Hume Against the Infinite Divisibility of Space

Kexin Yu
Department of Philosophy, University of Rochester, Rochester, USA 14627
yukexin1707@126.com

Keywords: ideas, infinite divisibility, mereological nihilism, copy principle

Abstract. Hume argues for the finite divisibility of space by starting with proving against the infinite divisibility of space. First and foremost, according to Hume, the mind’s capacity is finite so that it is impossible to contain an infinite number of ideas. However, in terms of his concept of general ideas, Hume admits that the mind has the ability to call up an infinite number of ideas, if provoked. Next, through observations of the imagination and the impression, Hume demonstrates that there must be simple and minimal ideas which constitute the general ideas of space and time and are themselves indivisible. The fact that one might be able to perceive a smaller thing with the help of telescopes does not show that our mind is incapable of the smallest ideas, but that the mind could be mistaken to take what is not the smallest as the smallest. One question which is discussed most is whether Hume is legitimate to make the reference from impressions and ideas to things themselves. However, Hume is less concerned with the so-called nature of things and focuses on how things appear to human beings. On the other way, Hume builds his argument upon the existence of minima. For Hume, all ideas are imagistic and extended and if added up ad infinitum, they would result in infinitude. Therefore, no finite extension could be said to be constituted by infinite ideas. Since only what is resolved into units really exists, an infinite extension does not exist at all. This statement maps onto the discussion of the special composition question. Like what general ideas are to Hume, composition is nothing but the arrangement of basic elements for the mereological nihilists.

1. Introduction
In the Treatise of Human Nature, Hume discussed his notions of space and time in the Second Part of Book I. He argues that space and time are not infinitely divisible. The infinite divisibility of space is necessary to be examined as Hume claims, “the infinite divisibility of space implies that of time, as is evident from the nature of motion. If the latter, therefore, be impossible, the former must be equally so” (T 1.2.2.5/31). This statement is significant because it implies that if space is infinitely divisible, time is infinitely divisible as well; therefore, if time is not infinitely divisible, space is not infinitely divisible, too. In this paper, I first re-construct Hume’s arguments against the infinite divisibility of space. Secondly, I follow Hume’s approach to 1) to examine the ideas of extension in general, 2) prove that there exist extended minima, and, 3) and unpack the arguments concerning space itself. When reconstructing Hume’s arguments, I illustrate objections raised against him, discuss their validity, and posit possible solutions, if any, to the objections.

2. The Main Ideas of Hume
Hume’s first argument is that any idea of a finite quality is not infinitely divisible. Particularly, ideas of space and time are not infinitely divisible. He starts with the premise that whatever is infinitely divisible consists of an infinite number of parts. Therefore, if something has a limited number of parts, its ability to be divided is limited. Different ideas of space and time, for Hume, are homogeneous, namely that any division of ideas does not affect the resultant ideas’ status as ideas. Parts of ideas are still ideas. On the other hand, since nothing is a part of itself, if the mind contains an idea, it contains all its idea-parts. Thus, if the mind contains an idea that is infinitely divisible, as
a result, it must contain an infinite number of ideas. However, it is evident that the mind’s capacity is finite. Therefore, no mind could have an infinite number of ideas. Therefore, no idea is infinitely divisible. Instead, there must exist simple, minimal, and indivisible ideas that are arranged to form complex, larger, and compound ideas. In this way, Hume regards space as extension, and time as duration, both of which are general ideas of indivisible objects arranged in their own manners. Based on this, he later reaches the conclusion that there is neither such thing as time without changing objects nor space unoccupied. In terms of space, minimal ideas are arranged one next to the other, to form an extension.

The idea that the mind’s capacity is limited is problematic because it is contradictory to Hume’s account of abstract ideas, which indicates that our mind has, to some extent, the power to produce an infinite number of ideas that are annexed to one single general term. Although this potentiality is often not actualized, it is still undeniable that the mind, at least, does have some type of quasi-infinite capacity. If so, it cannot be completely impossible for the mind to contain an infinite number of ideas. Therefore, the idea of an extension could possibly be infinitely divisible. It seems that, at least up to this point, if Hume wants to convince his readers that the ideas in the mind are all finite, he has to provide a more solid proof than this one: the limitation of the mind’s capacity in producing ideas.

