Kinematic Analysis of Hydraulic Excavator Working Device Based on D-H Method

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Abstract. To accurately describe the problem of excavator bucket teeth space trajectory in the mining process, is presented to describe the trajectory of excavator bucket teeth using D-H homogeneous coordinate method. Based on the principle of robotics, the excavator boom arm and bucket as three robot manipulators, through the driving of the boom cylinder and the arm cylinder and bucket cylinder, establish mathematical model of excavator working device pose space, using MATLAB/Simulink simulation to analyze the spatial motion, and fast get excavator mining envelope graph. The ZE-360 hydraulic excavator as an example, verify the scientific D-H homogeneous coordinate method in spatial motion analysis of excavator working device and adaptability, and provide a theoretical basis for the excavator trajectory and work space analysis.

Introduction

Hydraulic excavator is one of the most typical engineering machinery and the most complex, the most widely used structure. As a representative of engineering machinery products, the hydraulic excavator is widely used in construction, railway, highway, water conservancy, mining engineering construction etc. With the development of construction machinery, the working performance of the hydraulic excavator is put forward higher requirements. The excavator automation has become a focus of research in various countries, in this research, most work focused on working device of excavator control. At present, domestic and foreign research on excavator control, there are two kinds of methods, namely the excavator working device is split into two groups of link, kinematics analysis was carried out using bar group method [1], using D-H coordinate transformation method in which the kinematic inverse analysis of the excavator working device is performed. Although the analysis of the two-stage method is simple, it needs to be split into a number of two bar linkage mechanism, which cannot be analyzed systematically. Koivo uses the analytical method of solving the inverse problem of the robot kinematics [2], through the matrix multiplication calculation of the inverse kinematics of the excavator, not only the amount of computation, but also the solution process is not intuitive.

Aiming at the above problems, this paper, based on the working device of hydraulic excavator as the research object, regards the working device as the mechanical arm robot, and establishes the D-H mathematical model of four degree of freedom. Take the ZE-360 hydraulic excavator as an example, the mechanism parameter into the obtained spatial motion envelope diagram of working device. The whole working device is analyzed, the solving process directly, so as to solve the shortcomings of the literature research method.

Theoretical Model

Excavator Working Principle

The hydraulic excavator is composed of four parts, such as the working device, the upper frame, the driving room, and the walking device, as shown in Figure 1. The working device comprises a boom,
an arm, and a bucket, which are hinged by each other [3]. The boom is hinged with the upper frame, and the rotary platform of the movable arm is realized through the rotary platform. Boom arm and bucket realize the goal of mining by stretching each cylinder, driving the bucket according to a certain orbit.

Figure 1. Single bucket hydraulic excavator.

The significance of kinematic analysis of excavator working device is that the conversion between the joint angle and the position and attitude of the space and the variable of the driving oil cylinder and the joint angle are analyzed. Direct control object operation and control process of the actual working device in the cylinder of rotary motor and bucket and other related components, stretching through the cylinder to achieve the objectives and tasks. The kinematic analysis will build a driving mechanism (variable cylinder telescopic) and boom, the mutual angle between the arm and bucket (joint angle) and the relationship between the various components of the bucket angle and the spatial position. Through the analysis of the kinematics of the relationship between the realization of the control driven space variables, the final conversion to the spatial position and attitude variables, is the basis for further control of the excavator trajectory.

**Mathematical Modeling of Excavator Working Device in D-H Homogeneous Coordinate System.** Excavator Space Description: the rotary of platform, boom, arm and bucket rotating around each pin. Using the general method, by using the principle of robotics, excavator working device can be described as an open chain of four bar structure. This paper establishes the link coordinate system by using D-H homogeneous coordinates method [4].

