Algorithm of Contour Contour Contour of Complex Surface in NC Machining

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Abstract. A residual height trajectory algorithm of complex surface plane envelope is proposed for NC grinding of complex surface. Firstly, based on the differential geometric relationship between the grinding wheel end plane and the workpiece surface, the properties of the plane enveloping surface are analyzed; secondly, the residual height based on the space vector is calculated; Secondly, the algorithm is proposed by using the tangent characteristics of the contour line of the equal residual height; Finally, the tool path of the ellipsoid convex surface is calculated, and the characteristics of the two different tool paths are compared and analyzed, and it is proved that the proposed method is effective The tool path has higher machining efficiency at the residual height.

1 Introduction

When the surface of multi axis CNC machining workpiece is a complex surface, it is necessary to calculate the tool path on the surface. The above steps not only affect the efficiency of NC machining, but also have an important impact on the machining accuracy [1]. In actual production, CAD / CAM and other related software are generally used for tool path interpolation calculation. Common software includes UG, VERICUT and MasterCAM [2]. The general operation steps are as follows: firstly, the 3D surface model is established in the software (the model can fully represent the surface to be machined); secondly, the processing conditions such as processing parameters, tool model import and tool feeding mode are set; finally, according to the characteristics of the machine tool, the tool path can be calculated by post-processing in the software, and the NC machining program suitable for machine tool production is generated All steps of surface machining. Up to now, in order to improve the processing efficiency and quality, commercial software and scholars at home and abroad have given a variety of trajectory interpolation algorithms to adapt to different processing conditions such as different workpiece surface characteristics, tool profiles and machine parameters, such as the common parameter method and the constant residual height method [3].

However, the above processing methods are mainly for CNC turning or CNC milling, few companies or scholars focus on grinding, especially for variable positioning base circle grinding processing tool path interpolation algorithm research. Therefore, it is difficult to find relevant results on the market to improve the grinding efficiency and accuracy [4]. Therefore, in this paper, according to the actual production requirements, using the space
vector coordinates, a new planning and calculation scheme of plane envelope residual height trajectory is given.

2 Mathematical model of convex surface machining by plane enveloping

2.1 Properties of plane enveloping surface

The grinding tool is a grinding wheel, and the required curved surface needs to be swept by its end face along the preset machining path. The process can be regarded as the process of enveloping surface crossed by a certain curve of plane extended surface by using mathematical micro element analysis method. In this process, the plane is tangent to the cutting complex surface at all times, and the machined envelope surface coincides with the surface formed by the single parameter tangent plane family of curves on the complex surface \[4, 5\].

Take a characteristic line, the line is the envelope surface generated by the single parameter tangent plane family of complex surface and any plane in the plane family. All the characteristic lines form a set of envelope surface, so the solution of envelope surface can be obtained by calculating the characteristic line equation \[4\]. The envelope surface is transformed into feature line.

Arbitrarily select a curve, \( S = \tilde{S}(u, v) \), on the surface, \( s \) as arc parameter, which is related to the arc length, \( s \in (s_1, s_2) \). Assuming that the normal vector of the surface where each point of the curve, \( \tilde{S} \), is located is \( \vec{N} = \vec{N}(s) \), then a family of planes can be obtained tangent to this curve, \( \{ \pi_i \} (i = 1, 2, ..., n) \), and any plane can be represented by the following formula \[6\]:

\[
\vec{N}(s) \cdot (\vec{\rho} - \vec{C}(s)) = 0
\]

(1)

Where: point vector \( \vec{\rho} \) belongs to tangent plane.

An orthogonal shape \( \{ \vec{e}_1, \vec{e}_2, \vec{e}_3 \} \) is selected for the curve \( \vec{C} = \vec{C}(s) \) on the surface, \( \vec{e}_1 \) is tangent to the curve \( \vec{C} = \vec{C}(s) \). It can be concluded that:

\[
\vec{e}_1 = \frac{d\vec{C}(s)}{ds}
\]

(2)

\[
\vec{e}_2 = \vec{e}_1 \times \vec{e}_3
\]

(3)

\[
\vec{e}_3 = \vec{N}
\]

(4)

Where: the vector \( \vec{e}_3 \) belongs to the above curve, which is perpendicular to the surface at the corresponding point of the curve.

If the envelope is represented by vector, the envelope is:

\[
\vec{\rho}(s, t) = \vec{C}(s) + t\vec{\nu}
\]

(5)
Where: $\vec{v}$ is the feature vector of envelope plane and $t$ is the length of $\vec{v}$.