In proving the existence of minimal ideas, he puts forward two observations as further evidence for their existence. The first is observed through imagination. He finds that the ideas of a thousandth part and a ten-thousandth part of a grain are identical in the mind, while they are clearly mathematically different in proportion. In this experiment, it is shown that there are ideas which are too small to be divided further, because any smaller idea, if existing, can be too small to be perceived. However, the notion of any unperceived idea is, without a doubt, absurd, given Hume’s copy principle, namely that “all our simple ideas in their first appearance are derived from simple impressions, which are correspondent to them, and which they exactly represent” (T 1.1.1.7/4). The impression and the idea differ only in force and vivacity and thus the former always precedes the latter. Both ideas and impression are perceptions. Only when ideas are perceived can they be said to exist. Therefore, any idea that is too small to be perceived cannot exist. Contrarily, there must be ideas of minimal sizes and any idea that is smaller can neither be perceived by the mind nor actually exist.

Hume presents the other piece of evidence for minimal ideas through the impressions of senses. In this experiment, Hume draws an ink dot and moves away from it and at some point, he loses sight of it, but it’s plain, that the moment before it vanished the image or impression was perfectly indivisible” (T 1.2.2.4/27). This perfectly indivisible impression is a minimal impression. Both the ideas of the grain and the impressions of dot are evidences for minimal ideas. Hume thinks that impressions and ideas have parts in the same way because they are exactly the same sorts of things, except for their difference in force and vivacity. Similarly, the infinite divisibility of both could be disproved in the same manner.

However, Hume notices in the second experiment that, he is still able to see the ink dot with a telescope, while it is no longer visible to the naked eye; so, the dot does not actually disappear. Since this new impression of the dot perceived with the help of the telescope is smaller than the smallest extended impression that is perceived with the naked eye previously, it is naturally concluded that the mind is limited in displaying the smallest ideas to us while the telescope increases the mind’s capacity by enabling the eyes to have smaller visual impressions. Hume would definitely not agree with this view. On the contrary, he believes that nothing is in fact smaller than the smallest ideas that could be formed by the mind. The mistake of the senses is that they might represent what is large and composed of parts as small and simple. For example, when someone sees a mite and forms an idea that he or she believes to be the smallest one, he might assume that the impression is equal in size to the mite itself. Later when he or she finds that the mite thought to be the smallest has smaller parts, it seems plausible to conclude that there are things smaller than the smallest ideas one can form. However, there is not. A just notion of a mite would require having distinct ideas of each part of it, which is often an impossible task for the naked human eye.
Therefore, what is limited when we see a mite with naked eyes is not the mind’s capacity in forming the smallest ideas, but the sensation’s ability of just representations.

The case of the mite, together with that of the telescope, is an analysis of the common erroneous opinion that the idea of the space is thought to be infinitely divisible because humans can either, by some instrument such as a telescope, perceive something far beyond the capacity of bare eyes or any other sense organs, or, by reasoning, conceive of a thousandth of something that is really small, such as a mite. Therefore, we are prone to think that we can always have a smaller idea of the idea of anything however small it is. In response, Hume points out two things. First, our introspection is what determines the divisibility of ideas. Both the smallest ideas formed with and without the telescope are minimal ideas. In terms of the mathematical difference of ideas, Hume claims that the difference we conceive of by reasoning is actually inconceivable by senses and the imagination. Therefore, we cannot conceive of a thousandth of a mite by the imagination since we cannot conceive of all the other innumerable parts.

After the preliminary argument concerning ideas of space and time, Hume continues to argue that space and time themselves are not infinitely divisible. He starts with the premise that our simple ideas are adequate representations of the most minute objects and all “the relations, contradictions and agreements of the ideas are all applicable to the objects” (T 1.2.2.1/29). In addition, if something is impossible for our simple ideas of extension, it is not possible for any extension, too. It should be noted that the copy principle does not justify Hume’s move from ideas to things themselves; it only justifies the move from ideas to impressions. Given Hume’s doctrine that all knowledge is ultimately derived from impressions, one can hardly be justified in making a solid shift from impressions or ideas to things themselves. Logically, Hume does not have a perfect justification for this shift; instead, he considers the assertions about things themselves to be purely probable. However, the arguments Hume gives, in answer to objections of mathematicians as a whole provides a rather convincing and reasonable account for the transition from the ideas of space to space itself. As he puts it, “whatever the mind clearly conceives includes the idea of possible existence, or in other words, that nothing we imagine is absolutely impossible” (T 1.2.2.8/32). An infinitely small extension cannot be imagined, while finitely small atomic points can. Therefore, it is impossible that there is any infinitely small extension, while it is possible that there really are atomic points. Since it is not possible for our ideas of extension to be infinitely divisible; thus, neither is it for extension itself.

