Numerical Modeling of Shallow Foundation on Liquefiable Soil

Amrendra Kumar¹, Sunita Kumari² and Sanjeev Kumar Suman³

¹Research Scholar, Department of Civil Engineering, National Institute of Technology Patna, Bihar 800005, India; amrendraroy2k8@gmail.com
²³Assistant Professor, Department of Civil Engineering, National Institute of Technology Patna, Bihar 800005, India; ²sunitafce@nitp.ac.in; ³sksuman@nitp.ac.in

ABSTRACT: Liquefaction of soil is one of the most unpredictable event which causes failure of the structure during an earthquake which causes settlement, tilt and lateral sliding of structure and sub-structure. Therefore, it is essential to study the effect of an earthquake before the construction of any structure for a particular site, residing in high seismic zone. In the present paper, a shallow foundation resting on liquefiable is numerically modeled to find out displacement and excess pore pressure during and after for an earthquake like motion. The basic equation of Biot’s theory of porous media has been considered to analysis the shallow foundation using finite element method. The inelastic behaviour which include dilatancy and hardening has been modelled using Pastor–Zienkiewicz Mark III model. The effect of some key parameters like shear modulus and permeability has been also studied. Parametric study shows that chance of liquefaction becomes less as shear modulus increases. Permeability also play an important role in triggering the liquefaction phenomena.

Keywords: Liquefaction, shallow foundation, shear modulus, permeability

INTRODUCTION

Soil liquefaction phenomenon can be defined as the reduction of shear strength due to pore pressure buildup in the structure of soil. When loose and poorly graded saturated sands are subjected to earthquake loading, an upward propagation of shear waves occurs from bedrock which tend to densify and settle. In case of dry sand, understanding of densification and settlement is easy whereas in saturated sand reduction in volume occurs resulting in expulsion of pores water from voids. However, application of the cyclic stress time duration is very small as compared to the time required for water drainage from voids. In the meantime water cannot come out from pores, excess pore pressure rises at each load increment. When the pore pressure becomes equal to the total stress, effective stress reduces to zero. As a results, sands will temporarily or completely lose shear strength and stiffness. Such a state is referred to as initial liquefaction. At the onset of initial liquefaction, loose sands will undergo unlimited deformations or flow without mobilizing significant resistance to deformation. As a consequence, structures resting on the liquefied deposit undergo significant settlements and tilting. This phenomenon is referred as liquefaction. A number of failures of embankments, natural slopes, earth structures and foundations have been attributed to the liquefaction of sands caused by either static or seismic loading. A complete destruction is reported in some of the major earthquakes like El Centro 1940, Niigata 1964, San Fernando 1971, Kobe 1995, Bhuj 2001 due to liquefaction of soil. Considering dangerous effect of liquefaction phenomenon, attempts should be made to properly assess
the possibility of liquefaction for a given site before any type of major construction.

In the beginning researchers were focused on experimental work to understand the liquefaction phenomenon and cyclic mobility Seed and Lee, (1966), Seed and Idriss (1971), Castro and Poulos (1977), and Kramer (1996). At that time physical phenomenon was well understood, analytical modelling of soil liquefaction and numerical simulation remains a challenge. The development of appropriate constitutive models, capable of predicting soil liquefaction has been a challenging problem for the researchers. Oka et al. (1994) discussed the FEM–FDM coupled liquefaction analysis of a porous soil using elasto plastic model. Elgamal et al. (2003) developed a computational model for analysis of cyclic mobility scenario based on general fully coupled (solid–fluid) finite element formulations. Mesgouez et al. (2005) presented the applications of Biot’s theory in transient wave propagation in saturated porous media. Taiebat et al. (2007) worked on numerical analyses of liquefiable sand using critical state two-surface plasticity model and densification model for bounded soil domain. Dashti et al. (2010) conducted Centrifuge test and results shows that building settlement was not proportional to the thickness of the liquefiable layer and that most of this settlement occurs during earthquake strong shaking i.e. depend on intensity of earthquake. Dashti and Bray (2013) simulate fully-coupled numerical approach with the UBCSAND model implemented in FLAC-2D captured building settlements measured in these experiments reasonably well for one scaled input motion, mostly within factors of 0.7 and 1.8.

In the present study, a finite element based coupled algorithm is proposed for simulation of behaviour of shallow foundation on liquefiable soil domain. The generalized Biot’s theory is used to develop the governing equations, which couple the equilibrium and continuity equations for a deforming saturated porous medium. These are solved with Newmark-beta time marching scheme. The Pastor–Zienkiewicz Mark III model (Kumari et al. 2016) has been used to describe the inelastic behavior of soils under isotropic cyclic loadings.

MATHEMATICAL FORMULATION

The equilibrium and continuity equation have been used for the simulation of liquefaction. The primary variables in this form of equations are fluid pressure and solid displacement. Thus, this form is called \( U_c-P\) or for simplicity \( U-P\) formulation. Hence, displacements and pore pressures are calculated at the same time and interactively at each time step.

