Structure the Radiation Dose Curve of Continuous Slope

ShiWei Wen¹ and Zhang Min²*

Abstract

The traditional interpolation and fitting can not restore the radiation dose curve with the radiation source characteristic. Therefore, during the reduction the field of one dimensional radiation dose, the mathematical structure method was used to restore the curve with the dose value of the radioactive source is inversely proportional to the square of the distance from the radioactive source.

Key words: inversion, construct, one dimensional radiation dose, dose dominance of adjacent point.

INTRODUCTION

The rectangular area of containing the radiation source is divided into grid, and the corresponding grid node data is collected. We can structure dose function to fill the missing part of the data with the limited data. Consider horizontal rectangular area with point source pollution $\Omega = \{(x, y) | 0 \leq x \leq a, 0 \leq y \leq b\}$. In the X axis, range $[0, a]$, insert the $n-1$ point, $x_1, x_2, x_3, \cdots, x_{n-1}$ and let $x_0 = 0$, $x_n = a$. In the Y axis, range $[0, b]$, insert the $m-1$ point $y_1, y_2, y_3, \cdots, y_{m-1}$ and let $y_0 = 0$, $y_m = b$, so the search area is evenly divided (see Fig. 1).

$$E_{n+1} : 0 = x_0 < x_1 < \cdots < x_n = a,$$
$$E_{m+1} : 0 = y_0 < y_1 < \cdots < y_m = b,$$

$$x_{i+1} - x_i = a/n, \quad i = 0, 1, \cdots, n-1, \quad y_{j+1} - y_j = b/m, \quad j = 0, 1, \cdots, m-1.$$ 

Then $E_{n+1} \times E_{m+1} = \{(x_i, y_j) | i = 0,1,2,\cdots, n; j = 0,1,2,\cdots, m\}$ is the rectangular point set (see Fig. 1), the grid node value $(x_i, y_j)$ is $f_{i,j}, i = 0,1,\cdots, n, \quad j = 0,1,\cdots, m$ (see Table 1).

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¹School of Computer Science and Technology Software School, University of South China, Hengyang 421001, China
²School of Math and Physics, University of South China, Hengyang, 421001 China. *Corresponding Author
The idea of the inversion of the nuclear radiation dose field is that the dose of any point is obtained by using the cubic spline interpolation with the data of grid node, which is based on collected grid node data. For interior grid points, we can get the value of any point by structure. Therefore, the dose of any point in the region can be obtained.

The source of continuous slope change structure is the continuous concentration function curve with concave function characteristic in the range of grid data. Taking fully into account the characteristics of the radioactive source, the radioactive dose is inversely proportional to the square of the distance from the nuclear sources\[1\]. Based on the dose dominance of adjacent point, the radiation dose was restored and dose curve of one dimensional is obtained.

In order to construct the radiation dose curve has characteristics of concave function of the grid line, we use the method of function structure, cleverly constructed radiation dose curve continuous radiation dose curve to restore the grid line. The idea of slope continuous construction is to construct a continuous radiation dose function curve with concave function in the range of grid data. As shown Fig. 2.

When \( y = y_j, \ j = 1,2,\ldots,m \), The interval \([x_i, x_{i+1}]\) is divided into \( n \) interval, each interval length \( l = \frac{x_{i+1} - x_i}{n} \), radiation dose \( f_{i,j}, i = 1,2,\ldots,n \).

Let
\[
\begin{align*}
k_1 &= \frac{f(x_i) - f(x_{i+1})}{x_i - x_{i+1}}, \\
k_2 &= \frac{f(x_{i+2}) - f(x_{i+1})}{x_{i+2} - x_{i+1}}
\end{align*}
\]

\( k_1, \ k_2 \), respectively, the two endpointsslope of the \([x_i, x_{i+1}]\).

Because of the positive and negative of the two endpoints, we discuss the following,

(1) When \( k_1k_2 > 0 \), we suppose that the rate of change in each neighborhood is equal. Based on the above considerations, the change rate of radiation dose per cell segment is constructed.
\[
f(x_i + k\cdot l) = \frac{|k_1 - k_2|}{n} l + f[x_i + (k - 1)l], \quad k = 1,2,\ldots,n.
\]

According to the formula, the radiation dose value of each node can be transmitted.

When \( k_1k_2 < 0 \), It is shown that there exists a maximum or minimum value between \( x_i \) and \( x_{i+1} \).

