Control Design for Mobile Robot Based on Step-by-step Point Stabilization

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ABSTRACT

In this paper the control object is non-holonomic mobile robot and the control problem focus on point stabilization which is one of the most important issues on non-holonomic mobile robot motion control in view of both theory and application. The integrity of mobile robot kinematic model and the position error formula is established under the polar coordinates for point stabilization problem. Then the step-by-step point stabilization control problem is studied where the method of kinematic control design is given. By successively changing the state variables, the mobile robot will get the expecting position and will be step-by-step convergence. On the basis of this point stabilization control law the simulation is carried on by MATLAB. The results prove the feasibility of the controller.

INTRODUCTION

Industrial robots have been widely used in industrial handling, assembly, testing and other work. In the field of logistics, food and medicine, industrial robots have been gradually replacing manual operation [1]. Tile fixing is one of the Construction works which have the characteristics of high demand and high quality requirements. In normal conditions professionals are needed to complete the job, which have the disadvantage of high cost and long working time. Therefore there are important significances on researching robotic tiling machine.
The kinematics calibration motion, planning and controlling of the industrial robot research has mature control scheme [2]. So moving body of robotic tiling machine will be studied in this paper, which is seen as a mobile robot with the independent movement characteristics to complete a various of tasks in the environment. Its movement is pure rolling on the ground and it is non-holonomic constraint [3]. The research of non-holonomic constraint robot focus on the time-varying feedback control law which is smooth or piecewise smoothly and the feedback control law that is discontinuous and do not vary with time [4]. In order to overcome this difficulty three main control some methods are put forward smooth time varying control law, discontinuous and mixed feedback control law [5]. Lina Tang [6] and Xue Yang [7] respectively design the trajectory tracking control law based on the fuzzy system and neural network which realize the position tracking. In this paper the mobile body step-by-step point stabilization control law is designed and simulation is carried out by MATLAB.

The Kinematics Model of Mobile Robot

Figure 1 is the overall structure of the robotic tiling machine, where the upper part is 6-dof robotic manipulator and the lower part is the mobile robot. Position adjustment of the end effector rely on the 6-dof joint is realized. Depending on the sucker installed on the end effector or the robot can grab tiles and complete the job of pasting. The waist is on the top of the moving body that uses the motor driver to realize rotary motion. The arm connects with the waist that is derived by motor to achieve the rotation around the waist. The transmission of wrist is belt to realize pitch motion by the gear and the synchronous.

In the research of the mobile robot, the mobile robot kinematics characteristics are comprehensively understood, which can ensure that the designing robot can complete the expecting work according to the requirement. So, before mobile robot motion control is studied, the kinematics characteristics should be further researched. The mobile robot moves from the current position to the expecting position by adjusting the velocity and the course angle. The position of the robot described by Cartesian coordinate system, and the position of the mobile robot is described by the position of the barycenter in the coordinate system.
In the figure 2, X1OY1 represents the global coordinate system, XRMYR represents local part reference frame. The original point of part reference frame is on M that is the midpoint between the two wheels. Because of the barycenter of the mobile robot is changing with time, M is considered as the barycenter for the convenient. \( \theta \) represents the angle between the global coordinate system and local part reference frame, which is the course angle of the mobile robot. \( \rho \) represents the distance between the target location and mobile robot axle center. \( \alpha \) represents the angle between \( X \) and the line from robot axle center to target position. \( \beta \) represents the angle between \( X \) and the vector from axle center to target position.

In the model, \( r \) represents the radius of the drive wheel, and \( D \) represents the distance between the two wheels. \( \omega_L, \omega_R \) respectively represents the angle velocity of two wheels, and \( v_L, v_R \) respectively represent the velocity of two wheels. The kinematic relation is as equation (1).

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{\theta}
\end{bmatrix} = \begin{bmatrix}
\frac{r}{2} \cos \theta & \frac{r}{2} \cos \theta \\
\frac{r}{2} \sin \theta & \frac{r}{2} \sin \theta \\
\frac{r}{D} & -\frac{r}{D}
\end{bmatrix} \begin{bmatrix}
\omega_L \\
\omega_R
\end{bmatrix}
\tag{1}
\]

The position of the mobile robot is \([x, y, \theta]^T\) in global coordinate system. The movement of the mobile robot is controlled by the angle velocity of two wheels that are \( \omega_L, \omega_R \).

Getting to the target point problem is the underlying control basic problem of robot, which is the stabilization problem [8]. Point stabilization diagram is established according to the non-holonomic mobile robot kinematics model, as show
in figure 2. In the global reference frame, the current mobile robot position is $P = [x, y, \theta]^T$, and $x, y, \theta$ respectively represent X coordinates, Y coordinates, and heading angle under the current mobile robot position. The expecting position is $P_r = [x_r, y_r, \theta_r]^T$, and $x_r, y_r, \theta_r$ respectively represent X coordinates, Y coordinates, and heading angle under the expecting position. The given position error under rectangle coordinate system is $p_e = [x_e, y_e, z_e]^T$, $\theta_e \in [-2\pi, 2\pi]^T$, $x_e, y_e, z_e$ represent the error of the current position and expecting position on the x axis, the y axis and the z axis. Its corresponding expression is as equation (2).

$$p_e = P - P_r = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} = \begin{bmatrix} x_r - x \\ y_r - y \\ \theta_r - \theta \end{bmatrix}, \quad (2)$$

The target is located in the origin of the coordinate system and target position is known in the point stabilization control. Position error is set up under the polar coordinate, that is $X_R$ and the robot moving speed are both in the same direction, then set two drive wheels axle center to the target location as a vector $x$ and its direction is from the current position to the target location.

