time model, which models the lead-time as a random variable (e.g. Zipkin [13]), and (iii) the supply disruption model, which typically models the uncertainty of a supplier as one of two states: “up” or “down” (see Arreola-Risa, DeCroix [14]; Meyer, Rothkopf, Smith [15]; Parlar, Berkin [16]; Song, Zipkin [17]; Tomlin [18]). Our proposed research builds upon the supply disruption model.

Another stream of related research is the combination of supply uncertainty and pricing strategies. Daniel Granot etc.[19] analyzed the effect of price and order postponement in a decentralized newsvendor mode with multiplicative and price-dependent demand, while we build a decentralized newsvendor model with additive and price-dependent demand. Furthermore, they show the optimal policy in a buyback contract, but we will choose the wholesale contract. Qi Feng [20] presents a dynamic programming formulation for the profit maximization problem that simultaneously optimizes the pricing and inventory decisions under both supply and demand uncertainties. Tao Li etc.[21] show sourcing and pricing decisions of a firm with correlated suppliers and a price-dependent demand. Therefore, under stochastic demand, to combine uncertain supply and pricing postponement in the supply chain system in the wholesale price contract model is the significant difference and the main innovation of this paper.

The results show that when retailers adopt pricing postponement, the retail price will not lead to excessive demand, and the expected profit of the members are greater than without delaying pricing. Pricing postponement can effectively avoid the risk
from market and supply, which greatly improve the efficiency of the whole supply chain.

The Model

Model Description

Consider a decentralized supply chain with price-dependent (PD) demand, including one retailer and one supplier, wherein the supplier with random supply sells a single product to an independent retailer who is facing stochastic demand from the end-market. Randomness in supply is up or down, but in demand is price-dependent. In this paper, we adopt the additive demand model; $D = d(p) + \varepsilon$, which is commonly used in the economics and operations literature. $d(p)$ is the deterministic part of $D$ which decreases in the retail price $p$; $d(p) = a - bp$, and $\varepsilon$ captures the random factor of the demand model, wherein we assume that $d(p) = 1 - p$, the analysis can be easily extended to any positive values of $a, b$. Let $F(\cdot)$ and $f(\cdot)$ be the distribution and density functions of $\varepsilon$ respectively. $\varepsilon$ has a support on $[A, B]$ with $A \geq 0$, $F(\varepsilon) = 0$ for $\varepsilon \in [0, A]$ and $F(\varepsilon) = 1$ for $\varepsilon \in [B, \infty]$. Without loss of generality, we normalize the mean of $E(\varepsilon) = 1$.

The decision sequence in this model is as follows. $s$ (Supplier), who has limited supply capacity and can produce the items at a fixed marginal cost $c$ with no production lead time, is a Stackelberg leader. $s$ initiates the process by offering a limited supply quantity $K$ and a linear wholesale price $w$ ($w \geq c$). $r$ (Retailer) then commits to an order quantity $Q$ ($Q \geq 0$) and selling price $p$ ($p \geq w$), but the delivery quantity is $\min\{K, Q\}$ and the retail price affects the expected demand function. Demand uncertainty is realized thereafter. It is assumed that the salvage value of unsold inventory is zero for both $s$ and $r$, unsatisfied demand is lost, and there is no penalty cost for unmet demand.

We will study the PD newsvendor model, wherein all decisions are made before demand uncertainty is resolved, as the PD newsvendor model under no postponement, or the N-postponement model. We will refer to the model wherein $r$ only postpones his decision on the price.

