Dynamic Analysis of Rigid Pavement Resting on two Parametric Foundation

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ABSTRACT: In our present work we have considered the effect of time varying sinusoidal load generated through the quarter truck model moving with the constant velocity on the rigid pavement. The effect in the terms of deflection generated on the pavement is considered on the two and one parameter model for both dynamic and the static case and the results are compared. The finite element procedure is applied for the dynamic analysis of the rigid pavement in which the pavement is represented as finite beam elements of different lengths. The underlying soil medium is represented by the Pasternak model in which soft subgrade soil is modeled by the spring and Dashpot elements and the base course is modeled by the shear layer. The trapezoidal method is used for the solution of the dynamic equation with the help of which nodal displacements were generated. Furthermore, a time dependent response is also generated to compare the displacement with and without considering damping in the structure. Software like Mat Lab, Etabs and Sap2000 were used for the analysis purpose. The conclusion of our work is that, the maximum deflection of the pavement decreases with the increase in the subgrade modulus and thickness of the beam for same length and shear modulus value. Similarly, the maximum deflection also decreases for the same subgrade modulus, increasing shear modulus and keeping the length constant. The pavement response generated in terms of displacement for damped and un-damped case clearly showed that the displacement was more in later case. The maximum deflection generated in the dynamic case is more than the static case thus we can say that the dynamic analysis is important for the design of the pavement

INTRODUCTION

The core purpose of this paper is to highlight the importance of dynamic loads on the rigid pavement by comparing the displacement generated for static and dynamic conditions. To give weightage to our results pavement displacement is calculated for various combinations of length, thickness, shear modulus and subgrade modulus. The initial development in the procedure of the pavement analysis is based on the closed form solution which is obtained from the static analysis of structure of the pavement in which the pavement loads are assumed to be static and constant. However, there exist considerable differences between the static and dynamic responses generated. Engineers have been investigating the potential hazards of the dynamic load in the recent years which is now a problem of widespread practical significance. The importance of moving mass found practical application in the field of transportation, aerospace engineering, railways, bridges, etc.

The majority of engineering structures are subjected to the load which vary with time and space. Increase in the traffic load and the speed of the vehicle has caused the engineer for the more detailed study of the pavement structures. The effect of dynamic load is more apparent in the case of the thin slab. The
simplest case of the moving load is the case in which the concentrated load is moving over the simple beam and is represented by the fourth order differential equation.

**METHODOLOGY**

The methodology of the present work include the application of the finite element technique to determine the dynamic equation for the Pasternak model and its solution with the trapezoidal method. The Quarter truck model is used to study vehicle responses. The entire methodology can be represented in the flow chart as described below.

![Flow Chart](image)

**Formulation of quarter truck model equation**

Referring to the fig. 2, the formulation of the model is done by assuming that the response generated in the vehicle is of the form;

\[ x = x_0 e^{i\omega t} \]  \hspace{1cm} (1)

where, \( x \) is the amplitude of the vehicle, \( x_0 \) is the maximum amplitude of the vehicle, \( t \) is the time and \( \omega \) is the natural frequency of the vehicle. Now to analyze the behavior of the suspension system use the displacement \((x_1-x_2)\) of the tire, as \((x_1-w)\) is hard to measure and \((x_2-w)\) is negligible.
Using Newton’s 2nd and 3rd law of motion we get,

\[ M\ddot{x} + C_s (\dot{x}_1 - \dot{x}_2) + K_s (x_1 - x_2) = 0 \]  
(2)

\[ m\ddot{x} + C_s (\dot{x}_1 - \dot{x}_2) + K_s (x_1 - x_2) + C_i \dot{x}_2 + K_i \ddot{x}_2 = 0 \]  
(3)