D-H coordinate method sets that, the $z$ axis of the joint $i$ is coincident with the axis of the joint $i+1$, the $x$ axis is the $z$ axis of the adjacent common vertical line and the $y$ axis is determined by the right-hand rule. As shown in Figure 2, in the coordinate system, each link is described by $a_i$, $\alpha_i$, $d_i$, $\theta_i$. Among them, $a_i$, $\alpha_i$ is used to describe the characteristics of the connecting rod $i$ itself. $a_i$ refers to the distance measured from $z_{i-1}$ to $z_i$ along the axis $x_i$. $\alpha_i$ refers to the angle from $z_{i-1}$ to $z_i$ rotated around the axis $x_i$. $d_i$, $\theta_i$ is used to describe the link between the connecting rod $i$ and $i+1$. $d_i$ refers to the distance measured from $x_{i-1}$ to $x_i$ along the axis $z_i$. $\theta_i$ refers to the angle from $x_{i-1}$ to $x_i$ rotated around the axis $z_i$. For the rotational joint $i$, the $z$ axis along the joint axis direction, only $\theta_i$ is the joint variable. And the counter clockwise direction is the positive direction, the
The clockwise direction is the negative direction, and the other three parameters are fixed. For mobile joint, z axis along the rod stretching direction, only \(d_i\) is the joint variable, the other three parameters are fixed [5].

According to this principle, the connecting rod coordinate system of the excavator working device is established, and the coordinate system of the working device of the excavator is shown in Figure 3. The \(\theta_i\) is the rotary joint variable, and \(\theta_2\) is the boom joint variable, and \(\theta_3\) is the arm joint variable, and \(\theta_4\) is the bucket joint variable.

According to this principle, the connecting rod coordinate system of the excavator working device is established, and the coordinate system of the working device of the excavator is shown in Figure 3. The \(\theta_i\) is the rotary joint variable, and \(\theta_2\) is the boom joint variable, and \(\theta_3\) is the arm joint variable, and \(\theta_4\) is the bucket joint variable.

The connecting link coordinate system \(i\) with respect to \(i-1\) transform \(M_{i}^{i-1}\) is called the connecting rod transformation [6]. After the establishment of the link coordinate system, the relative relationship between two adjacent bars is established, then make the \(i\) coordinate system around the \(i-1\) axis, move parallel the \(x_{i-1}\) axis to the \(x_i\) axis in the same plane. And then move parallel \(d_i\) length along the \(z_{i-1}\) axis, which makes the \(x_{i-1}\) axis moved parallel to the same line with the \(x_i\) axis. Then move parallel \(\alpha_i\) along the \(x_i\) axis to make the \(i-1\) coordinate system and the \(i\) coordinate system coincide. Rotate \(\alpha_i\) angle around the \(x_i\) axis, so that makes the \(z_{i-1}\) axis and the \(z_i\) axis coincidence, namely the two coordinates system coincidence. A modified transformation is obtained from the Robot Theory:

\[
M_{i}^{i-1} = 
\begin{bmatrix}
\cos \theta_i & \cos \alpha_i \sin \theta_i & \sin \alpha_i \sin \theta_i & a_i \cos \theta_i \\
\sin \theta_i & \cos \alpha_i \cos \theta_i & \sin \alpha_i \cos \theta_i & a_i \sin \theta_i \\
0 & \sin \alpha_i & \cos \alpha_i & d_i \\
0 & 0 & 0 & 1
\end{bmatrix}.
\]

Transformation matrix multiplication between different coordinate systems gets the transformation matrix of the working device of \(M_{n}^{0}\):

\[
M_{n}^{0} = \Pi M_{i}^{i-1}.
\]

For the excavator working device, the establishment of the figure to simplify the coordinate system, you can draw a simplified model of the excavator. Among them, the origin of the zeroth coordinate system takes the intersection point between the rotary axis and the ground, the \(x\) axis is the datum plane, the direction of the working device is positive, and the \(z\) axis is perpendicular to the reference plane, and the orientation is positive. In the first coordinate system, the hinge point \(o\) of the boom and the arm is selected as the origin, and the \(x\)-axis direction is the same as that of the \(\theta\)-coordinate system, the \(z\) axis is the pivot axis of the hinge. The second and third coordinate system is similar. The hinge point of the boom and the arm is taken for the origin, so as the hinge point of the arm and the bucket. The \(z\) axis is the rotation axis of \(F, Q\) hinge point. The \(x\) axis respectively in the connection of the boom-arm hinge point and arm-bucket hinge point. The fourth coordinate system, with the \(x\) axis in the connection between bucket teeth and arm-bucket hinge point, locates in the bucket tip position.

