Solving the Nominal-the-Best Decision-Making Problem by Grey-Fuzzy Linguistic Method

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Abstract. This article combines two well-developed notions: the grey relations method (GRM) and fuzzy linguistic scoring technique, for solving the problem of qualitative and quantitative criterions coexistence due to the original GRM being unable to effectively handle the simple qualitative-orientated solution. Further, an empirical process adopting GRM with the fuzzy technique has successfully proved the ratings of every alternative with two types of objectives can be an optimal approach. This conclusion can be utilized to help solve other related computing issues.

1 Introduction

So far, the Multiple Criteria Decision-Making (MCDM) approach has been rapidly expanding and applied in many fields due to MCDM procedures broadly conforming to the resolution of conflict situations. Academics have been committed to improving MCDM since Zadeh [1] in the mid-1960’s proposed the ‘fuzzy set’ that considered the things in themselves are a set with chaotic and hard to quantify qualities. Pawlak also mentioned the ‘rough set’ [2] that primary classifies the group with numerous elements. Further, Deng’s ‘grey theory’ [3] modified Zadeh’s ‘fuzzy’ conception [1], recognizing the things themselves are a ‘hazy set’ containing fewer entities. Thus, the wide development of MCDM has become a contentious topic in academia. The existing literature has discussed solutions using qualitative judgments. However, there is a lack of effectively dealt with improvement numerous commonly used MCDM methods with simplicity pattern.

Therefore, this study combines two mature notions-the grey relations method (GRM) and fuzzy linguistic variable scoring technique, to deal with the coexisting situation of qualitative and quantitative criterions. The fundamentals of GRM are illustrated in the next section, and a case with fuzzy semantic judgment in GRM is shown. Finally, a conclusion is given in the last section.

2 Preliminaries

GRM, proposed by Deng [3], has had numerous applications in the last three decades. Compared with statistical methods requiring relatively large sample sizes and a crisp result, the concept of GRM is based on small samples and a hazy set with an arbitrary distribution. GRM basically solves unknown issues or incomplete problems. Liu and Forrest [4] found GRM is widely used in academic and practical issues. In these years, the development of
GRM has extended its use, for example, emphasizing the interval numbers of the grey relations decision-making issue [5], applying the multi-attribute COmplex PRoportional ASsessment of alternatives (COPRAS) into GRM [6], and added an intuitionistic fuzzy setting notion [7]. This section briefly introduces the principles of GRM.

Definition 1. According to Deng [8], four axioms must be satisfied in the grey relationship:

1) Interval norming: \(0 < \gamma(a_0, a_n) \leq 1, \forall m\); \(\gamma(a_0, a_n) = 1 \iff a_0 = a_n\);

2) Symmetric duality: In grey relations, if there are only two series of set alternative, then \(\gamma(a_0, a_n)\) could be recognized as a comparison of symmetric duality. \(a_0, a_n \in X\);

3) Wholeness: The comparison does not hold all the time in symmetry due to the differences of each sequence.

4) Approachability: While \(\gamma(a_0(q), a_n(q))\) increases, at the same time, \(a_0(q), a_n(q)\) decreases.

Once it meets the above-mentioned conditions, the grey relations illustrated as following terms is then proposed:

\[
\gamma(a_0(q), a_n(q)) = \min_{q} \min \left[ \left| x_0(q) - x_n(q) \right| + \zeta \max_{q} \left| x_0(q) - x_n(q) \right| \right] \leq \left| x_0(q) - x_n(q) \right| + \zeta \max_{q} \left| x_0(q) - x_n(q) \right| \tag{1}
\]

where \(x_0(q) - x_i(q) = \Delta_i(q)\), and \(\zeta \in [0, 1]\) which is commonly used 0.5.

Definition 2. \(\gamma\) can be considered the grey relationship index if \(\gamma(x_0, x_n)\) meets the grey relations axioms.