Finite element method for spatial discretization, \( U-P\) formulation is as follows:

\[
\begin{align*}
[M][\vec{U}_c]+[K][U_c]-[Q][P_c] &= \{f_U\} \\
[G][\vec{U}_c]+[Q]^T[\vec{U}_c]+[S][\vec{U}_c]+[H][P_c] &= \{f_P\}
\end{align*}
\]

Generalized Newmark-beta method is employed for integration in time. The primary unknowns are incremental displacements \( \Delta q \) and pore pressure \( \Delta p \) respectively.

\[
\begin{align*}
(L_1[M]+[K])\{\Delta q\}-[Q]\{\Delta p\} &= \Delta F_q + [M](L_2 \dot{q}_{i-1} + L_3 \ddot{q}_{i-1}) \\
(L_4[G]+L_4[Q]^T)[\Delta q_i]-(L_2[S]+[H][\Delta p_i]) &= \Delta F_p + [G](L_2 \dot{q}_{i-1} + L_3 \ddot{q}_{i-1})+[Q]^T(L_5 \dot{q}_{i-1} + L_6 \ddot{q}_{i-1})
\end{align*}
\]

\[
L_1 = \frac{1}{\beta \Delta t^2}; \quad L_2 = \frac{1}{\beta \Delta t}; \quad L_3 = 0.5/\beta; \quad L_4 = \alpha/(\beta \Delta t); \quad L_5 = \alpha/\beta; \quad L_6 = 0.5/\beta - 1
\]
In which, $\alpha$ and $\beta$ are the parameters of the generalized Newmark constant and $\Delta t$ is the time step.

The vectors $\dot{q}_t$, $\ddot{q}_t$ and $\dot{p}_t$ can be evaluated explicitly from the information available at time $t_n$. The surface footing is assumed to be at top surface resting on centre of domain. The load vector is distributed as follows:

$$\{Q\}_k = \int_{-1}^{1} \{Q\}_z \begin{bmatrix} \frac{1}{2}(\xi + \xi^3) \\ (1-\xi^2) \\ \frac{1}{2}(\xi - \xi^3) \end{bmatrix} d\xi$$

(5)

In which, $Q$ is applied surcharge per unit length. $\xi$ is the Cartesian coordinate in $x$ direction.

Pastor-Zienkiewicz Mark III model (Kumari et al. 2016) has been used to define the inelastic behaviour of soil which include dilatancy and hardening.

$$D^{ep} = D^e - \frac{D^e n_{g/L} n^T D^e}{H_{L/U} + n^T D^e n_{g/L/U}}$$

(6)

Where, $D^{ep}$, $D^e$, $n$, $n_{g/L/U}$ and $H_{L/U}$ are the elastoplastic constitutive matrix, elastic constitutive tensor, loading direction vector, flow direction vector under loading or unloading conditions, and loading or unloading plastic modulus, respectively.

$$g = q M_g p^e \left(1 + \frac{1}{\alpha} \right) \left[1 - \left(\frac{p^e}{p_{g}^e}\right)^\alpha \right], d_g = (1 + \alpha). (M_g - \eta)$$

(7)

In which, $p$ is mean confining stress; $q$ is deviatoric shear stress; $M_g$ is slope of the critical state line; $\alpha$ are constants; $p_{g}^e$ are size parameters.

**VALIDATION**

![Figure 1. Excess pore pressure verses time at 2 m depth.](image)

For soil liquefaction behavior and modeling, the centrifuge modeling has been considered among the best experimental methods. The stress conditions generated during the said phenomena can be closely simulated to the full-scale prototype model. Hence, the correctness and accuracy of the proposed finite element based solution algorithm are authenticated with centrifuge test results. Figure. 1. and figure. 2. Shows the validation of excess pore water pressure and displacement respectively.

The soil used in the validation model is a fine, uniform Nevada sand with $D_{50} = 0.13$ mm. The permeability of the sand calculated by standard ASTM code in the laboratory at $1 \text{ g}$ is $k = 0.0021$ cm/s. It was subjected to a field
acceleration of 80 g with the centrifuge model test results, conducted at the RPI centrifuge facility.

Figure 2. Variation of settlement centre of foundation at surface.

NUMERICAL ANALYSIS

A saturated soil domain of loose sand deposit having depth 15 m and width 24 m is considered for modeling the present problem (figure 3) for plane-strain conditions. On top of the soil deposit atmospheric pressure is applied whereas at bottom, drain boundary exist. On the left and right of soil domain, Kelvin elements are used to simulate infinite boundary. The width of footing is considered as 4 m.

The FEM code has been written in FORTRAN 90. The variation of both parameters excess pore pressure and displacement with time was considered for comparing the response.

A static analysis is performed to apply the gravitational forces due to self-weight of the soil and foundation before cyclic excitation. The resulting initial stress is evaluated throughout the considered domain due to hydrostatic pressures of fluid and used as initial conditions for the subsequent dynamic analysis. The equilibrium condition is attained after evaluating initial stress condition. Then, a nonlinear analysis is performed for the harmonic load with the supplied horizontal and vertical cyclic acceleration $a = a_0 \sin \omega t$. The dynamic analyses are performed using a Generalized Newmark scheme with nonlinear iterations by taking initial linear elastic tangential global matrix. The numerical integration parameters of the generalized Newmark’s method are selected as $a = 0.60$ and $b = 0.3025$ for the dynamic analysis.

