The Displacement Solution of Existing Tunnel and Ground by Traversing Construction by Stochastic Medium Theory

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Abstract: Tunnel excavation will cause the displacement of ground, which will lead to the displacement of the existing tunnel. Because of the great difference between the tunnel’s rigidity and the ground’s rigidity, the conventional analytical method cannot calculate the existence of different stiffness of the stratum displacement field. The stiffness of the existing tunnel is equivalent by stochastic medium theory. A computing model which includes the new tunnel and rock mass and the existing tunnel is build. Taking Beijing metro line 10 crossing metro line 1 as an example, with this method, the influence of the existing and ground is studied. The results show that the calculated results are in good agreement with the experimental results, and the analytical solution can well explain the barrier and diffusion of the existing tunnel. The research results provide a new analytical method for the calculation of the existing tunnel and stratum settlement caused by the construction of the tunnel.

Keywords: Crossing engineering; existing tunnel; stochastic medium theory; analytical solution

INTRODUCTION

In the construction of Beijing Rail Transit, with the increase of the density of the network, the new lines are more and more intersecting with the existing lines. Due to the different construction period, it’s inevitable to disturb the existing tunnel when new tunnel excavated closely.¹,²,³

Take Chongwenmen station of line 5 as an example, in which pioneer heading-column method was applied and perpendicularly under cross the interval of line 2. The existing tunnel settlement reached 31.26mm after the construction of middle hole which had a greater impact on the normal operation of the train.⁴ Therefore, correct analysis and reliable prediction of the deformation of existing tunnels caused by the construction of new tunnels has become a hot topic in the study of urban rail transit construction.⁵,⁶

Shallow tunnel excavation under the existing tunnel will cause the formation displacement, which will lead to deformation of the existing tunnel. At present, tunneling and deformation prediction are mostly calculated by finite element method, and the conventional analytical method could not be applied.⁷ The reason is that there is a huge difference between the stiffness of the existing tunnel and the strata and the analytic method could not calculate the displacement field of the strata with different stiffness.
This paper introduces the equivalent layer method in order to solve the problem of the mismatch between the existing tunnel and strata stiffness. When the elastic modulus of the upper and lower layer materials and Poisson's ratio are different, the thickness of the upper layer material can be equivalent by the stiffness after that the material become a homogeneous body and bilayer material is transformed into a homogeneous material. Combined with the analytical solution of the tunnel, the displacement analytical solution of the shallow tunnel excavation can be solved.

Based on stochastic medium theory, the tunnel is considered as a kind of hard rock material stratum, and the solution of displacement induced by the excavation of shallow tunnel in the three-layer material medium is deduced by the combination of the equivalent layer method and the mirror method principle.

FORMULA DERIVATION OF THE EQUIVALENT LAYER METHOD

**Principle of the equivalent layer method**

In geotechnical engineering, the formation is often composed of multiple layers of materials with different physical and mechanical properties. Due to the existence of existing tunnels, the heterogeneity of formation stiffness is more obvious in crossing engineering.

The basic idea of the equivalent layer method is: When the elastic modulus and Poisson's ratio of the upper formation are different from the elastic parameters of the lower formation, the thickness of the upper formation can be replaced by the equivalent thickness corresponding to the elastic modulus of the lower formation and the thickness of the upper formation. This method was proposed by Paklovsky and Babcov. In the domestic study, the equivalent layer method was applied by Ai [8] and Ouyang [9] to investigate the differential deformation of stratum and ground structures induced by tunnel construction. The formula of the equivalent layer method is denoted as:

\[
h = h_1 \left( \frac{E_1 \rho_2}{E_2 \rho_1} \right)^a \quad (1)
\]

Where \( h_1 \) is the thickness of the material 1 and \( h \) is the thickness of the equivalent material 2, \( E_1 \) and \( E_2 \) are the elastic modulus of the material 1 and the material 2 respectively; \( \rho_1 \) and \( \rho_2 \) are the densities of the materials 1 and 2; \( a \) is the index to take 0.5.

The under crossing construction of new tunnel would cause formation deformation. The process of upward propagation of formation deformation would have an impact on existing tunnels. Assuming that the stratum does not separate from the existing tunnel and coordinate deformation, the existing tunnel can be considered as an overlying hard rock and soil material of which the stiffness could be considered. The calculation diagram is shown below.
Formula Derivation of the bilayer material

The diagram of the equivalent layer method is shown in Fig 1, specific derivation process is as follows.

