A Study on the Kinematics of the Model Support System of the Dual-Machine Wind Tunnel Test

Kaiming Xu\textsuperscript{1,2}, Zhu Rao\textsuperscript{1}, Zhonghua Liu\textsuperscript{1}, Hong Chen\textsuperscript{1,*}, and Dapeng Gao\textsuperscript{1}

\textsuperscript{1}Low Speed Aerodynamics Research Institute, China Aerodynamics Research and Development Center
\textsuperscript{2}School Electronics Engineering and Computer Science Peking University

Abstract. In the dual-machine wind tunnel test, the rear machine model support system is an open chain mechanism composed of rigid links in series with rotation and translation functions, and belongs to a multi-rigid body link system. The kinematic analysis of the system is to use the centroid of the posterior model as the end position of the mechanism, and analyze the relationship between the position and posture of the centroid of the model and the space of each joint variable.\textsuperscript{[1]} Therefore, in order to study the motion control system of the model support system and the dual-camera pose simulation, the kinematics analysis of the back-machine model support system is first required. Based on this analysis, a cloud image of the working space of the model support mechanism is established to ensure that it does not collide with the side wall of the wind tunnel.

1 Solution to the kinematic equations of the supporting system

The structure of the back machine model support device is shown in Figure 1. The movement mechanism includes:
1) Movement mechanism between the height direction;
2) Yaw mechanism 1, the yaw angle adjustment is realized by the electric cylinder through the rod mechanism;
3) Yaw mechanism 2, the yaw angle adjustment is realized by the electric cylinder through the rod mechanism;
4) Pitch motion mechanism 3, the angle is adjusted by the electric cylinder through the rod mechanism;
5) The horizontal linear adjustment mechanism 4 uses a hydraulic cylinder to adjust the length of the rod through a rod mechanism. The end is the fixed surface of the airplane model, and here is the end of the mechanism in this paper.
Using the model data relationship, the support mechanism coordinate system shown in Figure 2 can be established, with the base point $O_0'$ of the mechanism in the wind tunnel as the basic coordinate system. However, based on the particularity of the wind tunnel test, the test model should be located at the center point of the cross section of the wind tunnel test section as much as possible to minimize the influence of the wind tunnel moving wall on the test data during the blowing process. In order to adapt to this feature and reduce the amount of calculation for solving the kinematic equations to a certain extent, the position of the axis center point $O_0$ of the yaw mechanism 1 when the Z-axis kinematic mechanism is at a specific point is used as the basic coordinate system. At this specific position, when the rest of the axis mechanisms are all zero, the model is at the center point of the cross section of the wind tunnel test section.

By analyzing the positional relationship between the joints in the figure, the D-H parameter table is obtained, as shown in Table 1.
Table 1. D-H parameter table of the back model support system.

<table>
<thead>
<tr>
<th>Link</th>
<th>Joint angle ( \theta_i(\degree) )</th>
<th>Joint offset ( d_i(\text{mm}) )</th>
<th>Joint length ( a_i(\text{mm}) )</th>
<th>Joint torsion angle ( \alpha_i(\degree) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>( d_1 )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>( \theta_1 )</td>
<td>0</td>
<td>3000</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>( \theta_2 )</td>
<td>-300</td>
<td>500</td>
<td>( \pi/2 )</td>
</tr>
<tr>
<td>4</td>
<td>( \theta_3 )</td>
<td>0</td>
<td>0</td>
<td>( \pi/2 )</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>( d_5 )</td>
<td>500</td>
<td>0</td>
</tr>
</tbody>
</table>

According to the structure, the following can be known:

1) Sideslip angle \( \beta = \theta_1 + \theta_2 \);

2) Angle of attack \( \alpha = \theta_3 \);

The frame transformation of the coordinate system \( \sigma_1(O_1x_1y_1z_1) \) to \( \sigma_0(O_0x_0y_0z_0) \) is shown as follows:

\[
T_1 = \text{Rot}(0,0,0)\text{Tran}(0,0,d_1) = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & d_1 \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (1)

The frame transformation of the coordinate system \( \sigma_2(O_2x_2y_2z_2) \) to \( \sigma_1(O_1x_1y_1z_1) \) is shown as follows:

\[
T_2 = \text{Rot}(z_2,\theta_2)\text{Tran}(a_2,0,0) = \begin{bmatrix}
\cos \theta_2 & -\sin \theta_2 & 0 & 0 \\
\sin \theta_2 & \cos \theta_2 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
1 & 0 & 0 & a_2 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (2)