No Postponement

In this section, we study the PD newsvendor model without any postponement, i.e. all decisions are made before demand is realized. Thus, in this case, the supplier ($s$) initiates the process by offering a limited supply quantity $K$ and a linear wholesale price $w$ (Stage 1), and then the retailer ($r$) commits to an order quantity $Q$ ($Q \geq 0$) and selling price $p$ ($p \geq w$), but the delivery quantity is $\min\{K, Q\}$ (Stage 2). Thereafter, demand is realized. We use backward induction to solve this two-stage Stackelberg game. Therefore the supplier’s and retailer’s expected profit functions are

$$E\pi_s = E\{\left(\left(w - c_s\right) S\right)\} = (w - c_s) \cdot S$$

$$E\pi_r = p\int_{A}^{S-(a-\beta p)} (\alpha - \beta p + x) f(x) \, dx + \int_{S-(a-\beta p)}^{B} S \cdot f(x) \, dx - w \cdot S$$

(1)

Note that the N-postponement model with a linear expected demand function under a wholesale price-only contract has been studied by Granot and Yin [19]. However, note that for a general distribution of $\varepsilon$, the supplier’s expected profit function in stage 1, taking into account the retailer’s reaction function of $(P', S')$ may not be well behaved.
**Price Postponement**

Under price postponement, the retailer postpones his retail pricing decision until after demand uncertainty is resolved. The sequence of events is as follows. The supplier initiates the process by offering a wholesale price \( w \) with uncertain supply (Stage 1). The retailer then commits to an order quantity \( Q \) (Stage 2) with the delivery quantity \( Q \) observed with \( \min\{K,Q\} \) (Stage 2). Demand uncertainty, \( \varepsilon \), is realized afterwards, and finally, after observing demand uncertainty, the retailer sets the retail price \( p \) (Stage 3).

In Stage 3, given \((w, S)\) and observing demand uncertainty, \( \hat{\varepsilon} \), \( r \) chooses \( p \) to maximize:

\[
\max_p \pi_r = p \cdot \min(S, D) - w \cdot S = p \cdot \min(\alpha - \beta p + \hat{\varepsilon}, S) - w \cdot S. \tag{2}
\]

**Theorem 1.** Given \((w, S)\) and observing demand uncertainty, \( \hat{\varepsilon} \), the retail price will not lead to excess demand, therefore we have \( \hat{\varepsilon} \leq S - d(p) \), and the retailer’s optimal retail price is

\[
p^* = \begin{cases} 
\frac{1}{2\beta} (\alpha + \hat{\varepsilon}), & \text{if } \hat{\varepsilon} \leq 2S - \alpha \\
\frac{1}{\beta} (\hat{\varepsilon} + \alpha - S), & \text{if } \hat{\varepsilon} > 2S - \alpha
\end{cases}
\tag{3}
\]

**Proof:** Given by (1), considering two options for \( p \).

Option 1, if \( S \leq \alpha - \beta p + \hat{\varepsilon} \), then \( p \leq \frac{1}{\beta} (\hat{\varepsilon} + \alpha - S) \), so \( r \)'s profit function is \( \pi_r = p \cdot S - w \cdot S \), which is decreasing with \( p \), then we have \( \pi_r = p \cdot S - w \cdot S \).

Option 2, if \( S > \alpha - \beta p + \hat{\varepsilon} \), then \( p > \frac{1}{\beta} (\hat{\varepsilon} + \alpha - S) \), so \( r \)'s profit function is \( \pi_r = p \cdot (\alpha - \beta p + \hat{\varepsilon}) - w \cdot S \), which is strictly concave of \( p \), then according to

\[
\frac{\partial \pi_r}{\partial p} = 0,
\]

we get \( p = \frac{1}{2\beta} (\alpha + \hat{\varepsilon}) \). Thus \( p^* = \max \left\{ \frac{1}{2\beta} (\alpha + \hat{\varepsilon}), \frac{1}{\beta} (\hat{\varepsilon} + \alpha - S) \right\} \).