On writing the above set of equations in the matrix form,

\[ \begin{bmatrix} M \end{bmatrix} \begin{bmatrix} \dddot{x} \end{bmatrix} + \begin{bmatrix} C \end{bmatrix} \begin{bmatrix} \dot{x} \end{bmatrix} + \begin{bmatrix} K \end{bmatrix} \begin{bmatrix} x \end{bmatrix} = 0 \]  
(4)

We, assumed here that the load transferred to the pavement is of Sinusoidal nature, \( F = F_0 \sin \omega t \)  
(5)

\[ \frac{d^4w}{dx^4} - \frac{G}{EI} bH \frac{d^2w}{dx^2} + \frac{kb}{EI} w = \frac{P}{EI} \]  
(6)

The equilibrium for a beam on the Pasternak foundation varies according to the above equation but when the concentrated load acts on the beam the above equation reduces to the form,

\[ \frac{d^4w}{dx^4} - \frac{G}{EI} bH \frac{d^2w}{dx^2} + \frac{kb}{EI} w = 0 \]  
(7)

**Finite Element Formulation**

We formulated the shape function of our model by taking a 2-noded beam element having 2 degrees of freedom per node namely transverse displacement and slope. The cubic displacement field \( v(x) \) is obtained for the above beam such that it covered the values of deflection and slopes at either end as given by the nodal values \( v_i, v_j, \Theta_i \) and \( \Theta_j \), the sign convention is as shown in the above figure.

\[ v(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 \]  
(8)

The shape function of the above beam element is as follows,
\[ N_1 = 1 - \frac{3x^2}{L^2} + \frac{2x^3}{L^3} \]
\[ N_2 = x - \frac{2x^2}{L} + \frac{x^3}{L^2} \]
\[ N_3 = \frac{3x^2}{L^2} - \frac{2x^3}{L^3} \]
\[ N_4 = -\frac{x^2}{L} + \frac{x^3}{L^2} \]

On writing the above set of equations in the matrix form we get,

\[
v(x) = \begin{bmatrix} N_1 & N_2 & N_3 & N_4 \end{bmatrix} \begin{bmatrix} v_i \\ \theta_i \end{bmatrix} \]

Dynamic equation formulation of the Pasternak model

The strain \( U \) of the system can be expressed in the following form,

\[
U = \frac{1}{2} \int_0^L \left( EI \left( \frac{d^2w}{dx^2} \right)^2 - GbH \left( \frac{dw}{dx} \right)^2 + kbw^2 \right) dx
\]

The element strain energy is given by,

\[
U_e = \frac{1}{2} [q]^T [K^e] [q]
\]

The beam element stiffness matrix is given by;

\[
[K^a_{beam}] = \begin{bmatrix}
12 & -6L_e & -12 & 6L_e \\
6L_e & 4L_e^2 & -6L_e & 2L_e^2 \\
-12 & -6L_e & 12 & -6L_e \\
6L_e & 2L_e^2 & -6L_e & 4L_e^2
\end{bmatrix}
\]

The subbase stiffness matrix is given by;

\[
[K_{subbase}] = \begin{bmatrix}
\frac{1}{5} & \frac{L_e}{20} & -\frac{3}{5} & \frac{L_e}{20} \\
\frac{L_e}{20} & \frac{L_e^2}{15} & -L_e & -\frac{L_e^2}{20} \\
-\frac{3}{5} & -\frac{L_e}{20} & \frac{3}{5} & -\frac{L_e}{20} \\
\frac{L_e}{20} & -\frac{L_e^2}{60} & -L_e & \frac{L_e^2}{20}
\end{bmatrix}
\]

The foundation stiffness matrix is given by;

\[
[K_{foundation}] = Kb \begin{bmatrix}
156 & 22L_e & 54 & -13L_e \\
22L_e & 4L_e^2 & 13L_e & -3L_e^2 \\
54 & 13L_e & 156 & -22L_e \\
-13L_e & -13L_e^2 & -22L_e & 4L_e^2
\end{bmatrix}
\]

