Delamination Initiation and Optimal Thickness of Reinforcement in Elastic-Plastic Pressure Vessels

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Abstract. In the present paper the effects of physical and non-linearities are taken into account their influence on the delamination initiation and limit states of composite pressure vessels reinforcement are discussed. The numerical examples deal with the behavior of the reinforcement of the junction of shells. The optimisation method of the reinforcement thickness is also formulated and solved herein.

Introduction

Some composite materials, such as AS4/PEEK, T300/1034-C and AS4/3501-6 composites, may exhibit nonlinearity or plasticity behavior. The application of elastic damage models (e.g. the Abaqus, NISA II built-in fibre-reinforced material damage model) for predicting the mechanical response of such composite laminates may be insufficient when plastic deformations are present under loading. It is well-known that the polymeric matrix of fibre reinforced plastics (FRP) exhibits a non-linear behavior understood in the sense of a physical non-linearity on the $\sigma$-$\varepsilon$ diagram. This effect may be additionally enhanced by fibre orientations especially for fibres oriented at $45^\circ$ where a typical plastic hardening is observed. However, in the majority of research works as well as in design codes it is a common practice to employ the geometrically and physically linear plate/shell theory in the analysis of composite structures. It may lead to the incorrect evaluation of failure (damage) index particularly for structural elements where a stress concentration occurs, e.g. holes, junctions of plates, shells and their reinforcement. In addition, in the case of reinforcements as a delamination failure becomes a dominant failure mode for composite structures possible stress relaxation due to the physical non-linearity may change completely design results.

In the present paper we extend previous studies and examine the effects of physical and geometrical non-linearities on failure modes of composite structures focusing our attention on the following problem: stress concentration and FPF studies at the junction of two cylindrical shells to model and analyse the stress concentration effects caused by nozzles placed in the cylindrical part of composite pressure vessels. It is assumed that the physical non-linearity is a result of fibre orientations and according to experimental investigations the highest effects of non-linearities is observed for fibres oriented at $45^\circ$. The numerical studies are conducted with the use of the FE package where

A laminate is built of a finite set of 2-D FE in the thickness ($z$) direction. Each 2-D FE corresponds to the assumed fibre orientations – groups of layers having the identical fibre orientations are described with the use of one 2-D FE along the thickness direction. The physical, non-linear law (in the form of plastic deformations) is created independently for each individual 2-D FE corresponding to the prescribed fibre orientation. In each of FE the initiation of non-linear deformations is described with the use of one parametrical plastic flow potential in the form proposed by Sun and Chen [1]. The form of physical relations used in the FE modeling of non-linear (plastic) behaviour of individual plies in the laminate is broadly discussed by Muc, Fugiel [2]. However, in the latter work the analysed problems deal with the elastic-plastic contact between composite pressure vessel body and the rigid foundation.
Physical Non-linearities

A great number of composite materials demonstrates physical non-linearities on the $\sigma - \varepsilon$ curves. Their origin is obvious – arising of the microcracks inside the matrix without any visible macrocracks. It leads directly to the stiffness degradation and results in the decrease of the appropriate value of the stiffness matrix. Those effects are directly connected with the fibre orientations Sun i Chen [1] have proposed the use of one parametrical plastic flow potential in the following form:

$$f = (\sigma_2^2 + 2a_0 \sigma_6^2) / 2$$ \hspace{1cm} (1)

Owen, Li [3] have applied the plastic potential similar to the Huber-Mises-Hencky yield condition which has six parameters of anisotropy:

$$f = a_1 (\sigma_1 - \sigma_2)^2 + a_2 (\sigma_2 - \sigma_3)^2 + a_3 (\sigma_3 - \sigma_4)^2 + 3(a_4 \sigma_5^2 + a_5 \sigma_8^2 + a_6 \sigma_9^2)$$ \hspace{1cm} (2)

The anisotropy parameters $a_1, ..., a_6$ are taken from experimental data.