In Fig 1, layer 1 is the material with relatively larger stiffness \( E_1, \mu_1 \) while layer 2 is the material with relatively smaller stiffness \( E_2, \mu_2 \).

It is assumed that a concentrated force acting periodically on the surface of the upper layer causes vibrations within the system and that the vibrating waves propagate in layers 1 and 2 at \( v_1 \) and \( v_2 \).

The vibration wave propagates in the first layer at a small angle of \( \alpha_1 \), and refraction occurs at the interface of the material, and propagates at an angle of \( \alpha_2 \) in the second layer. The size of \( \alpha_1 \) and \( \alpha_2 \) is related to the physical and mechanical parameters of soil layer.

\[
\alpha_2 = \alpha_1 \times \left( \frac{v_2}{v_1} \right) \quad (2)
\]

Assuming that the vibration wave is at a certain depth \( h_2 \) of layer 2, then according to Fig. 1:

\[
B = h_1 \tan \alpha_1 + h_2 \tan \alpha_2 = h_1 \alpha_1 + h_2 \alpha_2 \quad (3)
\]

When considering the vibration wave in a uniform layer of material in the spread, the vertical distance is \( h+h_2 \), and the width is \( B \). Here, \( h \) is the thickness of the layer corresponding to one layer having the thickness \( h_1 \) in terms of the propagation of the vibration in the same material as the two layers. \( B \) is denoted as:

\[
B = h \tan \alpha_2 + h_2 \tan \alpha_2 = (h+h_2) \tan \alpha_2 \approx (h+h_2) \alpha_2 \quad (4)
\]

According to equations (2), (3),

\[
h = h_1 \alpha_1 / \alpha_2 = h_1 v_1 / v_2
\]

According to Newton's formula, the vibration propagation velocity is:

\[
v = A \frac{E}{\rho} \quad (5)
\]

Where \( A \) is the coefficient related to the vibration direction and the lateral expansion coefficient of the material.

From the above:
If the density difference between the upper and lower layers is not large, then the type can be simplified as:

\[ h = h_1 \sqrt{\frac{E_1 \rho_1}{E_2 \rho_2}} \quad (6) \]

Formula Derivation of three-layer material

The rock mass exists above and below the existing tunnel in crossing engineering, so the calculation model is a three-layer material problem. The three-layer soil is deduced according to the calculation model of two-layer soil. Among them, layer 1 is the material with relatively smaller stiffness \( (E_1, \mu_1) \) while layer 2 is the material with relatively larger stiffness \( (E_2, \mu_2) \) and layer 3 is the same as layer 1. The propagation path of vibration under concentrated force is shown in Fig 1.

The vibration wave propagates in the layer 1 at a small angle of \( \alpha_2 \), refraction occurs at the first interface, propagates at an angle \( \alpha_1 \) in the layer 2, refraction occurs at the second interface, and then is reflected by the angle \( \alpha_2 \) at layers 3 to spread

\[ \alpha_2 = \alpha_1 \times \left( \frac{v_2}{v_1} \right) \quad (8) \]

It is assumed that the vibration wave is at a certain depth \( h_3 \) of layer 3, \( B \) is denoted as:

\[ B = h_1 \tan \alpha_2 + h_2 \tan \alpha_1 + h_3 \tan \alpha_2 = h_1 \alpha_2 + h_2 \alpha_1 + h_3 \alpha_2 \quad (9) \]

When considering the vibration in layer 1 with uniform material in the spread, the vertical distance of layers 2 is \( h \),

\[ B = h_1 \tan \alpha_2 + h \tan \alpha_2 + h_3 \tan \alpha_2 \approx (h_1 + h + h_3) \alpha_2 \quad (10) \]

According to equation (8), (9) and the foregoing analysis,

\[ h = h_2 \alpha_1 / \alpha_2 = h_2 v_1 / v_2 \]

From the above:

\[ h = h_1 \sqrt{\frac{E_2 \rho_1}{E_1 \rho_2}} \quad (11) \]

If the density difference between the upper and lower layers is not large, then the type can be simplified as:

\[ h = h_1 \sqrt{\frac{E_2}{E_1}} \quad (12) \]

Displacement analytical solution of shallow tunnel excavation by stochastic medium theory