The eq. no.11 can be written in the stiffness matrix form for any appropriately developed finite beam elements.
\[
\left[ [K_{\text{beam}}] + [K_{\text{subbase}}] + [K_{\text{foundation}}] \right] \{\delta\} = \{f\}, \text{ where } \{\delta\} \text{ and } \{f\} \text{ are the nodal displacements and nodal force vectors. The matrix formed by the combination of } \\
\left[ [K_{\text{beam}}] + [K_{\text{subbase}}] + [K_{\text{foundation}}] \right] \text{ is the stiffness matrix } K \text{ for the beam foundation system. In addition to this the mass matrix } M \text{ and the damping matrix } C, \text{ are needed for the dynamic analysis of the system. It may be mentioned here that all the matrix have been computed by the Gaussian integration and the consistent mass matrix has been used for } M. \text{ Once they are }
\]
formed the dynamic analysis can be performed in the time domain numerically. We have used here the \textbf{Trapezoidal method} for the solution of the dynamic equation in the time domain. The mass matrix for the beam element can be written as,
\[
[M] = \frac{\rho A L_e}{420} \begin{bmatrix}
156 & 22L_e & 54 & -13L_e \\
22L_e & 4L_e^2 & 13L_e & -3L_e^2 \\
54 & 13L_e & 156 & -22L_e \\
-13L_e & -13L_e^2 & -22L_e & 4L_e^2 \\
\end{bmatrix}
\]
The damping matrix for the beam element can be written as,
\[
[C] = \frac{c L_e}{420} \begin{bmatrix}
156 & 22L_e & 54 & -13L_e \\
22L_e & 4L_e^2 & 13L_e & -3L_e^2 \\
54 & 13L_e & 156 & -22L_e \\
-13L_e & -13L_e^2 & -22L_e & 4L_e^2 \\
\end{bmatrix}
\]
The dynamic equation for the beam in the matrix form can be written as,
\[
[M] \frac{\partial^2 \{q\}}{\partial t^2} + [C] \frac{\partial \{q\}}{\partial t} + K \{q\} = F
\]
Here, \( F \) is the external force acting on the system. Since, for the each time step (in present work, load is calculated for \( \Delta t = 1 \text{ sec} \) only) the load is acting on the central position of the beam.

**Static analytical solution of the Pasternak model**

For doing the static analysis we have solved the eq. no. 7 by considering the concentrated load to be acting at the central position of the beam and found the maximum deflection produced in the beam. The given equation is the fourth order differential equation which is solved by applying the following initial conditions given below,
\[
-EL \frac{d^2 w}{dx^2}(0) = 0, \quad -EL \frac{d^2 w}{dx^2}(l) = 0, \quad -EL \frac{d^4 w}{dx^4}(l/2) + \frac{G b}{dx^2}(l/2) = P/2
\]
and
\[
-EL \frac{d^3 w}{dx^3}(l) + \frac{G b}{dx^2}(l) = 0
\]
on putting the above conditions in eq. no. 7 we get the solution of the form,
\[
w(x) = e^{Al_1} \left( C_1 \cos(ABx) + C_2 \sin(ABx) \right) + e^{Al_2} \left( C_3 \cos(ABx) + C_4 \sin(ABx) \right)
\]
Where \( C_1, C_2, C_3 \) and \( C_4 \) are the constants which depends upon the initial conditions.

**RESULTS & DISCUSSIONS**

The natural frequency of the vehicle was determined by using the data of the table 1 which came out to be 30 rad/sec. Since we have assumed that the load transferred to the pavement from the vehicle is of the sinusoidal form thus finally load generated from the vehicle is \([F=60\sin(30t)] \) where 60 KN is the one-fourth weight of the vehicle.
Table 1. Vehicle parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>5500 kg</td>
</tr>
<tr>
<td>m</td>
<td>500 kg</td>
</tr>
<tr>
<td>(K_s)</td>
<td>150000 N/m</td>
</tr>
<tr>
<td>(C_s)</td>
<td>1120 N-s/m</td>
</tr>
<tr>
<td>(K_t)</td>
<td>310000 N/m</td>
</tr>
<tr>
<td>(C_t)</td>
<td>3100 N-s/m</td>
</tr>
</tbody>
</table>

Table 2. Pavement input parameters [Ref no. 22].

