A New Approach to Evaluating and Selecting Public-Private Partnership (PPP) Projects

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Abstract. Due to the popularity of Public-Private Partnership (PPP) projects and the observation that the evaluation and selection of projects can be regarded as a multiple criteria decision making problem, this paper proposes a new approach, namely, minimizing the total deviation from the ideal point, to decide on the weights associated with each criterion. We provide a numerical illustration to demonstrate the effectiveness of this approach.

Introduction

Public-Private Partnerships (PPPs) refer to innovative methods used by the public sector to contract with the private sector, which brings its capital and ability to deliver projects on time to the budget, while the public sector retains the responsibility to provide these services to the public in a way that benefits the public and delivers economic development and an improvement in the quality of life [1]. The worldwide popularity of PPPs projects is justified from the fact that PPPs can effectively avoid the often negative effects of either exclusive public ownership and delivery services, or outright privatization, on the other. Furthermore, PPPs combine the best of both entities: the public sector with its regulatory actions and protection of the public interest; and the private sector with its resources, management skills and technology. The main goals of PPPs include financing, designing, implementing and operating public sector facilities and services. The key characteristics associated with PPPs encompass [2,3,6]:

1) Long-term (typically up to thirty years) service provisions;
2) The transfer of risk to the private sectors;
3) Different forms of long-term contracts drawn up between legal entities and public authorities.

Although the types of PPPs vary due to the different needs of governments for infrastructure services, two broad categories of PPPs have been identified as: the institutionalized kind that refers to all forms of joint ventures between public and private stakeholders; and contractual PPPs. More specifically, PPPs can take many forms and may incorporate part or all of the following features [5]:

1) The public sector transfers facilities controlled by itself to the private sector (with or without payment in return) usually for the term of the arrangement;
2) The private sector builds, extends or renovates a facility;
3) The public sector specifies the operating features of the facility;
4) Services are provided by the private sector using the facility for a defined period of time (usually with restrictions on operations and pricing);
5) The private sector agrees to transfer the facility to the public sector (with or without payment) at the end of the arrangement.
In the presence of hundreds of PPP projects’ application, the fundamental decision of managers is to evaluate them and then select appropriate projects to implement. Motivated by the observations that the evaluating of PPP projects consists of multiple criteria, this paper proposes a decision approach that minimizes the total deviation from the ideal point to determine the weights associated with each criterion.

The rest of this paper proceeds as below. Section 2 proposes two approaches to making decisions of evaluating and selecting PPP projects. Section 3 provides a numerical illustration. Section 4 concludes this study.

Proposed Approach

The multiple criteria PPP projects evaluation and selection problem is formulated as follows:

\[ A = \{ A_1, A_2, \ldots, A_m \} : \text{a set of } m \text{ projects}; \]

\[ C = \{ C_1, C_2, \ldots, C_n \} : \text{a set of } n \text{ criteria}; \]

\[ Y_{mn} = \{ y_{ij} \} : \text{the decision matrix where } y_{ij} \text{ is the input data for project } i \text{ with respect to criterion } j, \ i=1,2,\ldots,m; \ j=1,2,\ldots,n; \]

The decision matrix \( Y_{mn} \) is normalized to the matrix \( X_{mn} \) using the following formula:

\[ x_{ij} = \frac{y_{ij} - y_{ij}^{\min}}{y_{ij}^{\max} - y_{ij}^{\min}}, \text{ for benefit criteria;} \]

\[ x_{ij} = \frac{y_{ij}^{\max} - y_{ij}^{\min}}{y_{ij}^{\max} - y_{ij}^{\min}}, \text{ for cost criteria;} \]

where \( y_{ij}^{\max} = \max \{ y_{1j}, y_{2j}, \ldots, y_{mj} \} \), and \( y_{ij}^{\min} = \min \{ y_{1j}, y_{2j}, \ldots, y_{mj} \} \).

\[ W = (w_1, w_2, \ldots, w_n) : \text{a set of criteria weights, and } \sum_{j=1}^{n} w_j = 1. \]

Minimizing the total deviation from the ideal point is intuitively appealing, due to the principle behind this approach is seeking to make all derived scores as close to the ideal point as possible. In line with [4], the matrix \( X_{mn} \) is transformed to a weighted decision matrix \( \Psi_{mn} = [z_{ij}] \), where

\[ z_{ij} = x_{ij} w_j, \ i=1,2,\ldots,m; \ j=1,2,\ldots,n. \]

The ideal point is defined as \( \Omega^* = \{ z_{1j}^*, z_{2j}^*, \ldots, z_{nj}^* \} \), where

\[ z_{j}^* = \max \{ z_{1j}, z_{2j}, \ldots, z_{mj} \} \]

\[ = \max \{ x_{1j} w_j, x_{2j} w_j, \ldots, x_{mj} w_j \} \]

\[ = \max \{ x_{1j}, x_{2j}, \ldots, x_{mj} \} w_j \]

\[ = x_j^* w_j, \]

and \( x_j^* \) is the ideal value under criterion \( j \). It is straightforward because the maximal scores obtained from certain criterion are reasonably regarded as the target to reach. The discrepancy between the performance under certain criterion and the ideal point can be measured by the following function:
\[ D_i = \sum_{j=1}^{n} (z_{ij} - z_j^*)^2 = \sum_{j=1}^{n} (x_{ij} - x_j^*)^2 w_j^2 \] (3)