The current position of the displacement error in polar coordinates is expressed as the equation (3).

$$e_p = \begin{bmatrix} \rho \\ \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \sqrt{\Delta x^2 + \Delta y^2} \\ -\theta + \arctan(\Delta y / \Delta x) \\ \theta + \alpha \end{bmatrix}, \quad \Delta x = x_r - x, \quad \Delta y = y_r - y \quad (3)$$

When equation (4) is taken derivative, the expression of system is as follow. When $\alpha \in (-\pi/2, \pi/2]$, the equation is shown as (4).

$$\dot{e}_p = \begin{bmatrix} \dot{\rho} \\ \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -\cos \alpha & 0 \\ \sin \alpha & -1 \\ -\sin \alpha & 0 \end{bmatrix} \begin{bmatrix} v \\ \omega \end{bmatrix}, \quad (4)$$

The movement direction of the mobile robot is from the initial point to the desired location.

When $\alpha \in (-\pi, -\pi/2] \cup (\pi/2, \pi]$
When the movement direction of the mobile robot is opposite to desired position, the velocity direction of the robot is redefined as $v = -v$.

**Step-by-step Point Stabilization Control**

Because there are three states quantity $[x, y, \theta]^T$ in the mobile robot kinematics model, that means changing one of the state can make it convergence, and control the rest of the state to make it convergence. Finally, it arrives the expecting position [9]. If the initial velocity of the mobile robot is $v=0$, and the corresponding angle velocity will be $\omega$. The course angle is equal to the angle between the line from robot axle center to the target location and $X1$. Then the velocity of the mobile robot is given, and the robot move towards the expecting position. Then the robot arrives at the expecting location. Finally, controlling angle velocity $\omega$ to make the angle $\theta$ is equal to $\theta_r$, the design requirements angle.

When $\alpha \subset (-\pi/2, \pi/2)$, the control law design is as follow.

In the first stage, the angle velocity is controlled to make the course angle is equal to the angle which is between $X1$ and the line from the axle center to the target location. And the equation is shown as (6).

$$\begin{cases}
\omega(t) = f_1(\bar{\theta} - \theta) \\
v(t) = 0
\end{cases} \quad \theta \neq \bar{\theta}$$  \hspace{1cm} (6)

In the second stage, the velocity of the mobile robot is controlled to make it move toward the target. And the equation is shown as (7).

$$\begin{cases}
v(t) = f_2 \sqrt{(x_c - x)^2 + (y_c - y)^2} \\
\omega(t) = 0
\end{cases} \quad \theta = \bar{\theta}$$  \hspace{1cm} (7)

In the third stage, the angle velocity is controlled to make the angle $\theta$ equal to the given angle. And the equation is shown as (8).

$$\begin{cases}
\omega(t) = f_3(\theta, -\bar{\theta}) \\
v(t) = 0
\end{cases} \quad \theta, \neq \bar{\theta}$$  \hspace{1cm} (8)
Where $\theta$ is the angle between the global coordinate system X1 and the line from the axle center to the target position. $f_1$ and $f_3$ represent the course angle function, and $f_2$ represent function of the distance. $\theta_\alpha$ represents expecting position heading angle.

When $\alpha \in (-\pi,-\pi/2] \cup (\pi/2,\pi]$, the movement direction of the robot will be $v = -v$.

According to the above design, simulation of the control law is taken. Initial position of mobile robot is (0,0,0), and the expected position is (2,3,\pi/2). Because of short distance and velocity and angle velocity are equal to zero at start and end of each phase. The cubic polynomial interpolation function is used to design the control law.

$$
\begin{align*}
\theta_i &= a_{i0} + a_{i1}t + a_{i2}t^2 + a_{i3}t^3 & 0 < t < t_i \\
S &= a_{s0} + a_{s1}t + a_{s2}t^2 + a_{s3}t^3 & t_i < t < t_2 \\
\theta_2 &= a_{s0} + a_{s1}t + a_{s2}t^2 + a_{s3}t^3 & t_2 < t < t_3
\end{align*}
$$

$$a_{s0} = 0 , a_{s0} = 0 , a_{s0} = 0 , a_{s0} = 0 , a_{s1} = 0 , a_{s1} = 0 , a_{s2} = \frac{3}{t_1^3} (\theta_f - \theta_i ) , a_{s2} = \frac{3}{(t_2 - t_1)^3} (s_f - s_i ) ,$$

$$a_{s2} = -\frac{3}{(t_1 - t_2)^3} (\theta_f - \theta_i ) , a_{s3} = -\frac{2}{t_1^2} (\theta_f - \theta_i ) , a_{s2} = -\frac{2}{(t_2 - t_1)^2} (s_f - s_i ) , a_{s3} = -\frac{2}{(t_2 - t_1)^2} (\theta_f - \theta_i )$$

$\theta_f, \theta_i$ represents the end angle of the first stage and start angle of first stage. $s_f, s_i$ represents the end position of second and start position.

**The Simulation Analysis**

The time of the first stage is 2 seconds, the second is 5 seconds, and the third is also 2 seconds. The motion curve can be gotten by calculation. The position error curve the angle velocity and the velocity are as follows.
In the first stage only the angle is changed, and in the second stage the distance between the current position and expecting position is adjusted. Finally, the angle is adjusted to get the expecting position. As show in the velocity curve in figure 3 angle velocity in figure 4 completely conforms to the design requests. The error curve in the figure 5 indicates the position error can equal to zero in 10s. the control law is effective and can realize the mobile robot point stabilization.

CONCLUSIONS

In this paper the point stabilization of the robotic tiling machine moving body is researched, and the step-by-step control law is used to make simulation analysis of the process. The result shows that the control law has the character of direct response as well as the whole process is simple. When the angle has been adjusted the robot will move to the expected position and stop at the target point, and adjust the course angle.

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REFERENCES