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Length of the beam (L)</td>
<td>5m, 10m, 15m and 20m</td>
</tr>
<tr>
<td>2. Thickness of the pavement (t)</td>
<td>100 mm, 200mm, 300mm and 400 mm</td>
</tr>
<tr>
<td>3. Subgrade modulus (K)</td>
<td>10000, 20000 and 30000 KN/m^3</td>
</tr>
<tr>
<td>4. Shear modulus (G)</td>
<td>0, 10000 and 20000 KN/m^2</td>
</tr>
<tr>
<td>5. Width of the beam (b)</td>
<td>1 m</td>
</tr>
<tr>
<td>6. Damping constant (C)</td>
<td>600 N-s/m</td>
</tr>
<tr>
<td>7. Shear layer (H)</td>
<td>1 m</td>
</tr>
<tr>
<td>8. Modulus of elasticity (E)</td>
<td>36050000 KN/m^2</td>
</tr>
<tr>
<td>9. Density ((\rho))</td>
<td>2400 kg/m^3</td>
</tr>
</tbody>
</table>

We have taken our beam to be simply supported thus eq. 10 reduces to the form,

\[ v(x) = N_2 \ast \theta_i + N_4 \ast \theta_j \]  \hspace{1cm} (20)

Solving the above equation by the method and procedure described above we get the following equation, solution of which gave the maximum deflection generated in the beam for both the static case and the dynamic case.

\[ v(x) = 4.756 \times 10^{-5} x^3 + 3.549 \times 10^{-4} x^2 + 0.5856 \times 10^{-3} x \]  \hspace{1cm} (21)

Now the results are plotted for the different cases and the comparative study was done to check the difference in the maximum deflection generated on both the static and the dynamic case.

Figure 5. PLOT OF DISPLACEMENT v/s TIME FOR THE DAMPED CASE.
Figure 6. PLOT OF DISPLACEMENT v/s TIME FOR THE UNDAMPED CASE.

From the above graphs we can see that the deflection generated in the undamped case is more than the damped case. Maximum deflection was calculated for the different thickness of 100mm, 200mm, 300mm and 400mm of pavement to get the clear idea of the difference in the deflection of the static and the dynamic cases and the results were plotted to get the clear idea. The similar trend were observed in all the cases so we have plotted the graph for the single case only in the present paper.

Figure 7. PLOT OF DYNAMIC CASE FOR L = 5m and G = 10000 KN/m².
CONCLUSIONS

The maximum deflection of the pavement decreases with the increase in the subgrade modulus and thickness of the beam for same length and shear modulus value. Similarly, the maximum deflection also decreases for the same subgrade modulus, increasing shear modulus and keeping the length constant. The pavement response generated in terms of displacement for damped and undamped case clearly showed that the displacement was more in later case which can be probably due to the free un-damped vibration taking place in the pavement. But in real situation damping is always present thus it is important to consider damping while performing structural analysis of the pavement. The maximum deflection generated in the dynamic case is more than the static case thus we can say that the dynamic analysis is important for the design of the pavement. It was also observed that the difference between the static and dynamic value is more prominently visible for the thin beam, as the thickness goes on increasing the percentage difference of deflection between dynamic and static case goes on decreasing.

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REFERENCES


F. Van Cauwelaert, Editor: Marc Stet, a textbook on “Pavement Design and Evaluation, the required mathematics and its applications”, Federation of the Belgian cement industry.


P. Seshu, Professor, Department of mechanical engineering, IIT BOMBAY, Mumbai, a text book of finite element analysis, PHI Learning Private Limited, New Delhi, 2012.