In flow theory the material behaviour is described by three conditions: the initial yielding condition (described above), the flow rule and the hardening rule. Let the total strain component is the sum of the elastic strain and the plastic one:

$$\{d\varepsilon\} = \{d\varepsilon^\text{el}\} + \{d\varepsilon^\text{pl}\}$$ \hspace{1cm} (3)

The elastic strain increment is given from the Hook law, whereas the plastic one $\{d\varepsilon^\text{pl}\}$ is derived from the flow rule:

$$\{d\varepsilon^\text{pl}\} = d\lambda \frac{\partial f}{\partial \{\sigma\}}$$ \hspace{1cm} (4)

and $d\lambda$ is a proportionality constant. After some manipulations that parameter may be expressed as follows:

$$d\lambda = \frac{\left(\frac{\partial f}{\partial \{\sigma\}}\right)^{\frac{3}{2}} \mathbf{C} \left\{d\varepsilon\right\}}{\frac{4}{3} 3 f \mathbf{H}_p + \left(\frac{\partial f}{\partial \{\sigma\}}\right)^{\frac{3}{2}} \mathbf{C} \left(\frac{\partial f}{\partial \{\sigma\}}\right)}$$ \hspace{1cm} (5)

$\mathbf{H}_p$ is a plastic modulus. Using the above relations, one can obtain finally the incremental elastic-plastic flow rule that has the similar manner as for isotropic materials, i.e.:

$$\{d\sigma\} = \left[C\right]\left(\{d\varepsilon\} - \{d\varepsilon^\text{pl}\}\right) = \left[C\right] \left[1 - \frac{\partial f}{\partial \{\sigma\}} \frac{d\lambda}{d\varepsilon^\text{pl}}\right] \{d\varepsilon\} = \left[C^\text{pl}\right] \{d\varepsilon\}$$ \hspace{1cm} (6)

Delamination Initiation and Propagation Criteria

In structural applications of composite materials, delamination growth often occurs under mixed-mode loading and delamination onset may happen before any of the traction components reach their corresponding maximum stress allowables. In this work, a quadratic nominal stress criterion is used to predict the onset of mixed-mode delamination. It allows for the effect of the interactions of the traction components on the delamination initiation. Considering that compressive normal traction does not normally contribute to the delamination onset, the quadratic nominal stress criterion is written in the following form as [4]:

$$F = \left(\frac{\sigma_1}{\sigma_0}\right)^2 + \left(\frac{\tau_{yx}}{\tau_{yx}^0}\right)^2 + \left(\frac{\tau_{yz}}{\tau_{yz}^0}\right)^2 = 1$$ \hspace{1cm} (7)

where $<\text{Macaulay bracket}>=(|°|+°)/2$ is the Macaulay bracket; $\sigma_0$, $\tau_{yx}^0$, $\tau_{yz}^0$ denote the interface Mode I, II and III failure strengths, respectively. Once the mixed-mode delamination initiation criterion is met,
delamination begins to propagate. The material stiffness of the cohesive layers starts to degrade for each mode in such a way that the total fracture energy absorbed per unit area under mixed-mode loading is governed by a delamination propagation criterion. The mixed-mode propagation criterion defines the state of complete delamination for different mixed-mode ratios. In this study, the power law criterion is adopted:

$$\left( \frac{G_I}{G_{Ic}} \right)^{\gamma} + \left( \frac{G_{II}}{G_{IIc}} \right)^{\gamma} + \left( \frac{G_{III}}{G_{IIIc}} \right)^{\gamma} = 1$$  \hspace{1cm} (8)$$

where $G_{Ic}$, $G_{IIc}$, $G_{IIIc}$ are the interfacial Mode I, II and III critical fracture energies. In this work, the one with $\gamma=1$ is adopted as such selection has been found to be suited to predict the complete delamination failure and accurately capture the dependence of the mixed-mode fracture energy on the mode ratio of AS4/PEEK thermoplastic composite materials [5].

