Approximating Function in the Application of the Approximate Calculation

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Abstract: Approximate calculation in engineering technology, production and living and so on all aspects and its important role, the specific production technology of the values are approximation, it always inseparable from the approximation calculation. We extrapolated the calculation examples of approximation into research. This paper expounds the approximate calculation of the approximate function of the basic important role and its basic types.

1. Introduction
In the engineering technology, the measurement of surveying and mapping, manufacturing, and other applications of science and technology, almost cannot leave the approximate calculation [1-3], because accurate only exists in theory, can't get accurate value [4-5] in a specific application. Approximate calculation we all want to get as close as possible to the result of the accurate value, in order to achieve this goal, we all need to be used in the calculation of the approximate function approximation precision value as much as possible, finally, the paper it can be seen, the most common basic approximation function is a rational function and polynomial function [8-9].

2. Approximation of a Value
It have been said that numerical analysis is concerned with the solution of mathematical problems by arithmetic processes. Clearly, then, the need to approximate non-arithmetic quantities by arithmetic quantities and to ascertain the errors associated with such approximations lies at the heart of much of numerical analysis. In a given situation there will usually be several possible methods of obtaining the desired approximation. Which of these to choose depends upon which of various possible criteria are used to judge how efficacious a give: approximation is. The following simple example will illustrate what these criteria are and how they affect the choice of an approximation.

Suppose that we are given a value of $x$ and we wish to calculate $\sqrt{x}$. The following are among the possibilities open to us:

(a). Use the classic method learned by most people in grammar school which begins by pairing off digits on either side of the decimal point.

(b). Look up $x$ in a table of square roots and if $x$ lies between two arguments in the table, interpolate to find $\sqrt{x}$.

(c). Use any one of a number of iterative techniques to compute. Our object here is not to decide which of these methods should be used but to discuss the considerations that must precede any such decision. We naturally assume that all the methods "work", i.e. that they all lead to a result which can reasonably be considered an approximation to $\sqrt{x}$. The basic question we must answer is: What error can we tolerate in the result?

This question not only recognizes the importance of the errors, both truncation and roundoff, incurred in an approximation but, more subtly, implies the importance of being able to estimate or
bound these errors. The latter is a consideration of first importance in choosing a method for the
solution of a problem. Only when we have methods where it is possible to estimate or bound the
error can we then try to compare these methods on the basis of the magnitudes of the errors to
which they lead. Each of the above methods for computing the square root has an error which can
be estimated or bounded and for which the truncation component, at least, can be made arbitrarily
small by carrying the computation far enough, e.g. by on the basis of error considerations, each is a
reasonable candidate for computing the square root.

We may have noted that the question above is ambiguous. What do we mean by error in the
result? Absolute error or relative error? Do we wish to bound the error for all \( x \) in some interval, or
shall we be satisfied with a small average error (where now "average" is ambiguous)? These queries
need to be answered in practice, but our purpose here has been to point out that the primary aim of
any approximation is to achieve some desired degree of accuracy and implicit in this aim is the
assumption that the accuracy can indeed be estimated.

In one important sense our approach to numerical analysis in this paper will be basically
pragmatic; i.e., except for techniques of special theoretical interest, we shall concentrate on methods
which are usable in practice. Thus, for a given method, we shall usually also wish to answer the
question: how fast can a solution be computed using a given method?

In the case of the first method above for the calculation of \( \sqrt{x} \), this question is easily answered
since given the number of decimal places desired in the square root this is a finite process with the
number and type of calculations strictly determined. Using the second method, interpolation, we
must first choose an interpolation formula which will achieve the desired accuracy. Having done
this, however, the amount of calculation is again strictly determined. (This is really a simplification
of the truth in the further research.) But using iterative processes; the situation is different. Our
assumption that the method would "work." is equivalent to assuming that the iteration converges.
But the amount of computation required to achieve the desired degree of accuracy depends on the
rate of convergence. Therefore, determining rates of convergence in iterative processes well always
be of importance to us.

Generalizing then from our example of \( \sqrt{x} \). We conclude that the primary aim of an
approximation is to achieve some desired degree of accuracy and to be such that the accuracy can
be estimated. Second, we are also interested in the amount of computation required to achieve the
approximation. With these heuristic notions behind us, we now proceed to consider the general
problem of approximation.

3. Classes of Approximate Functions

Much of the approximation done in numerical analysis consists of approximation a function \( f(x) \)
by some combination—most often a linear combination—of function drawn from some particular
class of function. The most familiar example of this is the approximation of \( f(x) \) by the first
terms of its Taylor-series expansion. Another familiar example occurs in the trapezoidal rule, in
which \( f(x) \) is approximation by a sequence of straight lines. In the former example and for each
straight lines in the trapezoidal rule the approximation is a linear combination of functions from the
class \( \{ p_n(x) \} \), where \( p_n(x) \) is a polynomial of degree \( n \). Another class which is suggested by the
importance of periodic function is the class of Fourier function \( \{ \sin nx, \cos nx \} \), \( n=0, 1, 2, \ldots \).
There are, of course, a number of other class of functions which would lead to useful
approximations in particular case. Especially worth mentioning are:

(I) Rational functions, which will play an important role in the research and which can be used,
whereas polynomials cannot, to approximate functions with poles.

(II) Piecewise polynomial functions, i.e., functions which are different polynomial on different
subintervals, which will be used Exponential functions.

(III) But for general application, polynomials and the Fourier functions are by far the most
important with the former predominating. Since this assertion about polynomial approximations is basic to our study of numerical analysis, in the next few pages we shall attempt to justify it.

4. Types of Approximations

Let $f(x)$ be a function which we wish to approximate using the class of functions $\{g_n(x)\}, n=0, 1, 2, \ldots$ Suppose we approximate $f(x)$ by the linear combination

$$f(x) \approx a_0g_0(x) + a_1g_1(x) + \cdots + a_mg_m(x)$$

(1)

where the $a_i, i=0, 1, \ldots; m$ are constants. We shall call (1) an approximations of linear type to $f(x)$. Because the analysis of approximations involving nonlinear combinations of the approximating functions, like most nonlinear analysis, is very difficult, we shall be concerned almost entirely with approximations of linear type.

5. Conclusions

We see from the above analysis, the approximate calculation must rely on approximating function, the most common basic approximation function is a polynomial function, and rational function, and their linear combination.

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Author introduction: Lijiang Zeng (1962-), male, born in Chishui of Guizhou Province, Professor of Zunyi Normal College, major research field: mathematics and applied mathematics, research direction: algebra and its application, number theory and its application, function theory and application. Have existed search results: CPCI-S(ISTP), CPCI-SSH(ISHHP), and EI 18 articles published, 38 English papers published, Email: ZLJ4383@sina.com .

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