Nonlinear Dynamical Analysis of a Breathing Crack Plate Based on Structural Intensity Method

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Abstract. A Breathing crack is one of the most common damage in plate structures. When a plate vibrates, the breathing crack will produce the open and closing movement, and the structure has nonlinear dynamic response characteristics. In this paper, the contact nonlinearity of a plate structure with a breathing crack is considered, and the nonlinear vibration of the breathing cracked plate is analyzed. The structural intensity is studied under dynamic excitation and the surface structural intensity vector field of the structure is determined. The position and shape of the crack are identified by the structural intensity maps, which provide a theoretical basis for the damage identification of cracked structures with a breathing crack.

Introduction

Structural intensity method is used to study the structure vibration energy propagation from the viewpoint of wave propagation, which is useful for identifying the vibration source in the structure, determining the energy propagation path in the structure, and diagnosing the structural defects.

Semperlotti F\textsuperscript{1-3} analyzed the nonlinear behavior of the orthotropic laminates with delamination damage, and the defects in the structure were accurately located by using the structural intensity vector image.

Li Kai\textsuperscript{4-5} visualized the vector field of the structural intensity vector by using the vector map and the streamline map, and the effect of the damper arrangement on the energy propagation of the composite plate structure was analyzed.

Schmidt W T\textsuperscript{6} obtained the structural intensity vector field of a cracked aluminum plate by using Laser Doppler Vibrometer, and the region of damage was determined.

Zhang Ling\textsuperscript{7} analyzed the power flow of a plate through the visualization method, and the vibration of a functionally graded materials plate was described by the structural intensity vector field.

Pang Z H\textsuperscript{8} studied the input power flow characteristics of the breathing cracked plate under resonant excitation.

However, studies on breathing cracked plate’s structural intensity field have not been carried out yet. This research, based on the structural intensity method, will focus on the nonlinear dynamic behavior of a breathing cracked plate, and the work is carried out from the aspect of damage identification.

Structural Intensity Concept

In this study, solid elements are used to model the plate structure. Here, the structural intensity expression of the solid element is given:
\[ I_x = -\frac{1}{2} \text{Re}(\sigma^* \nu_x^* + \tau_{xy}^* \nu_y^* + \tau_{xz}^* \nu_z^*) \]
\[ I_y = -\frac{1}{2} \text{Re}(\sigma^* \nu_y^* + \tau_{yx}^* \nu_x^* + \tau_{yz}^* \nu_z^*) \]
\[ I_z = -\frac{1}{2} \text{Re}(\sigma^* \nu_z^* + \tau_{xz}^* \nu_x^* + \tau_{yz}^* \nu_y^*) \]

In Eq. 1, \( I_x \), \( I_y \), and \( I_z \) are the structural intensity in the \( x \), \( y \), and \( z \) directions respectively; \( \sigma \) and \( \tau \) are the normal stress and shear stress of the structure; \( \nu \) is the speed response of the structure at a particular frequency; \( * \) represents conjugate operations on complex numbers; \( \text{Re} \) represents the real part of the complex number.

The Vibration Simulation of a Cracked Plate with Finite Element Method

The detailed description of the model is as follows: a rectangular plate with the length 0.7m, the width 0.5m, and the thickness 0.01m. There is a breathing crack with depth of 6mm, which is parallel to the short side of the plate, and the distance between the crack line and the short side is 0.3m. The plate’s material is with Young’s modulus \( 7e10 \) Pa, Poisson’s ratio 0.3, mass density \( 2100 \) kg/m\(^3\), and constant structural damping \( 0.005 \) kg·m/s. The plate is clamped at four sides. The singularity of the crack tip is simulated by quarter-node elements, the cross-section of the breathing cracked plate is shown in Figure 1. The second-order 20-node solid elements are used to simulate the plate, as shown in Figure 2, the total number of the solid elements is 25,650.

Since it is assumed that the breathing crack considered here does not slide, the frictionless standard contact pair is adopted. In this paper, on the basis of ensuring iterative convergence, it is necessary to increase the contact stiffness as much as possible in order to ensure the precision of the simulation analysis, the final scale factor of contact stiffness was chosen to be 0.15.

