Residual Stress Field of Triangular Symmetrical Composite Eutectic

Jinfeng Yu, Xiequan Liu, Xinhua Ni, Zhaogang Cheng, Wanheng He
Mechanical Engineering College, Shijiazhuang 050003, China

ABSTRACT: Composite ceramics by combustion synthesis under high gravity can acquire excellent static and dynamic mechanical properties. Composite ceramics were mainly composed of randomly orientated eutectics containing a triangular dispersion of orderly nano-submicrometer fibers. Our study is based on a model in which a micromechanical cell is embedded in an infinite effective medium. The micromechanical cell is composed of a fiber with eutectic interphase and matrix—atmosphere, and the mechanical property of the effective medium is same as the triangular region. This micromechanical cell is used to study residual stress field of triangular symmetrical composite eutectic. Considering fiber inclusion, eutectic interphase and matrix-atmosphere are isotropy, the residual stress distribution in the micromechanical cell is obtained. For $\text{Al}_2\text{O}_3/\text{ZrO}_2$ triangular symmetrical composite eutectic, the residual stress field is evaluated quantitatively. The results show that the residual stresses increase with reducing diameter of fiber inclusion. While the diameter of fiber inclusion is bigger than 100nm, the residual stresses in matrix–atmosphere are approximately constants. The analysis result will lay a foundation for stability research of flaws in composite ceramic mainly composed of triangular symmetrical eutectic.

1 GENERAL INSTRUCTIONS

The self-growing composite ceramic can acquire strong mechanical properties, excellent strength retention at high temperature and surprising fracture toughness at both room temperature and elevated temperature$^{[1,2]}$. Composite ceramic is mainly composed of triangular symmetrical eutectics with random orientation, in which orderly nano-submicrometer fibers with interphase are dispersed within the ceramic matrix. The interfaces between two phases in triangular symmetrical eutectics have homopolar surfaces where they share a common oxygen plane which makes the two phases bond strongly$^{[3]}$. There are residual compressive stresses in the matrix of triangular symmetrical eutectics$^{[4]}$. Hence, as the crack is met by the colonies, a shielding effect is induced caused by the interaction between the residual compressive field and the crack tip, leading to an increase in the applied stress intensity factor. The effect of the residual stresses on stability of flaws can be obtained according to the literature$^{[5]}$. Therefore, the evaluation of residual stress field is the basic problem which ascertains crack initiation.

2 EQUIVALENT STRESS FIELD

Three triangular regions in the triangular symmetrical eutectic have the same shape and trait, so we only choose a triangular region to analyze. Nano-submicrometer fibers with interphase are orderly dispersed within the ceramic matrix of the triangular region. A fiber with interphase and matrix-atmosphere form a micromechanical cell. The volume fractions of every phase in the composite ceramic mainly composed of triangular symmetrical eutectic.

Suppose the stiffness tensors of matrix, interphase, fiber and the triangular region are $C^m$, $C^i$, $C^f$ and $C$ respectively. Obviously, $S^m=(C^m)^{-1}$, $S^i=(C^i)^{-1}$, $S^f=(C^f)^{-1}$ and $S=(C)^{-1}$ are flexibility tensor of matrix, interphase, fiber and the triangular region. The thermal expansion coefficient tensors of matrix, interphase, fiber and the triangular region are $\alpha^m$, $\alpha^i$, $\alpha^f$ and $\alpha$ respectively. After inner temperature of the composite ceramic changed equably $\Delta T$, there are thermal stresses produced by the difference of thermal expansion coefficient. If the micromechanical cell is only composed of matrix, according the Eshebly theory, the equivalent stress...
field of the matrix in the triangular region is determined
\[ \sigma^m = \Omega^m (1 - \Omega^m H)^{-1} \epsilon^f \] (1)

Where, \( \Omega^m = C^m (I - M) \), \( M \) is the Eshelby tensor of the micromechanical cell; \( I \) is unit tensor; \( \epsilon^f = (\alpha - \alpha') \Delta t \). \( H \) is the compliance tensor increment of triangular region and given as follow[6]
\[ H = \sum H^f (1 - \Omega^m H)^{-1} \] (2)

Here
\[ H^f = \sum \{ f_i [\Omega^f]^{-1} + f_j [\Omega^f]^{-1} \} \] (3)

Here, \( f_i \) and \( f_j \) are the volume fractions of the interphase and the fiber in the triangular region respectively. \( \Omega^f = C^f (I - M) \), \( \Omega^f = C^f (I - M) \), \( H^f = S^f - S^m \). Making the additional surface force \( \sigma^h \) and \( \sigma^f \) separate act on the boundary of the interphase and fiber, the micromechanical cell could have homogeneous strains in the whole region. Evidently, the additional stresses in the interphase and fiber are separately as follows:
\[ \sigma = (C^f - C^m) S^m \sigma^m - (C^f - C^m) S^m \sigma^f \] (4)

We definite compliance waves of interphase and fiber as: \( H^f = S^f - S^m \) and \( H^f = S^f - S^m \). According to effective self-consistent method[7], the equivalent stress field of the fiber in the triangular region is given as follow:
\[ \sigma^f = -\Omega^f (1 + \Omega^f H^f)^{-1} \epsilon^f + (1 + \Omega^f H^f)^{-1} - \sigma^m \] (5)

Here, \( \epsilon^f = (\alpha^f - \alpha') \).

