Filter Algorithm Research of "Current" Statistical Mode Based on Multi-models Fusion

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ABSTRACT: In order to overcome the problem of what traditional "current" statistical filtering model is difficult to determine the self-related time constant as tracking maneuvering target, this paper advances improved "current" statistical filter model based on multi-models fusion. The algorithm firstly provides some "current" statistical filter model which has different self-related time constant, and then obtains filtering result by probability weighting the result of each model. The simulation result shows that the new algorithm has high tracking accuracy compared with traditional "current" statistical filtering model.

1 INTRODUCTION

The more representative maneuvering target tracking algorithm without maneuver detection includes Jerk model algorithm, Singer model algorithm, IMM and "current" statistical model algorithm. Jerk model takes the noise variance error matrix in the process as constant matrix, as the target high maneuvering, system process noise variance increases due to rate of change of maneuver acceleration dramatic changing, the hypothesis of setting noise variance matrix as constant is unreasonable[1]. Singer model has larger motor adaptability compared with "either-or" differential polynomial model. But for intense maneuvering, namely more than the uniformly accelerating range, using this model will lead to a larger model error[2]. For IMM model, as it uses multiple filters, with large calculation quantity, and the filtering accuracy is greatly influenced by the model probability transition matrix[3]. "Current" statistical model uses non-zero mean and Rayleigh distribution to express maneuver acceleration characteristics, so it is more realistic. Compared with other models, "current" statistical model is one of the preferred utility model[4].

In order to use "current" statistical model, it needs to calculate process noise variance matrix, self-related time constant first needs to be determined. There is no effective way to determine $\alpha$, mainly based on experience. If the value of $\alpha$ isn't appropriate, it will affect the performance of the algorithm, reducing the filtering accuracy. Aiming at this problem, this paper advances improved "current" statistical model filtering algorithm based on the fusion of multi-model.

2 "CURRENT" STATISTICAL MODEL FILTERING ALGORITHM

2.1 Construction of "current" statistical model

"Current" statistical model algorithm uses modified Rayleigh distribution to describe statistical characteristics of maneuvering acceleration, the supposed distribution has such characteristics: distribution changes with mean, and square variance is determined by mean. The discrete state equation and measurement equation in "current" statistical model algorithm is as equation (1):

$$
\begin{align*}
X(k+1) &= F(k)X(k) + G(k)\tilde{a}(k) + V(k) \\
Z(k) &= H(k)X(k) + W(k)
\end{align*}
$$

In equation (1), $X(k) = [x \dot{x} \ddot{x}]^T$, and state transition matrix $F(k)$ is as equation (2).

$$
F(k) = \begin{bmatrix} 1 & T & A \\
0 & 1 & B \\
0 & 0 & C \end{bmatrix}, \ C = e^{-\alpha T}
$$

$$
B = (1-C)/\alpha, \quad A = (T-B)/\alpha
$$

In equation (1), $\tilde{a}(k) (\tilde{a}(k) = \tilde{x}(k\mid k-1))$ is the mean of maneuvering acceleration, $V(k)$ is discrete time white noise sequence, and $E[V(k)V^T(j)] = Q(k)\delta_{k,j}$.

$$
Q(k) = \begin{bmatrix} q_{11} & q_{12} & q_{13} \\
q_{21} & q_{22} & q_{23} \\
q_{31} & q_{32} & q_{33} \end{bmatrix}
$$

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In equation (3), \(\alpha\) is self-related time constant. Usually, the value of \(\alpha\) is between 1/60 and 1, and the exact value is determined by real-time measurement. \(\sigma^2_a\) is square variance of acceleration, and \(Q(k)\) is symmetric matrix. The calculation formula of \(q_\sigma(i, j = 1, 2, 3)\) can be obtained from document [3]. In equation (1), \(H(k)\) is measurement matrix, \(H(k)=[1 0 0]\), \(\sigma\) is measurement error, measurement error square variance matrix is \(R\).

2.2 "Current" statistical model solution

1) Parameter initialization of "current" statistical model

Initial filter value \(X(2|1)\) of state vector \(X(k)=[x \dot{x} \ddot{x}]^T\) and initial value \(P(2|1)\) of filtering error variance matrix \(P\) can be obtained from document [3].

Setting the initial value of process noise variance matrix \(Q\) as \(Q_0\), setting the initial value of \(\alpha\) and sample interval \(T\).