Definition 3. While \(\Gamma\) is fully denoted as the grey relations axioms, \(\gamma \in \Gamma\) conforms to the grey relations axioms, and X is a factor of the set in grey relations, so \((X, \Gamma)\) can be seen as a grey value while \(\gamma\) is the actual value for \(\Gamma\).

Definition 4. Let \((X, \Gamma)\) be the grey value, and if the actual value \([\gamma(x_0, x_n), \gamma(x_0, x_p), \ldots \gamma(x_0, x_n)]\) is satisfied \([\gamma(x_0, x_n) > \gamma(x_0, x_p) > \ldots > \gamma(x_0, x_n)]\), then the grey relations order sequence is \(x_a > x_b > \ldots > x_n\).

3 Applying fuzzy linguistic variables in the grey relations ranking procedure

To better deal with the nominal-the-best issue, the GRM employs the following measurement:
\[
\gamma[a_0(q), a_n(q)] = \frac{\min_{all_n} \{\text{Objective}, x_n^{(0)}(q)\}}{\max_{all_n} \{\text{Objective}, x_n^{(0)}(q)\}}, \quad 1 \leq n \leq k.
\]

where \(x_n^{(0)}(q)\) is the value after grey relations generation; \(\min_{all_n}\) is the smallest value of \(x_n^{(0)}(q)\): \(x_1^{(0)}(q), x_2^{(0)}(q), \cdots, x_m^{(0)}(q)\); \(\max_{all_n}\) is the largest value of \(x_n^{(0)}(q)\): \(x_1^{(0)}(q), x_2^{(0)}(q), \cdots, x_m^{(0)}(q)\); Objective is the chosen value of \(x_n^{(0)}(q)\).

In other words, the element of the nominal-the-best problem in GRM seeks the difference value between each attribute and then decides the best solution (meet the requirements). However, if each attribute has no significant difference, or difficult to quantify (e.g. compactness, fashion, rosy), the existing measurement obviously cannot handle this problem effectively. To solve the problem, therefore, this article utilizes fuzzy linguistic variables in the GRM solution decision-making procedure. The basic concepts of the fuzzy linguistic variable and the procedure are given below.

**Step 1: Assign a fuzzy linguistic variable value**

Semantic assessment as a rating technique in fuzzy approach was developed by Zadeh [9]. In the process of the fuzzy decision-making method, people often use uncertain terms to describe things e.g. ‘nice looking, fat, colorful’ and quantify them as comparable numbers. This partly uses Delgado et al. [10] and Herrera and Martínez [11]’s definition as follows.

Let \(A = \{a_1, a_2, \cdots a_n\}\) is a term of an aggregated linguistic set, and the computation model is below:

First, the term with an aggregated set is given to \(A_{mn} = (a_{nm}, b_{nm}, c_{nm})\), where suppose \(c_{nm}\) is a functional attribute. For ascertaining a collective quantifying value for every alternative \(k_n\) with

\[
C_n = \left(\frac{1}{q} \sum_{m=1}^{q} a_{nm}, \frac{1}{q} \sum_{m=1}^{q} b_{nm}, \frac{1}{q} \sum_{m=1}^{q} c_{nm}\right),
\]

being \(q=\) decision maker numbers. Therefore, a fuzzy linguistic vector sets based on the Eucilidean distance to each \(C_n\) for obtaining the results in the primary term set \(A\) is represented as

\[
d(f_s, C_n) = \sqrt{w_1(a_s - a_n)^2 + w_2(b_s - b_n)^2 + w_3(c_s - c_n)^2}
\]

where \((a_s, b_s, c_s)\) and \((a_n, b_n, c_n)\) are the attribute functions of \(f_s\) and \(C_n\). In addition, \(w_1\) to \(w_3\) are denoted as the weights of each alternative solutions reflecting fuzzy set \(a, b\) and \(c\), notably, the weighting must meets \(w_q \in [0,1]; \text{and } \sum_m W_q = 1\).