3. The Problems of Hume’s Ideas

However, Hume’s inference from the ideas and impressions of space and time to what they themselves are seems less grounded. One might still question Hume by raising the possibility that even if the idea is not infinitely divisible, the thing which the idea represents may be. But the idea of the grain and the impression of the ink dot should not be confused with the grain or the dot itself. It is evident that Hume does not give any proof for the belief that ideas of X resembles X itself. “The shift is licensed by his presupposed account of representation” (Baxter 10). Actually, Hume himself totally agrees with anyone who finds out this problem, but he does not see it as troublesome. The reason is that he is less concerned with whether the dot per se resembles the impression or idea of it.

Hume’s theory can only be taken as reasonable once sufficient attention is paid to the fact that under his skepticism, all the arguments are built upon observation and experience of space and time as they appear. He does not endow any more knowledge than that. “As long as we confine our speculations to the appearances of objects to our senses, without entering into disquisitions concerning their real nature and operations, we are safe from all difficulties, and can never be embarrassed by any question” (T 1.2.5.26, n12/638). What Hume cares about is just how things are perceived by us, while this perception does not necessarily resemble reality. “Hume’s focus on appearance is the result of his version of the methodology for inquiry employed by the Royal Society. Granting the impossibility of refuting all skeptical challenges, the Society arrived at a mitigated skepticism with the goal of certainty beyond a reasonable doubt. Hume finds even this
requirement to mandate suspense of judgment” (Baxter 2). After a thorough inquiry, he realizes that he has to admit that all skeptical questions cannot be resolved and that he never has sufficient reasons to believe in anything. However, he still needs to bring about somewhat assurance. The last solution left for him is to only look at things as they appear. In other words, the appearance of things is not a distinct realm as opposed to the so-called worlds of real essence. He does not give firm belief in the essence of things at all. Instead, he confines all theories within the realm of appearance.

Another proof Hume puts forward starts with the existence of minima. Since all the minimal ideas are imagistic for Hume, they all have extension. If they are added up *ad infinitum*, the resultant idea will be infinite. However, nothing made up of an infinite number of minute extension is finite. In other words, whatever consists of an infinite number of parts is infinite in extension. Therefore, any finite extension cannot be infinitely divisible. The following argument, known as the Malezieu Argument, is concerned with the necessary existence of units, or minima; this is “another argument proposed by a noted author, which seems to me very strong and beautiful” (T 1.2.2.3/30). If there exists a part of extension that is infinitely divisible, it is never resolved into units. Contrarily, a finite extension is just a collection of extended parts. A collection of things exists only if the units exist. In the example Hume gives, it is absurd to imagine that twenty men exist, but not man one, two, three, etc. Because there does not exist a part of extension that is infinitely divisible, there must be real unities to ground the existence of extension. In this way, the collection of twenty people is not another thing added to the units for Hume. There is no another twenty-first object.

4. The Existence of Unites and Unity

Hume’s discussion of the existence of unites and unity is part of the lasting special composition question, namely how a number of “Xs” make a “Y”. What the Malezieu Argument presents is close to a mereological nihilist’s view of the composition question, i.e., there is no such thing as a composite object in the material world and that what common language means by the phrase “Xs compose Y” is nothing other than “Xs are arranged Y-wise”. Mereological nihilists do not consider a long table of plain wood standing on the floor as composed of small molecules or quarks, but a mere collection of quarks a1, a2, a3, etc. They would only admit that human perceptions of so-called composite objects come from impressions presented by the smallest indivisible elements arranged table-wise.

First, common opinions find mereological nihilism absurd because they often readily link composite objects, as they believe, with their functions. A table is designed to put stuff on. A knife is made for the purpose of cutting things. A chair is built in this particular way so that people can sit on it. Apparently, it makes no sense if those functions just come from nowhere. Therefore, it seems more reasonable to assign functions to the corresponding composition of simples and to grant that composite objects always have their own special structures, which allow composites to function as a whole. On the contrary, if mereological nihilism is taken as true theoretically, basic simple elements floating, scattering, or deposited around can be organized in an infinite number of ways. More than readily, based on the conventional observation of daily life, mereological universalists assert that simples are more often combined together in particular shapes than others, e.g., table-wise, chair-wise and knife-wise, because they compose something new beyond themselves, namely tables, chairs, and knives. Many opponents of mereological nihilism take this as crucial evidence for the existence of material composites.