2) Calculate the one step estimate value \(\hat{X}(k|k-1)\) of state vector

\[
\hat{X}(k|k-1) = F(k)X(k-1|k-1) + G(k)\ddot{a}(k) \tag{4}
\]

In equation (4), the calculation method of \(F(k)\) is as equation (2), \(\ddot{a}(k)\) is mean of maneuver acceleration, \(\ddot{a}(k) = \hat{X}(k|k-1)\), control matrix \(G(k)\) is as equation (5).

\[
G(k) = \begin{bmatrix}
\frac{1}{\alpha} \left( T + \frac{a_{\text{max}}^2}{2} + \frac{a_{\text{max}}^2}{\alpha} \right) \\
T - \frac{a_{\text{max}}^2}{\alpha} \\
1 - e^{-\alpha T}
\end{bmatrix} \tag{5}
\]

3) Calculate the filter gain \(K(k)\)

\[
P(k|k-1) = F(k)P(k-1|k-1)F^T(k) + Q(k-1) \tag{6}
\]

\[
S(k) = H(k)P(k|k-1)H^T(k) + R(k) \tag{7}
\]

\[
K(k) = P(k|k-1)H^T(k)S(k)^{-1} \tag{8}
\]

4) Calculate the filter value \(\hat{X}(k|k)\) of the state vector and filter error variance matrix \(P(k|k)\)

\[
v(k) = Z(k) - H(k)\hat{X}(k|k-1) \tag{9}
\]

\[
\hat{X}(k|k) = \hat{X}(k|k-1) + K(k)v(k) \tag{10}
\]

\[
P(k|k) = [I - K(k)H(k)]P(k|k-1) \tag{11}
\]

5) Calculate the error variance matrix \(Q(k)\) of process noise

The calculation formula of \(\sigma^2_a\) is as equation (12), among them, \(a_{\text{max}}\) and \(a_{\text{max}}\) respectively express max possible acceleration and min possible acceleration.

\[
\sigma^2_a = \begin{cases} \frac{1}{T^2}[a_{\text{max}} - \ddot{a}(k)] & \ddot{a}(k) \geq 0 \\ \frac{1}{T^2}[\ddot{a}(k) - a_{\text{max}}] & \ddot{a}(k) < 0 \end{cases} \tag{12}
\]

Calculated \(\sigma^2_a\), \(Q(k)\) can be calculated by using equation (3)

6) Iteration calculation

Taking the \(\hat{X}(k|k), P(k|k)\) and \(Q(k)\) as initial filter value, jump to step 2) and iterate. When the filter time or filter accurate meets the need, stops calculation.

3 IMPROVEMENT OF "CURRENT" STATISTICAL MODEL FILTERING ALGORITHM

It can be seen from the process and implementation step of the "current" statistical model filtering algorithm that in the calculation process of noise variance matrix, self-related time constant \(\alpha\) needs to be calculated. As how to calculate \(\alpha\), document [3] provides the following principle: as the maneuver style of goal is slow turn, the value of \(\alpha\) equals 1/60; as the maneuver style of goal is escaping maneuver, the value of \(\alpha\) equals 1/20. Document [3] doesn't provide appropriate idea about how to calculate \(\alpha\) in the circumstance of target making complex maneuver. In order to solve the problem of how to determine the value of \(\alpha\) as target making complex maneuvering, this paper improves the traditional "current" statistical filtering algorithm based on the idea of multi-model fusion: first, setting much of different self-related time constant; then respectively execute filtering; finally, the various result of each model will be probably weighted. Specific steps are as follows:

1) Initialize the self-related time constant vectors \(\alpha(0) = [\alpha_1(0), \alpha_2(0), \ldots, \alpha_n(0)]\) and the model probability vectors \(\mu(0) = [\mu_1(0), \mu_2(0), \ldots, \mu_n(0)]\), in which \(\sum_{i=1}^{n} \mu_i(0) = 1, n\) is the number of models.

2) Establishing filter model for each \(\alpha_i (i = 1, 2, \ldots, n)\), and then using formula (9) and (10) to calculate \(\hat{X}_i(k|k)\) and \(P_i(k|k) (i = 1, 2, \ldots, n)\).