Because the joint offset is \( d_4 \), the frame transformation of the coordinate system \( \sigma_3(O_3x_3y_3z_3) \) to \( \sigma_2(O_2x_2y_2z_2) \) is shown as follows:

\[
T_3 = \text{Rot}(z_3,\theta_3)\text{Tran}(a_3,0,0)\text{Tran}(0,0,d_3) = \begin{bmatrix}
\cos \theta_3 & 0 & \sin \theta_3 & a_3 \cos \theta_3 \\
\sin \theta_3 & 0 & -\cos \theta_3 & a_3 \sin \theta_3 \\
0 & 1 & 0 & d_3 \\
0 & 0 & 0 & 1
\end{bmatrix}
\] (3)

And the frame transformation of the coordinate system \( \sigma_4(O_4x_4y_4z_4) \) to \( \sigma_3(O_3x_3y_3z_3) \) is shown as follows:
\[ T_4 = \text{Rot}(z_4, \theta_4) = \begin{bmatrix}
\cos \theta_4 & 0 & \sin \theta_4 \\
\sin \theta_4 & 0 & -\cos \theta_4 \\
0 & 1 & 0
\end{bmatrix} \] (4)

The distance between the center line of the pitch joint axis and the male vertical line of the center axis of the horizontal adjustment mechanism is \( d_5 \), i.e., the joint offset is \( d_5 \). The frame transformation of the coordinate system \( \sigma_5(O_5x_5y_5z_5) \) to \( \sigma_4(O_4x_4y_4z_4) \), i.e., the coordinate transformation matrix is:

\[ T_5 = \text{Tran}(0,0,a_5)\text{Tran}(d_5,0,0) = \begin{bmatrix}
1 & 0 & 0 & a_5 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & d_5 \\
0 & 0 & 0 & 1
\end{bmatrix} \] (5)

Therefore, if the RPY angle of the posterior model of \( \sigma_5(O_5x_5y_5z_5) \) is \( \text{RPY}(\alpha, \beta, \gamma) \), then the 3×3 attitude matrix is:

\[ \text{RPY}(\alpha, \beta, \gamma)_{3\times3} = \begin{bmatrix}
\cos \alpha & \sin \alpha & 0 \\
-s\sin \alpha & \cos \alpha & 0 \\
0 & 0 & 1
\end{bmatrix} \]

\[ = \begin{bmatrix}
c\theta_4 (c\theta_3 c\theta_2 - s\theta_3 s\theta_2) & c\theta_3 s\theta_2 + c\theta_2 s\theta_3 & s\theta_3 (c\theta_3 c\theta_2 - s\theta_3 s\theta_2) \\
c\theta_3 s\theta_2 + c\theta_2 s\theta_3 & c\theta_3 c\theta_2 - s\theta_3 s\theta_2 & s\theta_2 s\theta_3 - c\theta_2 c\theta_3 \\
s\theta_3 & s\theta_2 s\theta_3 - c\theta_2 c\theta_3 & c\theta_2 c\theta_3 + s\theta_2 s\theta_3
\end{bmatrix} \] (6)

The position \( P = (p_x, p_y, p_z) \), then the pose matrix is:

\[ \text{RPY}(\alpha, \beta, \gamma) = \begin{bmatrix}
\cos \alpha & \sin \alpha & 0 & p_x \\
-s\sin \alpha & \cos \alpha & 0 & p_y \\
0 & 0 & 1 & p_z
\end{bmatrix} \]

\[ = \begin{bmatrix}
c\theta_4 (c\theta_3 c\theta_2 - s\theta_3 s\theta_2) & c\theta_3 s\theta_2 + c\theta_2 s\theta_3 & s\theta_3 (c\theta_3 c\theta_2 - s\theta_3 s\theta_2) \\
c\theta_3 s\theta_2 + c\theta_2 s\theta_3 & c\theta_3 c\theta_2 - s\theta_3 s\theta_2 & s\theta_2 s\theta_3 - c\theta_2 c\theta_3 \\
s\theta_3 & s\theta_2 s\theta_3 - c\theta_2 c\theta_3 & c\theta_2 c\theta_3 + s\theta_2 s\theta_3
\end{bmatrix} \]

\[ \begin{bmatrix}
a_2c\theta_2 + a_3c\theta_4 (c\theta_2 c\theta_3 - s\theta_2 s\theta_3) + d_3s\theta_4 (c\theta_2 c\theta_3 - s\theta_2 s\theta_3) + a_3c\theta_2 c\theta_3 - a_3s\theta_2 s\theta_3 \\
a_2cs + a_3c\theta_4 (c\theta_2 s\theta_3 + c\theta_2 s\theta_3) + d_3s\theta_4 (c\theta_2 s\theta_3 + c\theta_2 s\theta_3) + a_3\theta_2 s\theta_3 + a_3\theta_2 c\theta_3 \\
0 & 0 & 1
\end{bmatrix} \] (7)