5. The Main Mistake of Universalists

However, it is obvious that the mistake of universalists lies in their stereotyped observation of the material world. They look at the world in such a conventional manner that they become blind to any other way of thinking about the arrangements of simples. When examined, tables, chairs and knives stand out because they are always already accepted as composite material beings. Their existence is presupposed as if it needs no further investigation. Languages assign seemingly special meanings to
them by giving them names that other kinds of more “random” arrangements, which humans are not as attuned to, do not have. Moreover, if anyone wishes to disprove mereological nihilism in this way, he has to prove that these conventionalized compositions, as seen by them, do gain extra meanings by being considered as a whole instead of nothing but a set of simples. He will need to justify that a table is usable only when it has a name and that if there is no such a word in any language for tables, the equipment itself will disappear as well. However, despite the denial of material composites, mereological nihilists do not deny the corresponding relation between the function of a collection of simples and the innate arranged structure within this set. A knife is excellent at cutting things in that it has a sharp cutting edge. This sharp shape will not disappear if I stop using the word “edge” when referring to the structure. Therefore, basic simples arranged in certain ways are able to produce functional effects, but they do not need to compose anything new. Van Inwagen agrees that most functions produced by artifacts, as opposed to those which necessarily involve at least some kind of organism, can be understood as disguised cooperative activities: “all the activities apparently carried out by shelves and stars and other artifacts and natural bodies can be understood as disguised cooperative activities. And therefore, we are not forced to grant existence to any artifacts or natural bodies” (van Inwagen 122). In fact, functions performed by artifacts and natural bodies are cooperatively produced by simples.

Also, some find difficulty in interpreting arrangement. They think that “Xs are arranged Y-wise” is not a convincing answer to the special compositional question because its meaning seems extremely obscure. Yet, they believe mereological universalism avoids this problem, thus it is much more plausible. However, they fail to realize that this arrangement argument is as well a necessary proposition for mereological universalists. Mereological universalism allows people to talk about material objects at a larger compositional scale. Universalists add something new in addition to existing simple objects. But when they come down to the fundamental level, what they will have to look into is nothing but the simple arrangement patterns occurring. Indeed, the arrangement argument might be less clear with regard to its meaning, but it is not a weakness unique to nihilism. Non-nihilists account for the way material stuff is arranged without involving notions of part-hood or composites. Therefore, undermining the arrangement argument is not a sufficient weapon for them to win the battle and favor mereological universalism over mereological nihilism. Moreover, this arrangement argument, in fact, serves well in terms of its theoretical simplicity, according to Quine’s arguments about the ideological and ontological commitments of theories. As mereological nihilism, which helps to “see the strangeness of the contemporary view that the whole is a single thing in addition to its parts,” posits fewer objects, it is arguably more likely to be true than universalism (Baxter 9).

Another major paradox in Hume’s notion of space lies in his commitment to that ideas as images. For him, even the smallest parts of ideas are extended because if they were only small points not extended, they would not be able to be seen. However, if the minimal ideas are extended, they will readily be considered as, at least conceptually, divisible. There is a distinction between discernible parts and indiscernible, but conceptually separable, parts. Discernibility is the quality of ideas that can actually be pulled apart from each other. Yet, ideas do not necessarily need to be discernible to be conceptually separable. Even Hume himself allows for the distinction of reason in parts, as opposed to the real separability in thought. As Hume puts in the footnote: “This evident, that even different simple ideas may have a similarity or resemblance to each other, nor is it necessary, that the point or circumstance of resemblance should be distinct or separable from that in which they differ” (T 1.1.7.7, n5/637). The distinction of reason is a distinction the mind makes that is not grounded in any fact of reality. The example Hume uses to illustrate is the conceptual distinction between the body and its motion or figure. “Even in this simplicity there might be contained many different resemblances and relations…observing afterwards a globe of black marble and a cube of white, and comparing them with our former object, we find two separate resemblances, in what formerly seemed, and really is, perfectly inseparable” (T 1.1.7.18/25). These differential resemblances can only be found after seeing multiple objects and the difference between black and white marbles is only seen after they are compared to another white cube. The quality and the object
which means the blackness and the marble itself are still not separable in thought, but in the thought experiment, there are two evidently different resemblances, namely the color and the figure, which are then conceptually separable.