The position relationship among three crystal plane A, B and C of triangle symmetry eutectic is shown in Fig. 1.

Figure 1. Position relationship among crystal planes.

Fibers are triangle symmetrically distributed in the matrix and perpendicular to crystal plane A, B and C. \((x, y, z)\) and \((e_1, e_2, e_3)\) are the global and local Cartesian coordinate systems, respectively. \(x\)-axis is along the crystal axial \(c\) and \(e_1\) along a fiber. Without loss of generality, we let the \(e_2\)-axis coincides with \(y\)-axis. Assuming that \( \theta \) is the angle between \(e_1\) and \(x\)-axes, (i.e. the angle between a fiber and the crystal axial \(c\)). The transformation matrix is expressed as follow
\[
T = \begin{bmatrix}
\cos^2 \theta & 0 & \sin^2 \theta & 0 & \sin 2\theta & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
\sin^2 \theta & 0 & \cos^2 \theta & 0 & -\sin 2\theta & 0 \\
0 & 0 & 0 & \cos \theta & 0 & -\sin \theta \\
-\sin 2\theta/2 & 0 & \sin 2\theta/2 & 0 & \cos \theta & 0 \\
0 & 0 & 0 & \sin \theta & 0 & \cos \theta 
\end{bmatrix}
\] (6)

Here, \( \theta = 16.1^\circ \). The equivalent stress field of matrix and fiber in global coordinate system can be obtained
\[ \sigma^m_g = T \sigma^m 
\] (7)
\[ \sigma^f_g = T \sigma^f 
\] (8)

3 RESIDUAL STRESSES

Based on the equivalent stress field determined by equations (7) and (8), the average residual stresses in matrix and fiber can be computed. Matrix, interphase and fiber are isotropy, the elastic constants and thermal expansion coefficient tensors of matrix, interphase and fiber can be expressed as
\[
C^m = \begin{bmatrix}
\lambda_m & \mu_m & 0 & 0 & 0 & 0 \\
\mu_m & \lambda_m & 0 & 0 & 0 & 0 \\
0 & 0 & \mu_m & 0 & 0 & 0 \\
0 & 0 & 0 & \lambda_m & \mu_m & 0 \\
0 & 0 & 0 & \mu_m & \lambda_m & 0 \\
0 & 0 & 0 & 0 & 0 & \mu_m
\end{bmatrix}
\] (9)

\[
C^f = \begin{bmatrix}
\lambda_f & \mu_f & 0 & 0 & 0 & 0 \\
\mu_f & \lambda_f & 0 & 0 & 0 & 0 \\
0 & 0 & \mu_f & 0 & 0 & 0 \\
0 & 0 & 0 & \lambda_f & \mu_f & 0 \\
0 & 0 & 0 & \mu_f & \lambda_f & 0 \\
0 & 0 & 0 & 0 & 0 & \mu_f
\end{bmatrix}
\] (10)

Where \( \lambda_m, \mu_m \) and \( \alpha_m; \lambda_f, \mu_f \) and \( \alpha_f \) are Lamé’s constants and thermal expansion coefficients of matrix, interphase and fiber respectively. Substituting formula (9) and formula (10) into formula (1), the average residual stresses of matrix in local coordinates are obtained
\[ \sigma^m_{11} = \frac{E_m}{1 - \nu_m^2} N^m_{11} \Delta T 
\] (11)
\[ \sigma^m_{22} = \sigma^m_{33} = \frac{E_m}{1 - \nu_m^2} N^m_{22} \Delta T 
\] (12)

Here
\[
N^m_{11} = [A_{11} + 1 + \nu_m (A_{21} + 1)] f_r (\alpha - \alpha_m) + [A_{12} + 1 + \nu_m (A_{22} + 1)] f_r (\alpha - \alpha_m)
\] (13)
Where, \( E_m \) and \( \nu_m \) are elastic modulus and Poisson ratio of matrix; \( A_{1a}, A_{1b}, A_{2a} \) and \( A_{2b} \) are given as follow
\[
A_{1a} = \frac{2B_{1b}(C_{1b} + C_{5b} + C_{6b}) - (B_{1a} + B_{6a})(C_{1a} + C_{5a} + C_{6a})}{B_{1a}(B_{3a} + B_{5a}) - 2B_{2a}B_{4a}}
\]
\[
A_{1b} = \frac{2B_{2a}(C_{1a} + C_{5a} + C_{6a}) - (B_{1a} + B_{6a})(C_{1a} + C_{5a} + C_{6a})}{B_{1a}(B_{3a} + B_{5a}) - 2B_{2a}B_{4a}}
\]
\[
A_{2a} = \frac{B_{1b}(C_{1b} + C_{5b} + C_{6b}) - B_{1a}(C_{1a} + C_{5a} + C_{6a})}{B_{1a}(B_{3a} + B_{5a}) - 2B_{2a}B_{4a}}
\]
\[
A_{2b} = \frac{B_{1b}(C_{1b} + C_{5b} + C_{6b}) - B_{1a}(C_{1a} + C_{5a} + C_{6a})}{B_{1a}(B_{3a} + B_{5a}) - 2B_{2a}B_{4a}}
\]