So, we define \(A = \{a_1 : N, a_1 : VL, a_2 : L, a_3 : M, a_4 : H, a_5 : VH, a_6 : H, a_7 : P\}\),

each linguistic variable is labeled with a triangular function. The semantic values of each variable are assigned as follows:

\[
VP = (0,0,10); P = (0,10,30); MP = (10,30,50);
\]

\[
F = (30,50,70); MG = (50,70,90); G = (70,90,100);
\]

\[
VG = (90,100,100).
\]
where \( VP \) (Very Poor), \( P \) (Poor), \( MP \) (Medium Poor), \( F \) (Fairly), \( MG \) (Medium Good), \( G \) (Good), and \( VG \) (Very Good) are defined as linguistic variables as above.

**Step 2: Set a GRM matrix**

As the general type of other MCDM, the GRM matrix is expressed as below.

\[
X = \begin{pmatrix}
    a_{11} & \cdots & a_{1n} \\
    \vdots & \ddots & \vdots \\
    a_{m1} & \cdots & a_{mn}
\end{pmatrix},
\]

where \( a_{mn} \) is denoted as an evaluation of GRM to \( m \)-th alternative with orders of the index.

**Step 3: Change the “Nominal-the-better” value into a “Larger the better” value**

After obtaining the fuzzy rating numbers of each linguistic variable, we may directly compute every attribute in terms of the “Larger the better” measurement of the GRM.

\[
\gamma(a_0(q), a_n(q)) = \frac{x_n^{(0)}(q)}{\max_{n'} x_n^{(0)}(q)}, \quad 1 \leq n \leq k .
\]

**Step 4: Decide the relatively better solution**

### 4 A practical example of applying fuzzy semantic judgement by GRM

This section expresses a decision-making process example with fuzzy semantic judgment by GRM. Apparel firm X has five T-shirt products and one has to choose as a promotional good (denoted as \( T_1 \)-\( T_5 \)) to stimulate sales for Xmas (generally in November 7- December 25). Firm X applies four qualitative semantic criterions for the making decision makers’ choice \( C_1 \) (Suits younger generation), \( C_2 \) (Brightness), \( C_3 \) (Fashionable style), and \( C_4 \) (Identity). In addition, three quantitative dimensions \( C_5 \) (Sales amount), \( C_6 \) (Fabrication cost), and \( C_7 \) (Inventory turnover) have to be listed as the decision-making issue. Each criterion is assumed to have equal weight. Prior to forming a GRM ranking procedure, we must quantify \( C_1\)-\( C_4 \) to be the comparable value (denote as \( C_{qual} \)) in terms of the fuzzy linguistic scoring approach (Eq.2). Table 1-3 show the matrixes of the fuzzy linguistic scoring results by three industry professionals using qualitative judgment.

**Table 1. Fuzzy linguistic scoring results by decision maker 1.**

<table>
<thead>
<tr>
<th>DM</th>
<th>Linguistic ratings</th>
<th>Aggregated ratings</th>
<th>Fuzzy</th>
<th>Defuzzified</th>
<th>Normalized</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>G MG G VG</td>
<td>(56,70,78)</td>
<td>51.0</td>
<td>0.27</td>
<td></td>
</tr>
<tr>
<td>T2</td>
<td>G F MG G</td>
<td>(44,60,72)</td>
<td>44.0</td>
<td>0.26</td>
<td></td>
</tr>
<tr>
<td>T3</td>
<td>P VG G G</td>
<td>(46,58,66)</td>
<td>42.5</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>T4</td>
<td>MP MP MG</td>
<td>(28,44,58)</td>
<td>32.5</td>
<td>0.19</td>
<td></td>
</tr>
<tr>
<td>T5</td>
<td>F VP MG P</td>
<td>(16,26,40)</td>
<td>20.5</td>
<td>0.10</td>
<td></td>
</tr>
</tbody>
</table>