The time step used is usually governed by time of cyclic loading and frequency of the input motion. Rayleigh damping of 5 % is applied at the prevailing frequency in the earthquake like motion input to enhance the energy dissipation characteristic of the constitutive model. For 64 cycles of the loading motion, the numerical simulation has been performed. The amplitude and frequency of the cyclic loading were $a_0 = 0.2$ g and 1.5 Hz respectively. A surface footing of intensity 0.1 t/m² is assumed to be resting at top surface of the central position element of saturated sand layer.
RESULT AND DISCUSSION

This numerical model based on fully coupled formulation to analyse the liquefaction behaviour of shallow foundation in different situation. The results based on numerically simulation are discussed below.

Figure 3. and figure 4. shows the results of horizontal and vertical displacement below the footing at left, centre and right side. The maximum values of horizontal settlement of 4.32 cm and vertical settlement of 9.4 cm are predicted at the top of soil layer below the right of footing. The value obtained at right side is higher than the left side which may be due to amplification of the seismic wave. It has been also observed that most of the settlements occur during the shaking period that is before 15 cycle. A decreasing or constant trend is observed for settlement after completion of cyclic load. Generally, the horizontal settlement is less than vertical settlement at different depths.
Figure 4. Computed horizontal displacement with respect to time at below the footing.

Figure 5. Computed EPP at centre of footing with respect to time.

Figure 5 displays the computed excess pore pressure at different depth. Pore pressure began to increase once input cyclic load imparted to the soil domain. The computed pore pressure time histories indicate that soil at $Z/B$ is 1.0, 1.5, 2.0 and 2.5 is not liquefied because excess pore pressure (EPP) is less than initial vertical stress except at the value of $Z/B = 0$ and $Z/B = 0.5$.

**Effect of Shear Modulus**

The effect of shear modulus is also studied for shallow foundation on liquefiable soil at an acceleration of 0.2 $g$ m/s$^2$. The shear modulus ($G$) has been varied as 8 MPa, 12 MPa, 16 MPa, and 20 MPa respectively, while keeping other parameters constant. The relative density of the sand is taken as 54%. The study is carried out at two values of permeability $1.68 \times 10^{-4}$ m/s and $2.1 \times 10^{-6}$ m/s respectively. Table 1 shows the variation of displacement with respect to time, at different shear modulus.

A downward trend is noticed for both type of displacement as the value of shear modulus is increased. It is also observed that the maximum horizontal settlement (4.2 cm) and the maximum vertical settlement (9.55 cm) occur at 8 MPa respectively. This behaviour is occurring due to increases in stiffness of the soil. The vertical settlement is higher than that of horizontal settlement. The effect of permeability is also studied. A decreasing trend of displacement is seen...
with decrease in permeability due to increase in pore pressure. In vertical displacement about 16 % reduction observed with reduction of permeability from $1.68 \times 10^{-4}$ m/s to $2.1 \times 10^{-6}$ m/s, whereas the horizontal displacement variation is only 2 % to 4 % which is not significant.

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<th>Table 1. Variation of horizontal and vertical displacement.</th>
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<td>Frequency (Hz)</td>
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<td>$k=1.68 \times 10^{-4}$ (ms$^{-1}$)</td>
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<td>$k=2.1 \times 10^{-6}$ (ms$^{-1}$)</td>
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The effects of shear modulus on peak value of effective pore pressure, at different depth are shown in figure 6. It is seen that at 2 m depth, EPP variation with respect to shear modulus is negligible because of development of high pore pressure. At this depth, liquefaction is occurring at frequency 1.5 Hz for a particular value of cyclic loading. So, it may be concluded that for a particular value of dynamic loading, liquefaction may occur at any stiffness. It is also observed that effective pore pressure decreases with the increase in shear modulus. As the value of permeability decreases, increase in excess pore pressure.

Figure 6. (a) & (b) Variation of effective pore pressure (EPP) at different permeability and shear modulus value.
CONCLUSIONS

The model based on coupled formulation is able to simulate both settlement and development of pore pressure simultaneously. So it is more accurately estimated the liquefaction phenomena of shallow foundation on liquefiable soil. The maximum vertical settlement of 9.5 cm and horizontal displacement of 4.5 cm are observed at right of the footing surface of the soil domain respectively. At higher value of shear modulus ($G = 20 \text{ MPa}$), liquefaction does not occur within the soil domain except at $Z/B = 0$ and $0.5$. As the shear modulus is reduced, liquefaction of soil is observed because of generation of higher displacement and pore pressure. Due to cyclic inertial forces inside soil-structure also amplified cyclic pore pressure induced softening directly under the foundations. It is observed that maximum stress ratio $q/p$ is 0.98 at the depth of $Z/B = 0.25$, which declines with depth mainly due to effect of initial effective stress. The developed numerical formulation can be easily adapted to provide confidence for practicing engineers to use fully coupled procedures for predicting the dynamic performance of geotechnical site.

REFERENCES