For the vector $\vec{v}$ perpendicular to the envelope surface, if the lateral torsion $\tau_s$ is taken as zero, $\vec{v} = -k_n\vec{e}_2$, and $\vec{e}_2 = \vec{e}_1 \times \vec{e}_3$, in short, if the curve $\vec{C} = \vec{C}(s)$ belongs to a plane, then the tangent vector $\vec{e}_1$ and normal vector $\vec{N}$ of the points on the curve are the only two factors that affect the value of the eigenvector, so as to simplify the steps of calculating the characteristic line.

2.2 Calculation of equal residual height point

The essence of feature line expression of envelope surface is to select a group of feature lines, which can be regarded as the discrete state of envelope surface. A certain point on the feature line is named as equal residual height point (CSH – Point), and the unique point with equal residual height can be found to represent the line [7].

As shown in Figure 2, the characteristic line $\vec{C}(s)$ formed by a point $\vec{C}(s_i)$ on the curve is:

$$\vec{L}(t_i) = \vec{C}(s_i) + \vec{v} \cdot t$$

(6)

Where: $\vec{C}(s_i)$ is the value of $\vec{C}(s)$ in $s_i$.

If the value $t$ is selected $t = t_j$, the distance between a point on the feature line and the distance to be satisfied is the maximum limit value of machining residual height and the maximum allowable trajectory error value of machining, which is far less than the allowable contour error after machining. When the above conditions are true, the point is on the contour line; on the contrary, it is necessary to change the value and repeat the above calculation formula. Since the machining surface is convex surface, there must be a point on the feature line that can meet the requirements. According to the above method, the contour points on the feature line are derived in turn, then a set of isoresidual height points will be obtained, and the contour can be obtained by interpolation algorithm [4].

2.3 Tangent characteristics of isopleth line

As shown in Figure 3, it can be seen that the surface $\vec{S}(u,v)$ has an equidistant surface $\vec{S}^*(u,v)$ [8]:

$$\vec{S}^*(u,v) = \vec{S}(u,v) + d \cdot \vec{n}(u,v)$$

(7)

Where: $\vec{n}(u,v)$ is the normal vector of $\vec{S}(u,v)$ and is $d$ the distance between the surface $\vec{S}(u,v)$ and $\vec{S}^*(u,v)$. 
Figure 1. Characteristic lines of the envelop surface generated by a plane.

Figure 2. The algorithm for CSH-point.

Figure 3. Relationship between the normal vectors of convex surface and its offset surface.

Find the partial derivative of equation (7) to get the normal vector $\bar{n}^*(u, v)$ of the surface $\bar{S}^*(u, v)$. After sorting out, there are the following relations:
\[
\vec{S}_u^* \times \vec{S}_v^* = \vec{S}_u \times \vec{S}_v + d\mathbf{a}(\vec{S}_u \times \vec{n}_v + \vec{n}_u \times \vec{S}_v) + d^2 \mathbf{b}(\vec{n}_u \times \vec{n}_v)
\]  
(8)

According to Rodrigues theorem [9]:

\[
\vec{S}_u \times \vec{n}_v = \vec{S}_u \times k_v \vec{S}_v = k_v (\vec{S}_u \times \vec{S}_v)
\]  
(9)

\[
\vec{n}_u \times \vec{S}_v = k_u (\vec{S}_u \times \vec{S}_v)
\]  
(10)

\[
\vec{n}_u \times \vec{n}_v = k_u k_v (\vec{S}_u \times \vec{S}_v)
\]  
(11)

From formula (8), (9), (10) and (11), it can be concluded that:

\[
\vec{S}_u^* \times \vec{S}_v^* = [1 + d(k_v - k_u) + d^2 k_u k_v] (\vec{S}_u \times \vec{S}_v)
\]  
(12)

According to equation (12), it can be seen that the normal vectors of the surface \( S(u, v) \) and the surface \( S^*(u, v) \) are parallel at the corresponding points, \( \vec{n}(u, v) \parallel \vec{n}^*(u, v) \), as shown in Fig. 3.

The tangent vector \( \vec{H}(l) \) of any point in the line is expressed as \( \vec{H}(l_i) \), and the tangent \( \vec{H}(l_i) \) vector is \( \vec{T}(l_i) \). As can be seen from Figure 4, the envelope surface intersects the surface at \( \vec{H}(l) \).