All in all, by comparing these two options, we conclude that

\[
p^* = \begin{cases} 
\frac{1}{2\beta} (\alpha + \hat{\varepsilon}), & \hat{\varepsilon} \leq 2S - \alpha \\
\frac{1}{\beta} (\hat{\varepsilon} + \alpha - S), & \hat{\varepsilon} > 2S - \alpha
\end{cases}
\]

Then we can get the expected value of \( p \):

\[
\mathbb{E}p^* = \int_A^{2S - \alpha} \frac{1}{2\beta} (\alpha + \hat{\varepsilon}) f(\varepsilon) d\varepsilon + \int_{2S - \alpha}^{\beta} \frac{1}{\beta} (\hat{\varepsilon} + \alpha - S) f(\varepsilon) d\varepsilon \tag{4}
\]

In Stage 2, given \((w, p^*)\) and by (4), \( r \) determines his best order quantity \( Q \) before demand uncertainty is observed. Thus, \( r \) chooses \( S \) (or \( Q \)) to maximize
Theorem 2. The retailer’s expected profit function, given by (5), is strictly concave in $S$. Therefore the optimal order quantity satisfies

$$\int_{2S-a}^{h} (\alpha + \varepsilon - 2S) f(\varepsilon) d\varepsilon - \beta w = 0$$

(6)

At the same time we get

$$w = \int_{2S-a}^{h} \frac{(\alpha + \varepsilon - 2S)}{\beta} f(\varepsilon) d\varepsilon$$

(7)

Proof: substituting $E\pi^* = \int_{-\infty}^{-\alpha} \frac{1}{2\beta} (\alpha + \varepsilon)f(\varepsilon)d\varepsilon + \int_{2S-\alpha}^{h} \frac{1}{\beta} (\varepsilon + \alpha - S)f(\varepsilon)d\varepsilon$ into (5) and simplify, we can obtain

$$E\pi^* = \int_{-\infty}^{-\alpha} \frac{1}{4\beta} (\alpha + \varepsilon)^2 f(\varepsilon)d\varepsilon + \int_{2S-\alpha}^{h} \frac{1}{\beta} (\varepsilon + \alpha - S) \cdot f(\varepsilon)d\varepsilon - w \cdot S$$

Taking the first-order and second-order derivative of $E\pi^*$ with respect to $S$ gives us

$$\frac{\partial E\pi^*}{\partial S} = \frac{1}{\beta} \int_{-\infty}^{-\alpha} (\alpha + \varepsilon - 2S) f(\varepsilon) d\varepsilon - w, \quad \frac{\partial^2 E\pi^*}{\partial S^2} = -\frac{2}{\beta} \int_{2S-\alpha}^{h} f(\varepsilon) d\varepsilon < 0,$$

which implies that the retailer’s expected profit function is strictly concave in $S$. Thus we can obtain the optimal delivery quantity $S^*$ from

$$\int_{2S-a}^{h} (\alpha + \varepsilon - 2S) f(\varepsilon) d\varepsilon - \beta w = 0.$$ 

So the optimal order quantity $Q^*$ is

$$Q^* = \begin{cases} S', & K \geq S' \\
K, & K < S' \end{cases}$$

Stage 1, the supplier determines $w$ to maximize her expected profit function. To simplify the analysis, we use an alternative expression for the supplier’s problem in Stage 1. That is, instead of working with $S^*(W)$, we work with $W^*(S)$. Thus the alternative approach for the supplier’s problem is to choose $S^*$ that maximize

$$E\pi_s = (w - c_s)S - \left[ \int_{2S-a}^{h} \frac{(\alpha + \varepsilon - 2S)}{\beta} f(\varepsilon) d\varepsilon - c \right].$$

(8)

Taking the first-order, second-order and third-order derivation of $E\pi_s$ with respect to $S$ respectively gives us

$$\frac{dE\pi_s}{dS} = \int_{2S-a}^{h} \frac{(\alpha + \varepsilon - 4S)}{\beta} f(\varepsilon) d\varepsilon - c = 0,$$

$$\frac{d^2 E\pi_s}{dS^2} = \frac{4}{\beta} \left[ 2f(2S-\alpha) - \int_{2S-\alpha}^{h} f(\varepsilon) d\varepsilon \right] = \frac{4}{\beta} [Sf(2S-\alpha) - \overline{F}(2S-\alpha)],$$