### Table 1. Corresponding observation values of grid nodes.

<table>
<thead>
<tr>
<th>( y_0 )</th>
<th>( y_1 )</th>
<th>( y_2 )</th>
<th>\ldots</th>
<th>( y_m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_0 )</td>
<td>( f_{0,0} )</td>
<td>( f_{0,1} )</td>
<td>( f_{0,2} )</td>
<td>\ldots</td>
</tr>
<tr>
<td>( x_1 )</td>
<td>( f_{1,0} )</td>
<td>( f_{1,1} )</td>
<td>( f_{1,2} )</td>
<td>\ldots</td>
</tr>
<tr>
<td>( x_2 )</td>
<td>( f_{2,0} )</td>
<td>( f_{2,1} )</td>
<td>( f_{2,2} )</td>
<td>\ldots</td>
</tr>
<tr>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
<td>\ldots</td>
</tr>
<tr>
<td>( x_n )</td>
<td>( f_{n,0} )</td>
<td>( f_{n,1} )</td>
<td>( f_{n,2} )</td>
<td>\ldots</td>
</tr>
</tbody>
</table>
Increase the number of equal intervals, in order to ensure that the slope of the left and right ends of each cell is positive or negative, and then use the first case processing method to restore the radiation dose of any point in the cell. As shown in Figure 2.7 and 2.8.

(1) When $k_1 > 0, k_2 < 0$, As shown in Fig.2.7, the interval $[x_i, x_{i+1}]$ is divided into $n$ intervals. Each interval is $l = \frac{x_{i+1} - x_i}{n}$. If the radiation dose of $x_i + (k-1)l, x_i + k \cdot l, x_i + (k+1)l$ points are respectively $f_1, f_2, f_3$ then, $\forall x \in [x_i + (k-1)l, x_i + k \cdot l]$

$$f(x) = f_0 - \frac{f_1}{l} x + \frac{f_0 - f_1}{l} (x + k \cdot l).$$

(4)

(5)

(2) When $k_1 < 0, k_2 > 0$, As shown in Fig.2.8, the interval $[x_i, x_{i+1}]$ is divided into $n$ intervals. Each interval is $l = \frac{x_{i+1} - x_i}{n}$. In this case, there is a minimum point in the interval, the slope at the minimum value is 0. Therefore, the first method can be used to reduce the radiation dose at any point.

Algorithm: Structure the radiation curve of continuous slope

**Step1**  
Input $[x_i, x_{i+1}]$ and $n$, $[x_i, x_{i+1}]$ is divided into $n$ intervals. Calculate the results $l = \frac{|x_{i+1} - x_i|}{n}$.

**Step2** In the intervals $[x_i, x_{i+1}]$, Calculate the rate of change of $x_i, x_{i+1}$

$k_1 = \frac{f(x_i) - f(x_{i+1})}{l}, k_2 = \frac{f(x_{i+2}) - f(x_{i+1})}{l}$.

**Step3**  
Restore the radiation dose of any point in the interval $[x_i, x_{i+1}]$

**Step3.1** if $k_1k_2 > 0$, calculate $f(x_i + k \cdot l) = \frac{|k_1 - k_2|}{n} l + f(x_i + (k-1)l), k = 1, 2, \ldots, n - 1$.

**Step3.2** if $k_1k_2 < 0$ and $k_1 < 0, k_2 > 0$, then $\forall x \in [x_i, x_{i+1}]$,

Output $f(x_i + k \cdot l) = \frac{|k_1 - k_2|}{n} l + f(x_i + (k-1)l), k = 1, 2, \ldots, n - 1$.

**Step3.3** if $k_1k_2 < 0$ and $k_1 > 0, k_2 < 0$, then, $\forall x \in [x_i + (k-1)l, x_i + k \cdot l]$.
\[
    f(x) = \frac{f_0 - f_1}{l} x + \left[ f_0 - \frac{(f_0 - f_1)(x_i + k \cdot l)}{l} \right].
\]

**Step 3.4** if \(k_1, k_2 < 0\) and \(k_1 > 0, k_2 < 0\), then \(\forall x \in [x_i + k \cdot l, x_i + (k+1)l]\),

\[
    f(x) = \frac{f_0 - f_1}{l} x + \left[ f_0 - \frac{(f_0 - f_1)(x_i + k \cdot l)}{l} \right].
\]

**Step 4** Output \(f(x_i + k l), \ k = 1, 2, \cdots, n - 1\).

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**CONCLUSION**

This paper presents the traditional interpolation and fitting cannot restore the radiation dose curve with the radiation source characteristic. Therefore, during the reduction the field of one dimensional radiation dose, the mathematical structure method was used to restore the curve with the dose value of the radioactive source is inversely proportional to the square of the distance from the radioactive source.

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**REFERENCE**


