Study on the Symmetry of Vortex Structure Distribution in a Cavity

Shi-hua HE*, Chun-ying SHEN and Hong-xuan YANG

Faculty of Electric Power Engineering, Kunming University of Science and Technology,
Kunming, Yunnan, 650500, China
*Corresponding author

Keywords: Cavity flow, Vortex structure, Symmetry, Numerical simulation.

Abstract. The symmetric distribution characteristics of vortex structure of two-dimensional steady flow in a cavity are obtained by theoretical analysis and numerical verification. Based on establishing the non-dimensional governing equations and its boundary conditions used to describe the flow, the symmetric distribution law of vortex structure of cavity driven flow with different Reynolds numbers, depth-to-width ratios and speed ratios is explained theoretically. The computational fluid dynamics method (CFD) is applied to simulate cavity driven flow and to verify the theoretical analysis. For two equal speeds and opposite driving lids, the vortex structure is symmetric about the center of the rectangular cavity. For two lids driving with equal speeds and in the same direction, the vortex structure is symmetric about the horizontal centerline of the rectangular cavity. When Reynolds number approaches zero, the vortex structure is symmetric about the vertical centerline of the rectangular cavity.

Introduction

Cavity type flows exhibit complex vortex structure in simple geometries, and are often encountered in industrial applications and engineering fields. Meanwhile, cavity flow is also widely used in validation and comparison of the effectiveness of different numerical methods for solving incompressible Navier-Stokes fluid flows.

The emerging and evolution of the vortex structures in a cavity are mainly controlled by Reynolds number, aspect ratio and driving speed ratio. The Stokes flows of Reynolds number $Re \rightarrow 0$ in a cavity were solved by Joseph [1], Shankar [2] and Gaskell [3] et al with analytical methods. The actual cavity flow for $Re \neq 0$ is governed by Navier-Stokes equations, and can be simulated by means of CFD numerical methods. For the cavity flow with a single moving lid, Burggraf [4] calculated the two-dimensional steady state flow using finite difference method, and showed a significant feature of vortex structure which consists of a primary large vortex and two secondary vortices in a square cavity. Later, Benjamin [5], Schreiber [6] and Ghia [7] et al increased Reynolds number to simulate, further revealed the tertiary vortex near the left corner of the cavity at high Reynolds number. With the rapid progress of CFD, the effectiveness of different modern numerical technology, such as the spectral element method [8], meshless method [9] and adaptive grid method [10] et al has been confirmed in cavity driven flow. Especially, Cheng [11] numerated the flow configuration in a single-lid-driven rectangular cavity at different aspect ratios and Reynolds numbers using lattice Boltzmann method. He [12] investigated the effects of Reynolds number on vortex structure evolution in a double-lid-driven square cavity.

Because of the different combinations and values of the Reynolds numbers, the depth-width ratios and the driving speed ratios, the cavity flow has proved to be complex vortex structure distribution and flow behavior. In the present work, the common symmetrical characteristics of vortex structure distribution in cavity flow are revealed by both the theoretical analysis and numerical simulation results.
**Theoretical Analysis**

We consider 2-D, incompressible flows in a rectangular cavity of width $D$ and height $H$ with stationary side walls. The flow is induced by the tangential movement of two lids with speeds $U$ and $U_b$. If the length, velocity and pressure are scaled with $D$, $U$ and $DU/\mu$, then the steady dimensionless equations can be written as

\begin{align}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \tag{1} \\
Re(\frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y}) &= -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \tag{2} \\
Re(\frac{\partial v}{\partial x} + u \frac{\partial v}{\partial y}) &= -\frac{\partial p}{\partial y} + \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \tag{3}
\end{align}

where $u, v$ are the velocity components in $x$- and $y$-directions respectively, $p$ is pressure, $Re$ is the Reynolds number defined as $Re = \frac{\rho UD}{\mu}$, $\mu$ is dynamic viscosity and $\rho$ is density. The depth-to-width ratio and driving speed ratio are denoted respectively by $A = H/D$ and $S = U/U_b$. The origin of the coordinate system coincides with the center of the rectangular cavity. Boundary conditions are specified as Figure 1.

**Figure 1. Coordinates and boundary conditions of cavity flow.**

Firstly, let's investigate the symmetry for $S = -1$, that is the vortex structure distribution in a rectangular cavity with two opposite and equal speed moving lids.

According to the general rule of fluid mechanics, pressure pointing to the pressured surface is positive. If two sets of variables $(x, y, u, v, p)$ and $(-x, -y, -u, -v, p)$ are respectively substituted into Eq. (1), (2) and (3), we can find that the Eqs. (1), (2) and (3) are invariant. Thus, if we have a solution of Eqs. for variables $(x, y, u, v, p)$, then we can obtain another solution of Eqs. for variables $(-x, -y, -u, -v, p)$. If the solution is unique, then two sets of solutions must be the same. In addition, for $S = -1$, the applied boundary conditions are antisymmetric with respect to the center of the rectangular cavity, so that $u(x, y) = -u(-x, -y)$, $v(x, y) = -v(-x, -y)$ and $u(0, 0) = 0$, $v(0, 0) = 0$. Therefore, for $S = -1$, we have the conclusions that the center of the rectangular cavity is always a stagnation point, and the vortex structure distribution is symmetrical about the center of the rectangular cavity for all Reynolds numbers and aspect ratios.