The Kinematics Positive Solution of the D-H Method of the Excavator Working Device. According to the D-H coordinate method, the coordinate transformation is carried out on the adjacent
coordinate system, and the $M_i^{-1}$ is obtained, which is brought into the relevant data of the working device of the excavator, then the transformation matrix between adjacent coordinate systems is obtained [7]:

\[
M_i = \begin{bmatrix}
\cos \theta_i & 0 & \sin \theta_i & a_i \cos \theta_i \\
\sin \theta_i & 0 & -\cos \theta_i & a_i \sin \theta_i \\
0 & 1 & 0 & d_i \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  

(3)

\[
M_i = \begin{bmatrix}
\cos \theta_i & -\sin \theta_i & 0 & a_2 \cos \theta_i \\
\sin \theta_i & \cos \theta_i & 0 & a_2 \sin \theta_i \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  

(4)

\[
M_i = \begin{bmatrix}
\cos \theta_i & -\sin \theta_i & 0 & a_3 \cos \theta_i \\
\sin \theta_i & \cos \theta_i & 0 & a_3 \sin \theta_i \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  

(5)

\[
M_i = \begin{bmatrix}
\cos \theta_i & -\sin \theta_i & 0 & a_4 \cos \theta_i \\
\sin \theta_i & \cos \theta_i & 0 & a_4 \sin \theta_i \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]  

(6)

By formula (2) ~ (6), the transformation matrix of the excavator working device is obtained:

\[
M_4^0 = M_4^0 \times M_3^1 \times M_2^2 \times M_1^3
\]

(7)

In the above formula, $s_i = \sin \theta_i$, $s_{ij} = \sin (\theta_i + \theta_j)$, $s_{ijk} = \sin (\theta_i + \theta_j + \theta_k)$, $c_i = \cos \theta_i$, $c_{ij} = \cos (\theta_i + \theta_j)$, $c_{ijk} = \cos (\theta_i + \theta_j + \theta_k)$.

In the fourth coordinate system, the tip of the bucket teeth set unit vector coordinates for $P_4 = [0,0,0,1]^T$, so in the first coordinate vector coordinate $P_0 = [X,Y,Z,1]^T$ is:

\[
P_0 = M_4^0 \times P_4.
\]

(8)

Through formula (7), (8) to get the tooth tip of the position and attitude coordinate expression:

\[
\begin{aligned}
X &= c_i\left(a_1c_{234} + a_2c_{23} + a_3c_2 + a_4\right) \\
Y &= s_i\left(a_1c_{234} + a_2c_{23} + a_3c_2 + a_4\right) \\
Z &= a_1s_{234} + a_2s_3 + a_3s_2 + d \\
\varphi &= \theta_2 + \theta_3 + \theta_4
\end{aligned}
\]

(9)

The constant in the formula is the size parameter of the working device of hydraulic excavator. $a_i$ is the length of the boom-base hinge point along the $x_0$ axis at the origin of the $o$ coordinate system. $a_2$ is the length between the boom-base hinge point and the boom-arm hinge point. $a_3$ is the length
between the boom-arm hinge point and the arm-bucket hinge point. \( a_4 \) is the length between the hinge point of the arm and bucket rod and the bucket tip. \( d_i \) is the length between the boom-base hinge point and the origin of the \( o \) coordinate system in the direction of the \( z_o \) axis.

In the formula (9), the fourth equation value \( \varphi \) is the angle between the line of bucket tip and bucket pin connection and the horizontal line, which is the angle between the X axis of the fourth coordinate system and the horizontal line. \( \varphi \) can determine the position and attitude of bucket in the space effectively. The joint space variable \([\theta_1, \theta_2, \theta_3, \theta_4] \) can effectively determine the bucket position of bucket tooth, and describe it with the vector \([x, y, z, \varphi] \).