For the curve \( \vec{p}(s, t) \) on the enveloping surface \( \vec{H}(l) \), if the normal vector of the envelope surface where the point \( \vec{H}(l_i) \) is located is \( \vec{N}_\rho \), then:

\[
\vec{T}(l_i) \perp \vec{N}_\rho
\]  
(13)

If the envelope characteristic line \( \vec{L}(t) \) passing through a point \( \vec{H}(l_i) \) intersects \( \vec{C}(s) \) with \( \vec{C}(s_i) \), the envelope normal vector at the point can be expressed by equation(14):

\[
\vec{N}_S = \frac{\vec{S}_u(u(s_i), v(s_i)) \times \vec{S}_v(u(s_i), v(s_i))}{|\vec{S}_u(u(s_i), v(s_i)) \times \vec{S}_v(u(s_i), v(s_i))|}
\]  
(14)

On the enveloping surface \( \vec{p}(s, t) = \vec{C}(s) + t \vec{v} \), the normal vectors of the points on the characteristic line \( \vec{L}(t) \) are in the same direction or reverse direction. In the envelope then \( \vec{N}_S \parallel \vec{N}_\rho \), according to (13), there is:

\[
\vec{N}_{S(u, v)} \times \vec{N}_S = \vec{T}(l_i)
\]  
(15)
Equation (15) is the derivation result of tangent vector $\mathbf{T}(l_i)$ of a point on the contour line of equal residual height, $\mathbf{N}_{S(u,v)}$ and $\mathbf{N}_S$ is the normal vector of the surface $\mathbf{S}(u,v)$ to be machined, which simplifies the derivation steps. The location relationship is shown in Figure 4.

Figure 4. Tangent vectors of a CSH-curve.

3 Example of trajectory algorithm

It is assumed that the convex surface to be machined is an ellipsoid surface, the expression is as follows:

$$\frac{4}{16} + \frac{y^2}{9} + \frac{z^2}{4} = 1$$

(16)

The algorithm is checked in the above algorithm. When the plane $x = 2$ intersects with the ellipsoid surface, the first intersection line is the first grinding contour, which is taken as 0.5 mm as the unit length. The intersection line is discretized into the first grinding track along the axis $y$, and the value of equal residual height is taken as 0.1 mm. According to the above scheme, the calculation of the residual height line and grinding track of the whole ellipsoid contour is carried out. The grinding path and envelope surface are calculated and characterized in MATLAB, as shown in Fig. 5 and Fig. 6.

Figure 5. Grinding paths and CSH curves in work-piece coordinate system.

In grinding process, the machining contour is one of the processing methods which are greatly affected by the cutter. In this paper, the cup wheel and ball head grinding wheel (ball end milling cutter) are selected for comparative analysis. The generated tool path and equivalent residual height of the two are shown in Fig. 7 and Fig. 8 respectively.
Figure 6. Envelop surface composed of characteristic lines and grinding path points and CSH-points.

Figure 7. The grinding paths generated by a Cup grinding wheel Face grinding.

Figure 8. The grinding paths generated by a ball grinding wheel.

Figure 9. The relationship between intervals of grinding paths and radius of curvature.

According to Fig. 7 (cup wheel), when $x$ the grinding track changes from 2 to 2.6, the grinding track covers $4.576 \text{cm}^2$ the total length, $12.36 \text{cm}$, of four tracks (each containing 60 points). According to figure 8 (ball nose grinding wheel), the value $x$ is taken from 2 to 2.25, and its coverage area is $1.368 \text{cm}^2$, but the total length is $13.17 \text{cm}$, greater than that of cup-shaped grinding wheel, and the distance between adjacent tracks varies in the range of $[0.163, 0.1675] \text{cm}$. The distance between grinding tracks and the area covered by cup wheel are larger.
Similarly, Matlab is used to describe the curvature radius relationship of two points in the adjacent trajectory, as shown in Figure 9. It is not difficult to see that the distance between trajectories is positively related to the radius of curvature. It is proved that the curvature of the machined surface changes slowly, and the algorithm given above has obvious advantages.

4 Conclusion

On the basis of mathematical differential theory, the envelope surface swept along a line of convex surface is represented by vector. The envelope surface is analyzed from the perspective of differential, and the algorithm of equal residual height line is obtained. Through the calculation, it is found that the tangent of the contour is parallel to the tangent of the machining path. On this basis, the final trajectory algorithm is expressed by mathematical model.

In order to verify the algorithm, taking the ellipsoidal convex surface as the processing object, the grinding path and envelope surface are obtained based on the above algorithm, and the guide rails under different cutters using cup-shaped sand and ball head grinding wheel are compared to verify the advantages of the algorithm.

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