Similarly, for $S = 1$, that is the cavity with two equal speed moving lids in the same direction. If two sets of variables $(x, y, u, v, p)$ and $(x, -y, u, -v, p)$ are respectively substituted into Eqs. (1), (2) and (3), then the Eqs. (1), (2) and (3) are invariant and both solutions are the same. Additionally, for $S = -1$, the applied boundary conditions are symmetric with respect to the horizontal centerline of the rectangular cavity, so that $u(x, y) = u(x, -y)$, $v(x, y) = -v(x, -y)$ and $u(x, 0) = 0$, $v(x, 0) = 0$. Then $u$ must be an even function of $y$ and $v$ must be an odd function of $y$. Therefore, for $S = 1$, the vortex structure distribution is symmetrical about the horizontal centerline of the rectangular cavity for all Reynolds numbers and aspect ratios.
For the Stokes flow of $Re \to 0$ (discussed by $Re = 0$), the left-hand sides of Eqs. (2) and (3) are equal to zero. In the same way, Substitute two sets of variables $(x, y, u, v, p)$ and $(-x, y, u, -v, -p)$ respectively into Eq. (1), (2) and (3), both solutions are the same. For $S = -1$, the applied boundary conditions are symmetric with respect to the vertical centerline of the rectangular cavity, so that $u(x, y) = u(-x, y)$, $v(x, y) = -v(-x, y)$ and $v(0, y) = 0$. Then $u$ must be an even function of $x$ and $v$ must be an odd function of $x$. Therefore, for the Stokes flow of $Re \to 0$, the vortex structure distribution is symmetrical about the vertical centerline of the rectangular cavity for all driving speed ratios and aspect ratios.

According to the above conclusions, we can derive the following inference. For the cavity flow of $Re \to 0$, $S = -1$ or $Re \to 0$, $S = 1$, the vortex structure distribution is all symmetrical with respect to the vertical centerline, horizontal centerline and center of the rectangular cavity for all aspect ratios.

**Numerical Validation**

The flow configurations simulated by the differential quadrature method are used to verify the above theoretical explanation of the symmetry of the vortex structure in the cavity. About the computational principle and algorithm of differential quadrature method, see [13]. About the quality and accuracy of differential quadrature code used to solve two-dimensional incompressible flow in a cavity, see [12].

**Symmetry for $S = -1$.** Figure 2 shows the distributions of the vortex structures in a square cavity for driving speed ratio $S = -1$, aspect ratio $A = 1$ and different Reynolds number $Re$. Figure 3 shows the distributions of the vortex structures in a rectangular cavity for $S = -1$, different $A$ and $Re$. It can be seen that the center of the cavity is always a stagnation point which is a saddle point (Figure 2 (a), (b) and Figure 3 (b), (d), (f)), or the center of a vortex (Figure 2(c), (d), (e) and Figure 3 (a), (c), (e)). Meanwhile, all the vortex structures are symmetrically distributed about the center of the cavity.

![Figure 2](image2.jpg)

Figure 2. The distributions of vortex structures for $S = -1$, $A=1$ and different $Re$.

![Figure 3](image3.jpg)

Figure 3. The distributions of vortex structures for $S = -1$, different $A$ and $Re$.

**Symmetry for $S = 1$.** Figure 4 shows the distributions of the vortex structures in a rectangular cavity for $S = 1$, $Re = 100, 500$ and $700$ respectively, and different $A$. It can be seen that all the vortex structures are symmetrically distributed about the horizontal centerline of the cavity.
Figure 4. The distributions of vortex structures for S = 1, different A and Re.

Symmetry for \( Re \to 0 \). Taking the flow configurations of \( Re = 0.01 \) verifies the symmetric distributions of vortex structures in a cavity for \( Re \to 0 \). Figure 5 shows the distributions of the vortex structures in a rectangular cavity for \( Re = 0.01, S = 2 \), and different A. Figure 6 shows the distributions of the vortex structures in a rectangular cavity for \( Re = 0.01, S = 0 \), and different A. It can be seen that all the vortex structures are symmetrically distributed about the vertical centerline of the cavity.

Figure 5. The distributions of vortex structures for S = 2, Re = 0.01 and different A.

Figure 6. The distributions of vortex structures for S = 0, Re = 0.01 and different A.

In particular, when both the \( Re \to 0 \) and \( S = -1 \) are met (as shown in Figure 7 for \( Re = 0.01, S = -1 \)), or both the \( Re \to 0 \) and \( S = 1 \) are met (as shown in Figure 8 for \( Re = 0.01, S = 1 \)), It can be seen that all the vortex structures are symmetrically distributed about the vertical centerline, horizontal centerline and center of the cavity.
Conclusions

Based on theoretical analysis and numerical validation of the symmetrical distributions of vortex structures in a cavity, the following conclusions can be made.

For the cavity flow of $S = -1$, the vortex structures are symmetrically distributed about the center of the cavity. The center of the cavity is always a stagnation point.

For the cavity flow of $S = 1$, the vortex structures are symmetrically distributed about the horizontal centerline of the cavity.

For the cavity flow of $Re \to 0$, the vortex structures are symmetrically distributed about the vertical centerline of the cavity. For the cavity flow satisfying the $Re \to 0$ and $S = -1$, or $Re \to 0$ and $S = 1$, all the vortex structures are symmetrically distributed about the vertical centerline, horizontal centerline and center of the cavity.

Acknowledgement

This research was financially supported by the National Natural Science Foundation (51369013).

References