Nevertheless, this ambiguous use of the word “separable” in Hume’s interpretation of the distinction of reason seemingly contradicts the converse of the separability principle, i.e., that “whatever objects are different are distinguishable, and that whatever objects are distinguishable are separable by the thought and imagination” (T 1.1.7.3/17). According to this principle, the distinction of reason seems to indicate that the object and its quality are different things. This is definitely absurd for Hume as Hume places sense and imagination above reason obviously. So, distinction of reason does not imply any real distinction.

Another piece of tension arises from the incompatibility between Hume’s denial of infinite divisibility and theorems of Euclidean geometry, “because the infinite divisibility thesis itself is a theorem of Euclidean geometry” (Badici 232). The opinion of some schools considers mathematical points, namely what Hume calls indivisibles, to be non-identities. They hold that mathematical points are not able to form any real existence. According to Hume, if things are constituted by those indivisibles or mathematical points, then it makes sense for zeros to add up to a non-zero identity. In other words, extensionless things come to form something that is extended. Hume answers that those mathematical points have solidity and colors. The only way to measure the size of any extension is by the number of minima. Still, there is the problem that people might have differently sized ideas of minima, which means that the standard of measurement would not be uniform. Also, it is hard and arbitrary to decide the shape of minima. If minima do not have a determined shape, they could not be said to be extended. However, Hume thinks that “the success of applied geometry is no proof of infinite divisibility” (Baxter 8). He does not seem to be committed to the view that we actually can establish some rigorous standard. After all, for Hume, geometry itself is merely “built on ideas, which are not exact, and maxims, which are not precisely true” (T 1.2.4.17/45). It should be kept in mind that Hume criticizes the ability of reason in favor of the usefulness of customs and claims that any difficulty of this sort comes from the instability of human senses when it comes to such small points.

In addition, according to Hume, mathematical points are a good explanation of why a surface without depth and a line without breadth or depth exist. There are two kinds of objections. First, the objects of geometry do not exist in nature. Of course, this explanation is incompatible with Hume’s argument that “whatever can be conceived by a clear and distinct idea necessarily implies the possibility of existence” (T 1.2.4.11/43). Another objection is that, although the length could not be said to be separated from the breadth, they are separable by distinction of reason. At first, Hume refutes this objection and argues for the finite capacity of the mind. He further develops this refutation of the notion of termination – “if the ideas of a point, line or surface were not indivisible, it’s impossible we should ever conceive these terminations” (T 1.2.4.14/43). As terminations of finite quantities, “the ideas of surfaces, lines and points admit not of any division” (T 1.2.4.14/43-44). Or else, if the division goes on ad infinitum, there will be no such determined ultimate termination. Again, he suggests that even the most fundamental ideas in geometry are inaccurate and uncertain. Therefore, geometry is not sufficient as proof for the infinite divisibility of extension.

6. Conclusion

In this paper, I examine Hume’s arguments against the infinite divisibility of space in the Treatise of Human Nature. First, Hume claims that any idea of a finite quality is not infinitely divisible because the capacity of human mind is limited, thus the mind cannot contain an infinite number of ideas. However, the premise that the mind is limited in terms of the number of ideas it contains contradicts Hume’s interpretation of abstract ideas, which indicates that the mind does have some kind of quasi-infinite capacity. For Hume, space is extension made of minimal ideas that is simple and no longer indivisible. He uses the examples of imagining parts of a grain and perceiving an ink dot through senses as evidence for the existence of minimal ideas. Hume believes that the mind is
able to display the smallest possible ideas and is merely limited in representing a just notion of the object.

Afterwards, Hume makes the transition from ideas to space and time themselves and claims that the former is highly probably adequate representations of the latter. Additionally, as Hume believes, even if the minima are extended, they will result in an infinitely large extension if we add them up ad infinitum. Thus, a finite extension cannot be infinitely divisible. Yet because his notion of ideas is imagistic and always extended, Hume is troubled by the distinction between conceptual separability and real separability in thought. Any extended extension would be readily thought to be conceptually separable, although we are not able to actually pull them apart in our minds. Another problem that minimal extension has is the difficulty in determining their shape and size. If there is no such uniform standard, measurement of objects will be erroneous and absurd. Nevertheless, Hume never thinks that there is no such rigorous standard for the minima. Instead, he believes that human mind is unstable when thinking about these issues.

References