A multi-objective programming model is presented below to optimize the performance of all projects:

\[
\begin{align*}
    f_1 &= \min \sum_{j=1}^{n} (x_{1j} - x_j^*)^2 w_j^2 \\
    f_2 &= \min \sum_{j=1}^{n} (x_{2j} - x_j^*)^2 w_j^2 \\
    & \quad \vdots \\
    f_m &= \min \sum_{j=1}^{n} (x_{mj} - x_j^*)^2 w_j^2 \\
    \text{s.t. } & \sum_{j=1}^{n} w_j = 1,
\end{align*}
\] (4)

which is converted into a single-objective optimization model:

\[
\begin{align*}
    \min F &= \sum_{i=1}^{m} \sum_{j=1}^{n} (x_{ij} - x_j^*)^2 w_j^2 \\
    \text{s.t. } & \sum_{j=1}^{n} w_j = 1.
\end{align*}
\] (5)

In order to solve the quadratic programming (5), we construct a Lagrange function using a Lagrange multiplier \( \lambda \):

\[ L = \sum_{i=1}^{m} \sum_{j=1}^{n} (x_{ij} - x_j^*)^2 w_j^2 + \lambda \left( \sum_{j=1}^{n} w_j - 1 \right). \] (6)

The Hessian matrix of (6) with respect to \( w_j \) is a \( n \times n \) diagonal matrix and its diagonal elements are \( 2 \sum_{i=1}^{m} (x_{ij} - x_j^*)^2 \geq 0 \). Therefore, the Lagrange function has a minimum value, which is derived by differentiating (6) with respect to \( w_j \) and \( \lambda \), respectively:

\[
\begin{align*}
    \lambda^* &= \frac{1}{2 \sum_{j=1}^{n} \left[ \sum_{i=1}^{m} (x_{ij} - x_j^*)^2 \right]^{-1}}, \\
    w_j^* &= \frac{1}{\sum_{j=1}^{n} \left[ \sum_{i=1}^{m} (x_{ij} - x_j^*)^2 \right]^{-1} \sum_{i=1}^{m} (x_{ij} - x_j^*)^2}.
\end{align*}
\] (7)

Since the constraint of (5) is a non-empty convex set, and the objective function is convex, the optimal solutions (7) to (5) are the global optimal solutions.
Numerical Illustration

For the purpose of illustrating the effectiveness of the proposed approach, we consider the situation that PPP projects are involved with multiple criteria, namely, Indirect economic contribution ($y_{i1}$), Direct economic contribution ($y_{i2}$), Technical contribution ($y_{i3}$), Social contribution ($y_{i4}$) and Scientific contribution ($y_{i5}$). A summary of the input data is presented in the following Table 1.

<table>
<thead>
<tr>
<th>Projects</th>
<th>Criteria</th>
<th>Scores</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$y_{i1}$</td>
<td>$y_{i2}$</td>
<td>$y_{i3}$</td>
</tr>
<tr>
<td>P1</td>
<td>67.53</td>
<td>70.82</td>
<td>62.64</td>
</tr>
<tr>
<td>P2</td>
<td>58.94</td>
<td>62.86</td>
<td>57.47</td>
</tr>
<tr>
<td>P3</td>
<td>22.27</td>
<td>9.68</td>
<td>6.73</td>
</tr>
<tr>
<td>P4</td>
<td>47.32</td>
<td>47.05</td>
<td>21.75</td>
</tr>
<tr>
<td>P5</td>
<td>48.96</td>
<td>48.48</td>
<td>34.9</td>
</tr>
<tr>
<td>P6</td>
<td>58.88</td>
<td>77.16</td>
<td>35.42</td>
</tr>
<tr>
<td>P7</td>
<td>50.1</td>
<td>58.2</td>
<td>36.12</td>
</tr>
<tr>
<td>P8</td>
<td>47.46</td>
<td>49.54</td>
<td>46.89</td>
</tr>
<tr>
<td>P9</td>
<td>55.26</td>
<td>61.09</td>
<td>38.93</td>
</tr>
<tr>
<td>P10</td>
<td>52.4</td>
<td>55.09</td>
<td>53.45</td>
</tr>
<tr>
<td>P11</td>
<td>55.13</td>
<td>55.54</td>
<td>55.13</td>
</tr>
<tr>
<td>P12</td>
<td>32.09</td>
<td>34.04</td>
<td>33.57</td>
</tr>
<tr>
<td>P13</td>
<td>27.49</td>
<td>39</td>
<td>34.51</td>
</tr>
<tr>
<td>P14</td>
<td>77.17</td>
<td>83.35</td>
<td>60.01</td>
</tr>
<tr>
<td>P15</td>
<td>72</td>
<td>68.32</td>
<td>25.84</td>
</tr>
</tbody>
</table>

Using the formula (7), the weights associated with different criteria are:

$$W = (0.1786, 0.2487, 0.2222, 0.1521, 0.1985).$$

(8)

Therefore, the scores and rankings with respect to each project are reported in the seventh and eighth columns of the Table 1 and Figure 1 below.
It is observed that our approach, namely, minimizing the total deviation from the ideal point, considers the direct economic contribution as the most important criterion, and the social contribution as the least important one. Furthermore, our approach provides a complete ranking of all projects, which reflects the discriminatory power of the proposed approach.

Conclusions

PPP refers to innovative methods used by the public sector to contract with the private sector, which brings its capital and ability to deliver projects on time to the budget, while the public sector retains the responsibility to provide these services to the public in a way that benefits the public and delivers economic development and an improvement in the quality of life. This study develops a new approach that minimizes the total deviation from the ideal point, to determine the weights with respect to each criterion. A numerical example is presented to show the effectiveness of this approach, in terms of providing a complete ranking of all projects.

References