Quality Contracts in a Supply Chain under Different Control Setting

Pin ZHUANG*

College of Economics and Management, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China

*Corresponding author

Keywords: Game theory, Contract model, Supplier’s quality management.

Abstract. This paper investigated how to design quality contract models in a supply chain consisting of a single supplier and single manufacturer. We employed game theory to design quality contract models: wholesale price contract under the dominated supplier and profit margin contract under the dominated manufacturer in decentralized control situation.

Introduction

Supply chain contracts are considered as a useful tool to structure the costs and rewards of all of its members so as to achieve coordination in a decentralized situation. Studies of successful Supplier Quality Management (SQM) frameworks consistently showed that the orchestration of supplier alliances, supplier development and supplier monitoring are the three foundational elements to a globally scalable, secure supplier network. (I-Ki, Chin, 2004)[1].

Agrawal and Muthulingam (2015) investigated how the depreciation of organizational knowledge (organizational forgetting) affects quality performance. They analyze information on 2,732 quality improvement initiatives implemented by 295 vendors of a car manufacturer and find that organizational forgetting affects quality gains obtained from both learning-by-doing (autonomous learning) and quality improvement initiatives (induced learning); more than 16% of quality gains from autonomous learning and 13% of quality gains from induced learning depreciate every year. Their results highlight the ubiquity of organizational forgetting and suggest the need for continued attention to sustain and enhance quality performance in supply chains[2].

Yang et al.(2012) studied a buyer’s strategic use of a dual-sourcing option when facing suppliers possessing private information about their disruption likelihood and solved for the buyer’s optimal procurement contract. Their research showed that the optimal contract can be interpreted as the buyer choosing between diversification and competition benefits. Better information increased diversification benefits and decreases competition benefits[3].

Chaturvedi and Martínez-de (2011) considered a buyer facing multiple potential suppliers, each having an associated (exogenous) reliability that quantifies its risk of supply failure and designed optimal mechanisms that depend on the buyer’s level of information regarding the suppliers’ cost of production and reliability. Their specific contribution was to provide guidelines for designing optimal procurement mechanisms when there is a risk of supplier failure[4].

Liu and Wang (2015) developed a quality control game model for logistics service supply chain and analyzed the impact of different risk attitude of a logistics service integrator and a functional logistics service provider[5].

Quality Contracts in Decentralized Control Situation under Dominated Supplier

We consider the standard setting with a single supplier and single manufacturer who sells the supplier’s product to the final market. Manufacturer orders from supplier according to market demand \( Q \). The situation is described as follows. The market demand function is given by linear demand function in quality and price: \( q = Q(p,x) = \alpha + \varepsilon x - \beta p \), where \( \alpha > 0 \), \( \beta > 0 \) and \( \varepsilon > 0 \) are known
parameters, \( x \) denotes the supplier quality, and the retail price selling to the customer is \( p \). The supplier’s production cost is given by \( s \), the supplier quality control cost is \( \varphi x^2/2 \).

The dominated supplier, as the leader of the game, will declare a unit wholesale price \( w \) and the product quality \( x \), then the manufacturer, as the follower, decides the retail price \( p \).

The supplier’s profit function is \( \Pi_s(w, x) = \max_{(w, x)} (w - s)D - \varphi x^2/2 \). The manufacturer’s profit function can be written as: \( \Pi_m(p) = \max_{p} (p - w - m)D \).

From the first optimal condition \( \partial\Pi_s(p)/\partial p = 0 \), we obtain \( p(w, x) = \frac{\alpha + \varepsilon x + \beta(w + m)}{2\beta} \).

Substituting \( p(w, x) \) into the supplier’s expected profit function \( \Pi_s(w, x) \), we can obtain \( x(w) = \frac{e(w-s)}{2\varphi} \). Then \( p(w) = \frac{2\alpha e + e(w-s) + 2\beta e(w+m)}{4\beta} \).

Substituting \( x(w), p(w) \) into the supplier’s expected profit function \( \Pi_s(w, x) \), By calculating \( \partial\Pi_s(w, x)/\partial x = 0 \), we can derive Lemma 1.

Lemma 1. The wholesale price contract in decentralized control setting is following.