Formulation of the Optimisation Problem

Shape optimisation of the shell reinforcement is connected with searching for the maximal critical loads that can be carried out by laminated structures or it is introduced in order to equalize the stress (strain) failure criteria (in the sense of required yield conditions or FPF criteria) around the boundaries or for the whole axisymmetric structure. If we intend to equalize the values of the objective functional $U$ (stress/strain criterion) along the curve C then the optimisation problem differs qualitatively (the MinMax problem) from the typical formulations leading to the limitation of the functional value, i.e.: $\text{Max } U(s) \leq U_{\text{admis}}$, where $U_{\text{admis}}$ denotes the admissible, prescribed $a priori$ value of the functional.

Junction of Cylindrical Shells - Numerical Results

Let us consider the design of reinforcements of the junction of two cylindrical shells. It is assumed that the shell thickness $t$ is equal to 9.5 mm, and their outside radii $R = 305 \text{ mm}$ and $r = 160 \text{ mm}$, respectively. The geometry of the junction is presented in Figure 1 for a transverse and longitudinal cross-sections. The cylindrical shells is made of plies having the following stacking sequence $[0\pm 45_3]_S$, whereas its material and geometrical ratios are following: $L/R = 5$, $R/t = 10$, $G_{12}/E_2 = 0.5$, $\nu_{12} = 0.25$, $E_1/E_2 = 20$.

![Figure 1. Cross-sections of the junction.](image)

The dimensionless stress distributions are demonstrated in Figures 2, 3 and they show evidently the necessity of the junction shape optimization understood in the sense of adding an additional material called as reinforcement. Figures 2a, 3a present the stress distributions taking into account elastic deformations only, whereas Figures 2b, 3b exhibit the influence of plastic (non-linear deformations) of plies oriented at $45^\circ$. As it may be seen physical non-linearities result in the drastic reduction of the maximal stresses at the junction.
The optimisation problem has been solved for the functional $U$ in the form of the dimensionless stresses measured along the dimensionless distance $x/R$. $x$ equal to zero corresponds to the junction, and the sign + describes the distance measured along the cylinder having the greater radius $R$, and – the cylinder with the radius $r$. In fact, two independent optimisation problems have been solved: the first corresponds to the elastic deformations analysis only, whereas the second takes into account physical non-linearities in the form discussed in the section 2. Optimal design of the thickness reinforcement at the junction of two cylindrical shells (one of them represents a nozzle in a pressure vessel body) is conducted with the use of the Bezier spline functions. The broad discussion of the the optimization method as well as of the used genetic optimization algorithms is presented by Muc et al. [6, 7]. The objective of the optimization is following: to equalize the stress concentration factors around the nozzles including elastic and elastic-plastic deformation effects – it is formulated as the MinMax problem – see section 3. The results of the optimisation are plotted in Figure 4 independently in the transverse and longitudinal directions since the stress distributions are completely different. The above-mentioned Figures exhibit also differences between optimal reinforcements in the linear (elastic) and non-linear (elastic-plastic) cases.

As it may be seen physical non-linearities change completely the optimal reinforcement distributions since the stress distributions for elastic and elastic-plastic cases are completely different. However, on the other hand the results obtained with the use of non-linear, elastic-plastic
analysis demonstrate evidently the necessity of taking physical non-linearities into account in the design procedures of composite pressure vessels. For the optimal thickness distributions the value of the function $F$ (Eq.7) were evaluated to determine the possibility of the delamination initiation – Figure 5. As it may be seen the plastic deformation reduces the delamination development.

![Figure 5. Delamination initiation for the optimal thickness distributions.](image)

**Conclusions**

The necessity of taking into account of physical non-linearities in the design of pressure vessels made of fibre reinforced plastics is pointed out in the present paper. The present study shows evidently that the use of design codes result in the wrong prediction of the pressure vessel components thicknesses and the delamination initiation due to the incorrect application of experimental data demonstrating the existence of both physical and geometrical non-linearities. In our opinion, the proper conjunction of numerical analysis with the experimental data may result in the further weight savings of pressure vessels and in this way a better optimisation of such structures.

**References**