Transient finite element analysis is applied with a (time varying) vertical force \( F=\sin(20\pi t) \) located at the center of the upper surface, and transient calculation time is 1s with 320 steps, and the ramp loading is used.

Calculation of Structural Intensity

In recent years, surface structural intensity (SSI) has become a hot research topic, the structural intensity vector field of solid elements on the surface of the plate is considered in this section. The excited node’s vertical displacement-frequency curve of the crack plate is obtained by fast Fourier transform, which is shown in Figure 3.

From Figure 3, it can be found that the vertical displacement curve of the vertical excited node has the maximum displacement peak at the excitation frequency, which is the principal resonance peak. Due to the existence of the breathing crack in the plate, the periodic motion of open and closing can be generated in the process of vibration, which makes the curve has the peaks of the sub-harmonic resonance and the super-harmonic resonance. Time domain finite element calculation results by fast Fourier transform, according to Eq. 1, can be used to calculate the structural intensity vector field of the breathing cracked plate at these characteristic peak frequencies (in this case, 10Hz is the main
resonance frequency, 20Hz for the super-harmonic resonance frequency). It is also possible to plot amplitude maps and divergence maps by using the structural intensity vector field, the amplitude of the structural intensity vector is defined by Eq. 2, and the divergence of the structural intensity vector field is given by Eq. 3.

\[ |\mathbf{I}| = \sqrt{I_x^2 + I_y^2 + I_z^2} \]  

\[ \text{div}(\mathbf{I}) = \nabla \cdot \mathbf{I} = \frac{\partial I_x}{\partial x} + \frac{\partial I_y}{\partial y} + \frac{\partial I_z}{\partial z} \]  

Thus, the structural intensity vector field’s generation and visualization are realized by the theoretical algorithm. Figure 4 is the structural intensity maps, which are obtained by using the above method.

![Figure 4. SSI of a breathing cracked plate with a crack of 6mm deep.](image)

It is obvious from Figure 4 that there is a distinct crack at 0.3m from the left side of the plate. However, the 10Hz structural intensity map is applied by 10Hz excitation, with the whole plate as the dominant vibration response, and the vibration component of the crack is relatively small; and the super-harmonic resonance generated at 20Hz frequency is due to the open and closing periodic motion of the breathing crack, which overflows the energy, forcing other parts of the plate to "passively" do periodic vibration, while the vibration component of the crack is relatively large.

The structural intensity map at the excitation frequency can accurately reflect the shape and position of the breathing crack. However, at the super-harmonic resonance frequency, the structural intensity vectors near the breathing crack zone have a small range of distortion, which can not reflect the shape of the crack very accurately.

![Figure 5. 10Hz SSI vector image of a breathing cracked plate.](image)
In order to further analyze the influence of the depth of the breathing crack on structural intensity vectors, this section gives the SSI vector images with different crack depth, as shown in Figure 5. From Figure 5, it can be found that the recognizability of the breathing crack is affected by both the "degree" of the crack damage and the "distance" between the deepest part of the crack and the surface of the plate. In a certain range, its identification is mainly affected by the degree of the crack damage, as the depth of the crack increases, the energy spilled from the breathing crack to the surface of the plate gradually increases, making the identification increase; however, with the increase of the breathing crack damage, the identification is mainly affected by the distance between the deepest part of the crack and the surface of the plate, at this time, the energy spilled from the breathing crack to the surface of the plate is reduced, so the identification degree is reduced.

Summary

In this paper, the nonlinear dynamic response of a breathing cracked plate is studied from the viewpoint of damage identification by using the method of structural intensity.

In summary, a complete set of structural damage detection scheme can be summarized as follows: first of all, observing the location of the peak in the displacement-frequency curve of the structure, if there are secondary resonance peaks on any side of the main resonance peak, the structure can be judged to be damaged; then, the structural intensity map at the super-harmonic frequency can be used to lock the range of the crack; finally, the shape of the crack and its exact location can be determined by using the structural intensity map at the main resonance frequency.

References