The Inverse Kinematics Solution of D-H Method for Hydraulic Excavator Working Device. Similarly, the known spatial position variables \([x, y, z, \varphi] \) can be obtained by the joint space variable \([\theta_1, \theta_2, \theta_3, \theta_4] \). With the vector \([x, y, z, \varphi] \) to describe the bucket tooth tip position, that is only a space variable constraint joint solutions, only need to use graphical and geometric relations to get an angle value [8]. Without considering the rotary boom hinge point C for the origin of the first coordinate system as the base coordinate system, system will original work into a two-dimensional plane coordinate system, coordinate tooth tip hypothesis for \([x, 0, z, \varphi] \), as shown in figure 4.

\[
\theta_2 = \alpha + \beta + \gamma \quad (10)
\]
\[
\theta_3 = \angle CFQ - \pi \quad (11)
\]
\[
\theta_4 = \varphi - \theta_2 - \theta_3. \quad (12)
\]

V point two-dimensional plane coordinates is \((X, Z)\); Q coordinates \(\begin{cases} X_Q = X - QV \cos \varphi \\ Z_Q = Z - QV \sin \varphi \end{cases}\), equals that:

\[
\begin{cases}
X_Q = X - a_4 \cos \varphi \\
Z_Q = Z - a_4 \sin \varphi
\end{cases} \quad (13)
\]

In figure 4, \( \alpha, \beta \) are in the \( \triangle CFQ \) and \( \triangle CVQ \). The cosine theorem obtains the following:

\[
\alpha = \cos^{-1} \frac{CQ^2 + a_2^2 - a_3^2}{2CQ \cdot a_2} \quad (14)
\]
\[
\beta = \cos^{-1} \frac{CQ^2 + CV^2 - a_4}{2CQ \cdot CV} \quad (15)
\]
\[
\angle CFQ = \cos^{-1} \frac{a_2^2 + a_3^2 - CQ^2}{2a_2a_3}. \quad (16)
\]

Known V point coordinates, get \( \gamma \):
\[ \gamma = \tan^{-1} \frac{z}{x}. \]  

(17)

Considering the rotary of base, \( \theta \neq 0 \), \( Y \neq 0 \), for \( \alpha, \beta, \gamma, \angle CFQ \) will not change with the base rotation. Transform the above amount from first coordinate system into the zeroth coordinate system. Known \( a_1, d_1 \) just replace \( x \) coordinate with \( \sqrt{(x-a_1)^2 + y^2} \) in the formula (17). The \( z \) coordinate is replaced by \( z - d_1 \), \( \theta_1 = \tan^{-1} \frac{y}{x} \). Each joint angle of the end of the coordinates \([x, y, z, \phi]\) is get:

\[ \begin{align*}
\theta_1 &= \tan^{-1} \frac{y}{x}, \\
\theta_2 &= \alpha + \beta + \gamma, \\
\theta_3 &= \angle CFQ - \pi, \\
\theta_4 &= \phi - \theta_2 - \theta_3. \end{align*} \]  

(18)

Among them, \( CQ = \sqrt{(x - a_4 \cos \phi - a_1)^2 + y^2 + (z - a_4 \sin \phi - d_1)^2} \); \( CV = \sqrt{(x - a_1)^2 + y^2 + (z - d_1)^2} \).

By means of the method of geometric solution, given the unique coordinate vector \([x, y, z, \phi]\) at the end of the bucket, it is possible to determine the spatial coordinates of the working device, the joint angles \([\theta_1, \theta_2, \theta_3, \theta_4]\). The spatial coordinates of the working device can be determined, and achieve the positive and negative two-way solution of the joint angle and the space position.