The optimal wholesale price is:

\( w^* = \frac{2\alpha e^2 + 2\beta e s - e^2 s}{4\beta e^2 - e^2} \) (1)

The optimal quality level is:

\( x^* = \frac{e[\alpha - \beta(s + m)]}{4\beta e^2 - e^2} \) (2)

The optimal retail price is:

\( p^* = \frac{3\alpha e + (\beta e - e^2)(s + m)}{4\beta e^2 - e^2} \) (3)

The optimal order quantity is:

\( Q^* = \frac{\beta e[\alpha - \beta(s + m)]}{4\beta e^2 - e^2} \) (4)

The supplier’s expected profit is:

\( \Pi_s^* = \frac{\varphi[\alpha - \beta(s + m)]^2}{2(4\beta e^2 - e^2)} \) (5)

The manufacturer’s expected profit is:

\( \Pi_m^* = \frac{\varphi[\alpha - \beta(s + m)]^2}{4\beta e^2 - e^2} \) (6)

The supply chain system’s expected profit is:

\( \Pi^* = \frac{3\varphi[\alpha - \beta(s + m)]^2}{4(4\beta e^2 - e^2)} \) (7)
Quality Contracts in Decentralized Control Situation under Dominated Manufacturer

Just as a dominated supplier will declare a unit wholesale price \( w \), the economics and marketing literature has long recognized that a dominated manufacturer can declare a required profit margin. The manufacturer’s profit function can be written as: \( \Pi_n(p) = \max_{p} (p - w - m) D \). After the dominant manufacturer declares his required profit margin \( M \), i.e., \( M = \Pi_n(p)/D \), the supplier knows that, for whatever she quotes, the unit retailer price will be \( p = w + m + M \). Hence, the supplier’s profit function is \( \Pi_s(w,x) = \max_{x}(w - s)D - \varphi x^2/2 \). From the first optimal condition \( \partial \Pi_s(w,x)/\partial x = 0 \), and \( \partial \Pi_s(w,x)/\partial x = 0 \), we obtain \( w(M) = \varphi p - \beta \varphi (s + m - M) / \varphi 2 \beta \varphi - \varphi^2 \) and \( x(M) = \varphi p - \beta \varphi (s + m + M) / \varphi 2 \beta \varphi - \varphi^2 \).

Substituting \( w(M) \) into the manufacturer’s expected profit function \( \Pi_n(M) \), \( \Pi_n(M) = QM = (\alpha + \beta p - \beta)M \), we can obtain \( \Pi_n(M) = M \beta \varphi \left[ \alpha - \beta (s + m + M) \right] / \varphi 2 \beta \varphi - \varphi^2 \). Solving \( \partial \Pi_n(M)/\partial M = 0 \), we obtain optimal profit margin \( M \).

Lemma 2. The profit margin contract in decentralized control setting is following.

The optimal profit margin is
\[
M^* = \frac{\alpha - \beta (s + m)}{2 \beta}
\] (8)

The optimal wholesale price is
\[
w^* = \frac{\alpha p + (3 \beta p - 2 \varphi s - \beta \varphi m)}{2 (2 \beta \varphi - \varphi^2)}
\] (9)

The optimal quality level is
\[
x^* = \frac{\varphi p - \beta (s + m)}{2 (2 \beta \varphi - \varphi^2)}
\] (10)

The optimal retail price is:
\[
p^* = \frac{(3 \beta p - \varphi s) \alpha + \beta (\beta p - \varphi s) (s + m)}{2 \beta (2 \beta \varphi - \varphi^2)}
\] (11)

The optimal order quantity is:
\[
Q^* = \frac{\beta \varphi [\alpha - \beta (s + m)]}{2 (2 \beta \varphi - \varphi^2)}
\] (12)

The supplier’s expected profit is:
\[
\Pi^*_s = \frac{\varphi [(\alpha - \beta (s + m)]^2}{8 (2 \beta \varphi - \varphi^2)}
\] (13)

The manufacturer’s expected profit is:
\[
\Pi^*_m = \frac{\varphi [(\alpha - \beta (s + m)]^2}{4 (2 \beta \varphi - \varphi^2)}
\] (14)

The supply chain system’s expected profit is:
\[
\Pi^* = \frac{3\varphi \left[ \alpha - \beta (s + m) \right]^2}{8(2\beta \varphi - \epsilon^2)}
\] (15)

**Conclusion**

In this paper, we have investigated contract design in a supplier-manufacturer supply chain in decentralized setting and studied how the manufacturer control supplier’s quality. We employed game theory to design quality contract models: wholesale price contract under the dominated supplier and profit margin contract under the dominated manufacturer. From the analysis above, we can draw a conclusion that the quality contracts were the optimal strategy controlling supplier’s quality in decentralized setting.

**Acknowledgement**

This research was supported by the Fundamental Research Funds for the Central Universities, No. NR2017002.

**References**