According to the above analysis, the existing tunnels and surrounding rocks are considered as three-layer equivalent materials with different mechanical properties, and the newly built shallow tunnels are excavated under the existing tunnels. Figure 3 shows the relative position of the new tunnel-forming-existing tunnel. The radius of the new tunnel is \( R \), and the depth of the axis is \( h_1 + h_2 + h_3 \). Elastic modulus of the soil to take \( E_1 \), Poisson ratio \( \mu_1 \). Considering the existing tunnels as box-type structure, according to bending stiffness equivalent to equal size solid material, elastic modulus take \( E_2 \), Poisson ratio \( \mu_2 \). The Poisson's ratio is assumed to be \( \mu_1 = \mu_2 \).
The actual position coordinates are \((x', z')\), where \(x'\) is the horizontal distance from the new tunnel axis, and \(z'\) is the vertical distance from the ground surface. According to the layer method conversion, the coordinates become \((x, z)\).

\[
x = x'
\]

\[
z = \begin{cases}
  z', z' \leq h_1 \\
  h_1 + (z' - h_1) \frac{E_2}{E_1}, h_1 < z' \leq (h_1 + h_2) \\
  h_1 + h_2 + z' - (h_1 + h_2), z' > (h_1 + h_2)
\end{cases}
\quad (13)
\]

The upper and lower strata and the existing tunnel into a homogeneous stratum after the layer transformation. The analytical solution of homogeneous stratum displacement caused by excavation of new tunnel is obtained by stochastic medium theory.

For a single-hole tunnel in Random medium theory, the integral formula of surface settlement value of tunnel excavation is\[^{[10]}\] :

\[
W(X) = W_o(X) - W_v(X) = \int_{-\pi}^{\pi} \int_{-\eta}^{\eta} \tan \beta \exp \left[ -\frac{\pi \tan^2 \beta}{\eta} (X - \xi)^2 \right] d\xi d\eta
\quad (14)
\]

(1) Surface Deformation Caused by Circular Section Tunnel Construction

\[
W(X) = \int_{-\pi}^{\pi} \int_{-\eta}^{\eta} \omega(X, \xi, \eta) d\xi d\eta - \int_{-\pi}^{\pi} \int_{-\eta}^{\eta} \omega(X, \xi, \eta) d\xi d\eta
\quad (15)
\]

The integral limit is:
\[
\begin{align*}
  a &= H - A \\
  b &= H + A \\
  c &= -\sqrt{A^2 - (H - \eta)^2} \\
  d &= -c \\
  e &= H - (A - \Delta A) \\
  f &= H + (A - \Delta A) \\
  g &= -\sqrt{(A - \Delta A)^2 - (H - \eta)^2} \\
  h &= -g
\end{align*}
\]

Where the value of \( \tan \beta \) depends on the strata condition of the excavation, \( \Delta A \) is the shrinkage area of the section, which is related to the shrinkage of the excavation section.

After equivalent, the depth of new axis of the tunnel is denoted as:

\[
h' = H + h_z \frac{E_z}{E_i} - h_z \quad (16)
\]

CASES VERIFICATION

Calculate Assumptions

There are four hypotheses when stratum and existing tunnel displacements are analyzed by the equivalent layer method and stochastic medium theory: first, the stratum, both tunnels considered as isotropic elastic material; second, do not consider the contact surface problem, assuming that the stratum and the existing tunnel in the interface coordination; third, the deformation of existing tunnels accords with the law of formation deformation, the smaller the buried depth is, the smaller the settlement value is and the larger the influence range. Fourth, the existing cross-section of the tunnel is not deformed, only analyze longitudinal deformation. Assuming makes calculation results will be slightly inconsistent with the actual situation and in in the real world: first, the interface between the soil and existing tunnels may be dislocation and separation; Second, both the tunnel floor and roof settlement is basically the same.

The statistics of crossing engineering in Beijing area are carried out by author. The results show that the deformation of the under construction is well controlled, and the deformation of the existing tunnel is small. It can be considered that the existing tunnels are in coordination with the subsoil, and there is no dislocation and separation.

At present, the monitoring of displacement of existing tunnels is focused on the existing tunnels and tunnels. A number of crossing projects are calculated and compared with the measured data using the equivalent layer method and stochastic medium theory. According to the engineering experience in Beijing, when calculate \( U_z \) and \( U_x \), the value of \( z \) is to take the depth of the tunnel at the top of the floor.