In the above formula, \( \alpha = \cos \alpha, \beta = \sin \beta \), the same is true for angles \( \beta \) and \( \gamma \). Then in the base coordinate system \( \sigma_0(O_0x_0y_0z_0) \), the pose matrix of the posterior model is:

\[ \begin{bmatrix}
O_{3\times3} & P_{1\times3} \\
0 & 1
\end{bmatrix} = T TT T T T T T T T T RPY(\alpha, \beta, \gamma) = \begin{bmatrix}
0 & T RPY(\alpha, \beta, \gamma)
\end{bmatrix} \] (8)
2 Solution to support system workspace

The working space of the support system refers to the collection of points that can be reached in space by the reference point on the end effector of the support system. The working space of the support system represents the range of motion of the end vehicle model and is an important index for evaluating the performance of the model support system. In view of the particularity of the wind tunnel test and the space limitation of the wind tunnel test section, the working space of the support system is an important issue that must be considered.

The support system based on the posterior model is a multi-degree-of-freedom series open-chain structure, and there is a coupling relationship between the respective degrees of freedom, and because it has 5 degrees of freedom, the analytical calculation is more cumbersome, so the calculation of the workspace is more complicated. In order to solve this problem, many researchers have proposed many solutions, mainly including analytical methods, graphical methods and numerical methods. Among the numerical methods, the Monte Carlo method is solved by randomly sampling the motions of each joint, which is relatively easy to implement. Therefore, this paper uses the Monte Carlo method to solve the work space of the support system.

We call the space occupied by the reachable point of the aircraft model at the end of the support system the total working space, which is noted as $W(P)$. This relationship can be expressed as:

$$W(P) = \{P(q) : q \in Q\} \subseteq \mathbb{R}^3$$  \hspace{1cm} (11)

In the formula, $q$ is the joint variable, $Q$ is the joint space, and $P(q)$ is the generalized joint variable function. A certain amount of random quantities that meet the requirements of joint changes are assigned to the joint variables through uniform distribution, so as to obtain a cloud image of the workspace composed of random points, which constitutes the Monte Carlo workspace.\cite{6,7}

Solution to the support system Monte Carlo method work space mainly follows the following steps:

1) Analyze the mechanical configuration of the posterior support system and the space size of the wind tunnel test section, and define the joint space of each joint. Then define the same number of random values for each joint in the joint space through uniform distribution;

2) Solve each group of joint variable values through the positive kinematics equation of the support system to obtain the position value of the end execution point of the group in the base coordinate system;

3) Draw the positions of all the end execution points, and generate a cloud map of the workspace of the supporting mechanism.
3 Workspace cloud image simulation

After the analysis of the mechanism configuration of the rear machine support system and the space size of the wind tunnel test section, subject to the constraints of the mechanism and working conditions, the variable range and constraint relationship of each joint variable are obtained. Then use a uniformly distributed random function for each joint space to take 20,000 points, bring them into the first section of the kinematics positive solution equation and draw a cloud image. The following is the cloud image of the workspace simulation result.

![Flow direction of wind hole](image)

**Figure 3-6.** Work space of the support system for the five-degree-of-freedom posterior model.

![Projection of working space on XOY plane](image)

**Figure 3-7.** Projection of working space on XOY plane.
The simulation results show that the working space of the support system is obtained in the set joint space and after the joint constraints are added. The scope is controlled within the space allowed by the test section of the wind tunnel, which prevents the support system mechanism from touching the wall of the wind tunnel and realizes the function of software touch wall protection. The working space has a wide range, which meets the requirements of wind tunnel test sports, and the support position and angle are more flexible, which can meet the requirements of various models and tests.

4 Conclusion

This paper mainly analyzes the kinematics of the back-machine model support system. First, the kinematics modeling and analysis methods of the supporting mechanism are used to establish the coordinate system of the posterior model support system using the D-H table, and the D-H parameter table of the supporting mechanism is listed and the positive kinematics equation of the supporting system is derived. Finally, the Monte Carlo method
is used to solve the work space of the support system, and the work space of the support system is simulated through the back-machine model, which avoids the support system mechanism from touching the wind tunnel wall. This proves that the support system mechanism meets the movement requirements of the support system in the wind tunnel test.

References