**Table 2. Fuzzy linguistic scoring results by decision maker 2.**

<table>
<thead>
<tr>
<th>DM</th>
<th>Linguistic ratings</th>
<th>Aggregated ratings</th>
<th>Fuzzy</th>
<th>Defuzzified</th>
<th>Normalized</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>VG VP G MG</td>
<td>(42,52,60)</td>
<td>38.5</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>T2</td>
<td>MP F VP G</td>
<td>(22,34,46)</td>
<td>25.5</td>
<td>0.20</td>
<td></td>
</tr>
</tbody>
</table>
Thus we may ascertain the average fuzzy linguistic scoring result of five T-shirts as
\[ C_{\text{qual}} = \left\{ \left\| T_1 \right\| = 0.27, \left\| T_2 \right\| = 0.24, \left\| T_3 \right\| = 0.25, \left\| T_4 \right\| = 0.20, \left\| T_5 \right\| = 0.11 \right\}. \]

Subsequently, a GRM decision-making matrix \( U \) with qualitative (\( C_{\text{qual}} \)) and quantitative (\( C_5-C_7 \)) criterions has the following settings.
\[
U = \begin{bmatrix}
0.27 & 6.12 & 4.69 & 0.80 \\
0.24 & 4.38 & 5.00 & 0.60 \\
0.25 & 5.50 & 4.17 & 0.55 \\
0.20 & 7.04 & 5.82 & 0.73 \\
0.11 & 5.76 & 4.85 & 0.48 \\
\end{bmatrix}
\]

Compute the difference sequence between each alternative, and we obtain matrix \( D \) by applying matrix \( U \).
\[
D = \begin{bmatrix}
0.73 & 4.12 & 1.69 & 3.20 \\
0.76 & 2.38 & 2.00 & 3.40 \\
0.75 & 3.50 & 1.17 & 3.45 \\
0.80 & 5.04 & 2.82 & 3.27 \\
0.89 & 3.76 & 1.85 & 3.52 \\
\end{bmatrix}
\]

From matrix \( D \), we get matrix \( C \) of the grey coefficient result by Eq. (3):
\[
C = \begin{bmatrix}
1.0000 & 0.4895 & 0.7720 & 0.5682 \\
0.9909 & 0.6633 & 0.7190 & 0.5490 \\
0.9939 & 0.5399 & 0.8808 & 0.5444 \\
0.9789 & 0.4299 & 0.6086 & 0.5613 \\
0.9531 & 0.5175 & 0.7437 & 0.5381 \\
\end{bmatrix}
\]

Now, we may calculate the grey relations between every alternative result used by Eq. (1). In this case, \( \rho = 0.5 \), we get the ranking of \( T_3 (0.7397) \succ T_2 (0.7305) \succ T_1 (0.7074) \succ T_5 (0.6881) \succ T_4 (0.6447) \). Finally, \( T_3 \) is chosen to be the Xmas promotional goods for firm X.

<table>
<thead>
<tr>
<th>DM3</th>
<th>Linguistic ratings</th>
<th>Aggregated ratings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( C_1 )</td>
<td>( C_2 )</td>
</tr>
<tr>
<td>( T_1 )</td>
<td>G</td>
<td>VG</td>
</tr>
<tr>
<td>( T_2 )</td>
<td>P</td>
<td>G</td>
</tr>
<tr>
<td>( T_3 )</td>
<td>F</td>
<td>MP</td>
</tr>
<tr>
<td>( T_4 )</td>
<td>F</td>
<td>G</td>
</tr>
<tr>
<td>( T_5 )</td>
<td>VP</td>
<td>F</td>
</tr>
</tbody>
</table>

Table 3. Fuzzy linguistic scoring results by decision maker 3.
5 Conclusion

This article presented the mixed GRM and fuzzy linguistic scoring technique and successfully solved the problem of qualitative and quantitative criterions coexistence. Further, the empirical results strengthened the effectiveness and practical utility of our purposed model due to the nominal-the-best problem handled by original GRM not being able to resolve the qualitative issue, which is difficult to quantify. The following directions should be seeking further comprehensive applications in other computing fields.

6 Acknowledgments

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References