3) Using formula (1) to calculate the model probability vectors \(\mu(k) = [\mu_1(k), \mu_2(k), \ldots, \mu_n(k)]\) . In formula (13), the calculation formula of \(\lambda_j(k)\) is as formula (14), and the calculation formula of \(v_j(k), S_j(k)\) is as formula (7) and (9).
\[
\mu_j(k) = \frac{\lambda_j(k)\mu_j(k-1)}{\sum_{j=1}^{n} \lambda_j(k)\mu_j(k-1)}
\]
(13)

\[
\lambda_j(k) = \left[2\pi S_j(k)\right]^{\frac{1}{2}} e^{-\frac{1}{2}(x(k) - k_j)^T(k_j)^{-1}(x(k) - k_j)}
\]
(14)

4) Calculate \(\hat{x}(k|k)\) and \(P(k|k)\), the calculation formula is as (15) and (16).

\[
\hat{x}(k|k) = \sum_{j=1}^{n} \mu_j(k)\hat{x}_j(k|k)
\]
(15)

\[
P(k|k) = \sum_{j=1}^{n} \mu_j(k)P(k|k) + \sum_{j=1}^{n} \mu_j(k)\hat{x}_j(k|k)(\hat{x}_j(k|k) - \hat{x}(k|k))'
\]
(16)

5) It is need to judge whether the calculation result meet the requirement. If not, then jump step 2), else output \(\hat{x}(k|k)\) and \(P(k|k)\).

4 SIMULATION AND ANALYSIS

So as to verify the filtering effect of improving "current" statistical model proposed in this paper, compare the filtering effect of the new model with the traditional model.

4.1 Simulation conditions

The initial state of target is \(x_0 = [120000m, -426m/s, 2000m, 5m/s]\), the sampling interval \(0.2s\), target moving for 61 second. At the 31 second, the target maneuvering, the acceleration of X direction is \(5m/s^2\), and the acceleration of Y direction is \(-10m/s^2\); At the 38 second, the target maneuvering, the acceleration of X direction is \(-8m/s^2\), and the acceleration of Y direction is \(18m/s^2\); At the 49 second, the target maneuvering, the acceleration of X direction is \(10m/s^2\), and the acceleration of Y direction is \(-20m/s^2\).

4.2 Simulation experiment

Comparing the filtering effect of improved "current" statistical model with traditional "current" statistical model (\(\alpha\) respective equating \(1, 1/20, 1/60\) and \(1/90\)). Taking Y direction for example, the filtering effect is shown in Figure.1~Figure.4:

It can be seen from Figure.1~Figure.4 that the filtering effect of traditional filtering method is closer with new method, and it is difficult to distinguish the error between them. In order to quantify the advantage and disadvantage of the traditional methods and this method, filtering error parameter is calculated, and the specific result is shown in Table 1.

<table>
<thead>
<tr>
<th>method</th>
<th>Mean square variance</th>
<th>mean</th>
<th>error</th>
</tr>
</thead>
<tbody>
<tr>
<td>traditional method ((\alpha = 1))</td>
<td>6.76</td>
<td>-0.17</td>
<td>6.76</td>
</tr>
<tr>
<td>traditional method ((\alpha = 1/20))</td>
<td>6.66</td>
<td>-0.36</td>
<td>6.67</td>
</tr>
<tr>
<td>traditional method ((\alpha = 1/60))</td>
<td>6.94</td>
<td>0.21</td>
<td>6.94</td>
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<tr>
<td>traditional method ((\alpha = 1/90))</td>
<td>6.61</td>
<td>-0.06</td>
<td>6.61</td>
</tr>
<tr>
<td>this method</td>
<td>6.41</td>
<td>-0.24</td>
<td>6.41</td>
</tr>
</tbody>
</table>
It can be calculated from Table 1: the accuracy of this new method improves 5% compared with traditional method (as $\alpha = 1$); the accuracy of this new method improves 4% compared with traditional method (as $\alpha = 1/20$); the accuracy of this new method improves 8% compared with traditional method (as $\alpha = 1/60$); the accuracy of this new method improves 3% compared with traditional method (as $\alpha = 1/90$). The conclusion can be drawn: new method has higher accuracy than traditional method.

5 SUMMARY

The "current" statistical model filtering algorithm based on multi-model fusion proposed by the paper, effectively solves the calculation problem of self-related time constant $\alpha$ as in the circumstance of unknown target maneuver mode. The simulation result shows that the algorithm advanced by the paper improves the filtering accuracy compared with traditional algorithm which takes a single fixed value of $\alpha$, and can effectively achieve stable tracking for maneuvering targets.

REFERENCES