Substituting formula (11) and formula (12) into formula (5), the average residual stresses in matrix of the triangular symmetrical eutectic can be gotten from equations (15) ~ (18), and the relation between the residual stresses in matrix and fiber diameter is shown in Fig.2 and Fig.3. The results show that residual stresses in matrix are compressive stresses. The maximum residual stress is along y direction, and the residual stresses in matrix increase with reducing diameter of fiber inclusion. When \( d<50 \text{nm} \), the residual stresses increase quickly; When \( d>50 \text{nm} \), the residual stresses changes gently. There are residual shearing stresses in matrix of the triangular symmetrical eutectic, while the residual shearing stresses are nonexistent in matrix of the eutectic with lamellae and the eutectic with parallel fiber [6].

![Figure 2 Residual normal stresses in matrix as a function of fiber diameter along y direction.](image)

![Figure 3 Residual shearing stresses in matrix as a function of fiber diameter.](image)

For Al\(_2\)O\(_3\)–ZrO\(_2\) triangular symmetrical eutectic, \( \nu_m = 0.233 \), \( E_m = 402 \text{GPa} \), \( \alpha_m = 8.3 \), \( \nu_f = 0.31 \), \( E_f = 233 \text{GPa} \), \( f_f = 0.3 \), \( \alpha_f = 10.6 \), \( \nu_f = 0.233 \), \( E_f = 10E_m \).

\[
\sigma_{11}^m = \frac{E_m}{1 - \nu_m} N_{11}^f \Delta T
\]
\[
\sigma_{22}^m = \frac{E_m}{1 - \nu_m} N_{22}^f \Delta T
\]
And

\[
N_{11}^f = [A_{11} + 1] + [A_{12} + 1] (f_i - 1)(\alpha_f - \alpha_m) \\
N_{12}^f = [A_{11} + 1] + [A_{22} + 1] (f_i - 1)(\alpha_f - \alpha_m)
\]

(21)

Substituting formula (19) and formula (20) into formula (8), the average residual stresses of fiber in global coordinate system are as follow

\[
\sigma_{xx}^f = \sigma_{11}^f \cos^2 \theta + \sigma_{33}^f \sin^2 \theta
\]

(23)

\[
\sigma_{yy}^f = \sigma_{11}^f
\]

(24)

\[
\sigma_{zz}^f = \sigma_{11}^f \sin^2 \theta + \sigma_{33}^f \cos^2 \theta
\]

(25)

\[
\tau_{xz}^f = (\sigma_{11}^f - \sigma_{33}^f) \frac{\sin 2\theta}{2}
\]

(26)

The average residual stresses in fiber of the triangular symmetrical eutectic can be gotten from equations (23) ~ (26), and the relation between the residual stresses in fiber as the functions as the fiber diameter are shown in Fig.4 and Fig.5. The results show that residual stresses in fiber are compressive stresses, and the maximum residual stress is along y direction. The residual stresses in fiber increase with increasing diameter of fiber inclusion in other directions. When d<50nm, the residual stresses in fiber change relatively greatly. When d>50nm, the residual stresses change slowly and size-independence isn't obvious. There are residual shearing stresses in matrix of the triangular symmetrical eutectic while the residual shearing stresses are nonexistent in matrix of the eutectic with lamellae and the eutectic with parallel fiber, and the value of the residual shearing stress \(\tau_{xz}^f\) is much smaller than the value of residual normal stress \(\sigma_{yy}^f\).  

4 CONCLUSIONS

(1) The equivalent stress field of matrix and fiber in global coordinate system was obtained based on the Eshebly theory.

(2) The average residual stresses of matrix and fiber in global coordinate system were obtained according to the equivalent stress field of matrix and fiber. The residual stresses in fiber and matrix are compressive stresses. The size-dependent of residual stresses was calculated. The maximum residual stress is along y direction in fiber and matrix. And the value of residual shearing stresses is smaller than the value of the residual normal stresses.

5 ACKNOWLEDGMENTS

This work is supported by the National Natural Science Foundation of China under grant no. 11272355.

REFERENCES