Cases verification

Take the interval of Guomao-Shuangjing in Beijing subway line 10 under beneath the interval of line1, deformation response of construction stratum and existing tunnel is investigated. The new tunnel area of 52.7m², the radius of
approximately 4.1m, the axial depth of 22.25m. The depth of the tunnel roof is 12.3m, and the depth of the floor is 18m. The elastic modulus of the existing tunnel girder is 17.5GPa, the equivalent density is 1074kg/m$^3$, and the Poisson ratio is 0.3. Existing tunnel in which the main formation to the sand and gravel stratum, soil bulk density take 1800kg/m$^3$, modulus of elasticity take 150MPa, Poisson's ratio 0.3.

New tunnel construction convergence modes include radial contraction, elliptical deformation and vertical settlement. In practical engineering, the deformation of the tunnel could not be fully reflected by the settlement of the tunnel vault and the measured value of the convergence of the side wall because of the hysteresis of the support. Peck formula$^{[12,13]}$ is used to get the formation loss rate according to the strata deformation in order to calculate the amount of convergence deformation and thereby determining the value of the radial shrinkage. The loss rate of formation is 0.333% and $\Delta A$ is 0.09 in interval of Guo-Shuang according to the author's previous research results.$^{[11]}

(1) Vertical displacement distribution

According to equation (14) and (15), the displacement and distribution law of the strata and the existing tunnel are obtained. The vertical direction is positive, as shown in Fig. 3 to Fig. 6.

![Figure 3. The Floor deformation of existing tunnel.](image)

![Figure 4. The settlement curves of uneven ground layers.](image)
From the above deformation curve can be seen:

1) The maximum displacement of the existing tunnel is 4.77mm, the measured value is 4.7mm, and the calculated value agrees well with the measured value.

2) The maximum subsidence occurred at the top of the new tunnel centerline, 1.58mm in analytical solution. The maximum settlement of the existing tunnel floor is 4.77mm occurred above the centerline of the new tunnel. The maximum settlement of new tunnel vault is 7.63mm. With the increase of the buried depth, the maximum settlement gradually increases, and the influence range decreases gradually.

3) Under the same conditions, the maximum settlement of the ground surface is 3.52mm in the natural formation without existing tunnels, the maximum settlement is 7.78mm at the buried depth of the tunnel floor, the maximum settlement of the new tunnel is 8.68mm The maximum settlement value is increased and the influence range is reduced compared with the formation where tunnels exist.

4) The maximum subsidence values at the depths of the natural and heterogeneous strata were compared. In the natural strata, with the increase of burial depth, the maximum settlement gradually increased, the change was smooth and no obvious mutation. In the strata with existing tunnels, the maximum settlement increases with the increase of the buried depth, but the slope is different at different stratum boundaries. In the layer 1, with the change of depth, the maximum settlement change is small, in the layer 2, with the change of depth, the maximum settlement increased sharply. At the same depth, the settlement of the homogeneous stratum is larger than that of the inhomogeneous stratum, which indicates that the existence of the existing tunnel (big rigidity) hinders the upward development of the formation deformation.
Conclusion

The displacement analytical solution of tunnel and stratum in crossing engineering is deduced based on the equivalent layer method and stochastic medium theory, which also verified in the engineering example. Researches have shown that:

1) The displacement solution of shallow tunnel with shallow tunnel was deduced based on stochastic medium theory and the equivalent layer method, which provides a new method for calculating the displacement of construction stratum and existing tunnels.

2) The calculation method was validated in the Guomao-Shuangjing interval of Beijing Metro Line 10, and the calculated results were in good agreement with the measured deformation, which verified the effectiveness of the method.

3) The presence of existing tunnels has barrier and diffusion effects on formation deformation compared to natural formation deformation. The interaction relationship between the new tunnel, rock and soil and existing tunnel is expounded: new tunnel construction is the root cause of deformation, the formation is the deformation of the media. Existing tunnel (large stiffness) on the one hand occurred deformation and additional stress because of the formation deformation, on the other hand had an obstruction and diffusion on formation.

References


Yao Haibo. Research on construction technology of large section tunnel crossing under the existing subway tunnel by means of shallow tunneling method[ D]. Beijing: Beijing Jiaotong University, 2005.(in Chinese)

Bai Haiwei. Study on longitudinal deformation of existing metro tunnel caused by undercrossing construction and engineering measures [M.S. Thesis][D]. Beijing: Beijing Jiaotong University, 2008.(in Chinese)


