Exact Solutions for the (2 + 1)-Dimensional Nizhnik-Novikov-Veselov System

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Abstract. Finding the exact solutions of nonlinear partial differential equations plays an important part in nonlinear problems. We use the complete discrimination system for polynomials [1-6] to get exact solutions for the (2 + 1)-dimensional Nizhnik-Novikov-Veselov system in this paper. We divide different situations to discuss the solutions and get three kinds of solutions.

Keywords: (2+1)-dimensional Nizhnik-Novikov-Veselov system; complete discrimination system for polynomials; exact solutions.

1 Introduction

There are many areas need the travelling wave solutions of nonlinear equation, such as optical fibers[7-8], biology [9-10], solid state physics[11]. Base on many methods of nonlinear equation, such as the Hirota method and the Tanh function method[12-13], some exact solutions had be got. But it’s not complete. With the development of nonlinear scientific research, more and more people focus on the study of the (2+1)-dimensional Nizhnik-Novikov-Veselov system. Some researchers of nonlinear problems give some exact solutions of (2+1)-dimensional Nizhnik-Novikov-Veselov system[14-15]. In this paper we take the complete discrimination system for polynomials to get new exact solutions which is double periodic solution of elliptic functions.

2 (2+1) dimension nizhnik-novikov-veselov

In this paper we consider the asymmetry of (2 + 1) dimension Nizhnik-Novikov-Veselov

\[ u_t + u_{xxx} - 3v_xu - 3vu_x = 0 , \]  
\[ u_x = v_y . \]  

Take the traveling wave transform

\[ u(x,y,t) = u(\varphi), \ v(x,y,t) = u(\varphi), \ \varphi = x + ly - st . \]  

We get

\[ lv'''' - (3c_0 + sl)v' + 3l(v^2)' = 0 , \]
where \( c_0 \) is the integral constant.

We take integral to \( \varphi \),

\[
(\varphi - \varphi_0)^2 - (3c_0 + sl)v^2 - 2lv^3 + c = 0,
\]

where \( c \) is the integral constant. We take following transformation

\[
w = 2^{1/3}(v + 1/6), \quad \varphi_1 = 2^{1/3}\varphi .
\]

We have

\[
(w\varphi_1)^2 = w^3 + pw^2 + qw + r ,
\]

where

\[
p = \begin{cases} 
\frac{[3c_0+(s-1)]^2}{3} - \frac{1}{1}, \\
\frac{[-6c_0+(1-2s)]^2}{3} - \frac{1}{108}.
\end{cases}
\]

Then we can get the integral form

\[
\pm(\varphi_1 - \varphi) = \int \frac{dw}{\sqrt{F(w)}},
\]

where \( F(w) = w^3 + pw^2 + qw + r \).

The complete discrimination system for polynomials is given by

\[
\Delta = -27\left(\frac{2d_2}{27} + d_0 - \frac{d_1d_2}{3}\right)^2 - 4\left(d_1 - \frac{d_2^2}{3}\right)^3, D_1 = d_1 - \frac{d_2^2}{3}.
\]

According to the complete discrimination system for polynomials, we can get the exact solutions for the asymmetry of \((2 + 1)\) dimension Nizhnik-Novikov-Veselov.

### 3 The solutions of the asymmetry of \((2 + 1)\) dimension Nizhnik-Novikov-Veselov

#### 3.1 Case 1. Singular solutions

If \( \Delta = 0, D_1 < 0 \). We have

\[
F(w) = (w - \alpha)^2(w - \beta),
\]

then

\[
v_1 = 2^{-1/3}\left[(\beta - \alpha)\tan^2\left(\frac{1}{27}\varphi - \varphi_0\right) + \beta]\right] - \frac{3c_0+sl}{6},
\]

where \( \alpha, \beta \) are real numbers and \( \alpha < \beta \).

If \( \Delta = 0, D_1 = 0 \). We have

\[
F(w) = (w - \alpha)^3,
\]

then

\[
v_2 = 2^{-1/3}\left[4(2\varphi - \varphi_0)^{-2} + \alpha\right] - \frac{3c_0+sl}{6}.
\]
3.2 Case 2. Solitary wave solutions

If $\Delta = 0 , D_1 < 0$. We have

$$F(w) = (w - \alpha)^2(w - \beta),$$  \hfill (14)

then

$$v_3 = 2^{-\frac{1}{3}}[(\beta - \alpha) \tanh^2 \left(\frac{\sqrt{\beta - \alpha}}{2} \left(2^\frac{1}{3} \varphi - \varphi_0\right)\right) + \beta] - \frac{3c_0 + sl}{6},$$  \hfill (15)

$$v_4 = 2^{-\frac{1}{3}}[(\beta - \alpha) \coth^2 \left(\frac{\sqrt{\beta - \alpha}}{2} \left(2^\frac{1}{3} \varphi - \varphi_0\right)\right) + \beta] - \frac{3c_0 + sl}{6},$$  \hfill (16)

where $\alpha, \beta$ are real numbers and $\alpha > \beta$.

3.3 Case 3. Double periodic solutions

If $\Delta > 0 , D_1 < 0$. We have

$$F(w) = (w - \alpha)(w - \beta)(w - \gamma),$$  \hfill (17)

when $\alpha < \epsilon < \beta < \gamma$ , take $\epsilon = \alpha + (\beta - \alpha) \sin^2 \theta$, we get

$$v_5 = 2^{-\frac{1}{3}}[\alpha + (\beta - \alpha) \sn^2 \left(\frac{\sqrt{\gamma - \alpha}}{2} \left(2^\frac{1}{3} \varphi - \varphi_0\right), k\right)] - \frac{3c_0 + sl}{6},$$  \hfill (18)

when $\alpha < \beta < \gamma < \epsilon$ , take $\epsilon = \frac{-\beta \sin^2 \theta + \gamma}{\cos^2 \theta}$. we get

$$v_6 = 2^{-\frac{1}{3}}[\gamma - \beta \sn^2 \left(\frac{\sqrt{\gamma - \alpha}}{2} \left(2^\frac{1}{3} \varphi - \varphi_0\right), k\right)] - \frac{3c_0 + sl}{6},$$  \hfill (19)

where $\alpha, \beta, \gamma$ are real numbers and $k^2 = \frac{\beta - \alpha}{\gamma - \alpha}$

If $\Delta > 0$. We have

$$F(w) = (w - \alpha)(w^2 + pw + q).$$  \hfill (20)

then when $w > \alpha$ ,

$$v_7 = 2^{-\frac{1}{3}}[\alpha + \frac{2\sqrt{h}}{1 + \cn^2 \left(\frac{2^\frac{1}{3} \varphi - \varphi_0}{k}\right)}] - \frac{3c_0 + sl}{6},$$  \hfill (21)

where $h = \alpha^2 + p\alpha + q$ ; $k^2 = \frac{1}{2} \left(1 - \frac{\alpha + p}{\sqrt{h}}\right)$.

4 Conclusions

In this work, by using the complete discrimination system for polynomials, we can learn a new clear way to get the wave exact solutions of the (2+1)-dimension Nizhnik-Novikov-Veselov system. We divided different situations to get the solitary wave solution, rational solution and double periodic solutions. The complete discrimination system for polynomials to solve the problems of nonlinear partial differential equation to get the exact solutions should be realized.
References