$$\frac{d^2 E\pi_s}{dS^2} = \frac{4}{\beta} \left[ 3f(2S-\alpha) + 2f'(2S-\alpha) \right] > 0.$$
Therefore, only if \( f(\omega) \) is increasing,
\[
\frac{dE\pi_S}{dS} S = \frac{\alpha + \beta}{2} = -c < 0, \quad \frac{dE\pi_S}{dS}(S = 0) = \frac{\alpha + E(\varepsilon)}{\beta} - c > 0;
\]
\[
\frac{d^2E\pi_S}{dS^2} S = \frac{\alpha + \beta}{2} \frac{4S}{\beta} f(\beta) = \frac{2(\alpha + \beta)}{\beta} f(\beta) \geq 0, \quad \frac{d^2E\pi_S}{dS^2}(S = 0) = -\frac{4}{\beta} < 0.
\]

Thus we conclude that \( \frac{dE\pi}{dS} \) first decreases in \( S \) and then increases, and cross the horizontal line only once at the unique solution \( \hat{S}^* \in (0, \frac{\alpha + \beta}{2}) \) satisfying
\[
\frac{dE\pi_S}{dS} = \int_{2S^* - \alpha}^{\beta} \frac{(\alpha + \varepsilon - 4S)}{\beta} f(\varepsilon)d\varepsilon - c = 0.\]
Substituting the implicit expression for \( \hat{S}^* \) into (7) and simplifying, gives us
\[
\hat{W}^* = \frac{2\hat{S}^* - \alpha}{2\hat{S}^* - \alpha} + c, \quad \hat{E}p^* = \frac{\alpha + E(\varepsilon)}{2\beta} + \frac{S^* - \alpha}{2},
\]
\[
\hat{E}p^* = \frac{2(\hat{S}^*)^2 - \alpha}{\beta}, \quad \hat{E}\pi_r^* = \int_{\lambda}^{2\hat{S}^* - \alpha} \frac{1}{4\beta}(\alpha + \varepsilon)^2 \cdot f(\varepsilon)d\varepsilon + \frac{(\hat{S}^*)^2 - \alpha}{\beta}.
\]

So the decision problem is to solve (8), and combine (6) and (7), we can obtain the optimal value of decision variables. To evaluate the effect of price postponement, we next use the numerical examples which can be seen from the table 1.

**Numerical Examples**

In this section, we illustrate some numerical examples to gain further insights. We assume that the determination of the demand function is \( d(p) = \alpha - \beta p = 1 - p \), while the uncertainty part, \( \varepsilon \), follows a uniform distribution for any \( \varepsilon \in (0,2) \), which means \( A = 0, B = 2 \). And the basic parameter are set as follows: \( c \in [0,2], K \sim [0,1] \).

Our results are summarized in Table 1.

From Table 1, we find that, when the production cost increases, the optimal order quantity decreases and the corresponding retailer’s expected profit and supplier’s expected profit also decrease. But the optimal wholesale price and the retail price increases. While when the supply increases, we observe that the optimal order quantity, the optimal wholesale price and the retail price increase, as well as the corresponding retailer’s expected profit and supplier’s expected profit increase. In addition, compared to the case of no-postponement policy, the retailer’s profit and the expected profit of the supplier are higher under pricing postponement.
Table 1. Supply chain performance under p-postponement with uncertain supply.

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**Conclusion**

In this paper, we consider the effect of price postponement in a decentralized newsvendor model with additive and price-dependent demand under uncertainty supply. We assume the retailer’s demand and the supplier’s supply are all stochastic, and we derive the optimal order quantity and the price for each member. Finally, the impact of the model parameters on the decentralized supply chain is showed through numerical examples. The results show that pricing postponement can effectively avoid the risk from market and supply, which greatly improve the efficiency of the whole supply chain. This research can be directly extended by assuming the supply follows a random distribution function and considering more uncertainties such as the supplier’s lead time and the selling price.

**Literature References**