MATLAB/Simulink Simulation Model Establishment

According to the spatial position relation of excavator working device from formula (9) and (18), the simulation model was established by using MATLAB/Simulink. Taking ZE-360 hydraulic excavator as an example, the SimMechanics module in the Simulink to build a simulation model of the excavator, as shown in Figure 5 below. Among them, the simulation model is composed of 1 modules, 3 hinge point excitation module, 3 cylinder block module and detection module [9]. SimMechanics' Body module is used to simulate the boom, arm and bucket of the excavator in the process of modeling. The Joint Actuator module is used to simulate the driving of each hydraulic cylinder, so that the shape of the excavator is simplified, and the concrete excavator working device is abstracted as a mathematical model to facilitate the study. In order to facilitate the rapid drawing of the excavator working device envelope graph, the rotating part of the machine body is not considered, and the working device of the excavator is in one plane.
Enter the boom arm and bucket length values in the excavator working device in the SimMechanics model, as shown in table 1. And in the Body module set up its corresponding angle range, table 2 gives the relative value of each angle. Set a sine drive for the system (Subsystem):

\[ o(t) = Amp \times \sin(Freq \times t + Phase) + Baise. \]  

In the formula, \( Amp = (\theta_{\text{max}} - \theta_{\text{min}}) / 2 \), \( Phase = 0 \), \( Bias = -Amp \).

Table 1. Size parameters of excavator working device.

<table>
<thead>
<tr>
<th>Working device</th>
<th>Length [mm]</th>
<th>Arm length FQ [mm]</th>
<th>Bucket length QV [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length</td>
<td>6470</td>
<td>3188</td>
<td>1660</td>
</tr>
</tbody>
</table>

Table 2. Joint rotation angle of excavator working device.

<table>
<thead>
<tr>
<th>Rotation angle</th>
<th>( \theta_1 ) [°]</th>
<th>( \theta_2 ) [°]</th>
<th>( \theta_3 ) [°]</th>
<th>( \theta_4 ) [°]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle value</td>
<td>Min 90 Max 90</td>
<td>Min -39.70 Max 61.99</td>
<td>Min 35.24 Max 154.39</td>
<td>Min -56.35 Max 29.59</td>
</tr>
</tbody>
</table>

In order to make the excavator bucket teeth experience all work areas, the simulation time should be long enough. The simulation time is set infinitely long, namely inf. The following figure 6 simulation time is \( t_1 = 235s \), \( t_2 = 565s \), \( t_3 = 1163s \), \( t_4 = 4633s \), drawing the excavator working device envelope. From the four different time envelope diagrams, if the simulation time is long enough, bucket teeth theoretically will traverse all the work area. To further verify the effectiveness of the above method, the simulation model of \( x_{\text{max}}, y_{\text{max}}, y_{\text{min}} \) output value, which compared the maximum digging radius of hydraulic excavator, the maximum digging height, maximum digging depth with the same type of excavator working parameters, the related results are shown in table 3.

Table 3. Comparison of simulation value and real value of hydraulic excavator operation parameters.

<table>
<thead>
<tr>
<th></th>
<th>Maximum digging radius [mm]</th>
<th>Maximum digging height [mm]</th>
<th>Maximum digging depth [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation value</td>
<td>11385</td>
<td>8747</td>
<td>8913</td>
</tr>
<tr>
<td>Actual value</td>
<td>11075</td>
<td>9036</td>
<td>8730</td>
</tr>
<tr>
<td>Error rate</td>
<td>2.72%</td>
<td>3.30%</td>
<td>2.05%</td>
</tr>
</tbody>
</table>

Figure 6. Excavator envelope diagram.

Conclusion

According to the joint space variable \([\theta_1, \theta_2, \theta_3, \theta_4]\), known working device parameters, can effectively determine the bucket teeth position, and can use a vector \([x, y, z, \phi]\) to describe. Similarly, the known
space position function $[x, y, z, \phi]$ can be obtained in the working space of excavator working device different pose angle $[\theta, \theta, \theta, \theta]$, realize the mutual transformation between the two parameters.

According to the excavator bucket tip position matrix, in the MATLAB/Simulink environment, using SimMechanics module for modeling and Simulation of excavator working device of excavator working range can be obtained, and draw the mining envelope graph. The model improves the simulation efficiency, and provides a reliable basis and technical support for the analysis of the kinematics of the excavator and the improvement of the performance of the working device.

Reference


