Three-dimensional Robust Guidance Law with Impact Angle Constraint and Compensation of Channels Coupling

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ABSTRACT: To deal with the coupling between channels in the guidance system for a bank-to-turn missile and the uncertainty of the measurement signal, a novel three-dimensional guidance law with impact angle constraint and compensation of channels coupling is proposed based on the robust control theory. Firstly, line-of-sight rate is described through vector description, and the representation of channels coupling is presented. Secondly, a guidance law which fulfills $L_2$ performance index is designed by taking the measurement uncertainties as exogenous disturbance under the nonlinear robust control theory. A novel form of guidance law is obtained after joining the coupling term. Simulation results show that the proposed robust guidance law can guarantee the integrity of guidance information, also satisfy the requirements of guidance precision and impact angle constraint, and more robust than the traditional guidance law.

1 INSTRUCTIONS

As a modern control design, bank-to-turn (BTT) technique has better performance than traditional skid-to-turn (STT) technique with respect to large lift-to-drag ratio and high maneuverability. In a STT missile, roll rates is treated as 0 so that the cross-coupling between the pitch and yaw is negligible, and design of the three-dimensional guidance law is to decouple into longitudinal and latitude motions. However, in a BTT missile, high maneuverability can be expecting to cause the orientation of the acceleration changing rapidly, which means the cross-coupling is large and cannot be neglected.

One method is to treat the cross-axes couplings as unknown disturbances and the guidance law design problem is formulated as a disturbance attenuation control problem (Yang & Chen, 1998, Zhou et al., 2001, Hu, 2010). Yang and Chen proposed an $H_\infty$ robust guidance law and solved the associated Hamilton-Jacobi partial differential inequality (Yang & Chen, 1998). Zhou et al. designed a robust guidance law with $L_2$ gain performance and proved its robust stability, but the guidance law was under 2-dimensional mathematical model (Zhou et al., 2001).

Impact angle constrained guidance has become a necessity in modern warfare, and conventional guidance laws such as proportional navigation are silent about the impact angle constraints in general. Sliding mode control theory has been used to design three-dimensional (3D) impact angle guidance (Kumar & Ghose, 2014, Kumar et al., 2012, Kumar et al., 2014). There also exist some 3D impact angle guidance laws based on other techniques. Sun and Zheng proposed a three-dimensional guidance law for the impact angles control, using the variable structure control theory (Sun & Zheng, 2007). Oza and Padhi proposed a suboptimal guidance law in three-dimensional space with model predictive static programming technique (Oza & Padhi, 2010, Oza & Padhi, 2012). Wang proposed a partial integrated guidance law which satisfies terminal impact angle constraints in both azimuth and elevation for a STT missile (Wang & Wang, 2014).

The main contribution of this paper is to develop an impact angle constrained guidance law under the 3D engagement geometry. The guidance law regards the uncertainness of model parameters and the measure bias of target as disturbance, and the robust control theory has been used to guidance law $L_2$-gain performance, which ensures asymptotic and desired impact angle requirement.

2 PROBLEM STATEMENT

Consider a 3D engagement geometry as shown in Figure 1, where M stands for vehicle and T stands for target. The vehicle velocity is $\mathbf{v}_m$, the relative distance vector of the missile to the target is assumed to be $\mathbf{R}$ and define $\mathbf{e}$, be the unit vector of
R. Decompose the three-dimensional motion of the vertical plane motion and the bank plane motion. Let \( e_t \) be unit vector along the line TY and \( e_d \) be the unit vector along the intersection line of the bank plane and the XTZ-plane. The terms \( V_t \) and \( V_z \) are the projection vector of \( V_m \) in the vertical plane and the bank plane, and the terms \( q_{d} \) and \( q_{l} \) are the azimuths of LOS in the vertical plane and the bank plane respectively. \( q_{nt} \) is the angle between the LOS and TM', which is the intersection line of the bank plane and the YTX-plane.

![Figure 1. Engagement geometry in 3-dimensional space. Noted that \( e_h \) is the unit vector which perpendicular to the bank plane.](image)

Define \( q \) be the angle of sight, then \( \dot{q} \) is the angular velocity of sight which can be derived as the vector sum of and altitude angular velocity and azimuth angular velocity:

\[
\dot{q} = \dot{q}_d e_d + \dot{q}_i e_i
\]  
(1)

One guidance purpose is that the component of the angular velocity of sight which is perpendicular to \( \dot{q} \) tends to 0. Noted the component as \( \dot{q}_l \):

\[
\dot{q}_l = \dot{q}_d e_d + \dot{q}_i \cos q_d e_n
\]  
(2)

The projection vector of \( V_m \) in \( e_d \) can be expressed in two ways as following:

\[
r \cos q_d \dot{q}_i = r \dot{q}_n
\]  
(3)

Introduce(3) to, then

\[
\dot{q}_l = \dot{q}_d e_d + \dot{q}_n e_n
\]  
(4)

The time derivative of (4) is given by

\[
\ddot{q}_l = \ddot{q}_d e_d + \dot{q}_n \dot{e}_n + \dot{q}_d \dot{e}_d + \dot{q}_i \dot{e}_i
\]  
(5)

(5) Can be expressed by the spherical unit vectors \( (e_d, e_i, e_n) \):

\[
\dot{e}_d = \dot{q}_i \cos q_d e_i - \dot{q}_i \sin q_d e_n
\]

\[
\dot{e}_n = \dot{q}_i \sin q_d e_d
\]  
(6)

Assume the target is non-maneuvering, the equations of motion in two-dimensional are given by:

\[
\ddot{q} = \frac{2\dot{R}}{R} q + \frac{\dot{R}}{R} \dot{q}
\]  
(7)

\[
u = \dot{\theta}
\]

Where \( R \) is the relative displacement along the line of sight, and \( \dot{R} \) is the relative velocity, \( \dot{\theta} \) is the angular velocity of the missile velocity vector. We can extend (7) to 3D space:

\[
\dot{q}_l = \frac{2\dot{R}}{R} q - \frac{\dot{R}}{R} \dot{q} + q_d \dot{q}_i \cos q_d e_r
\]

\[
-\dot{q}_d \dot{q}_l \sin q_d e_n + \dot{q}_n \dot{q}_l \sin q_d e_d
\]

\[
u = \dot{\theta}_d e_d + \dot{\theta}_i e_i
\]  
(8)

Where \( \dot{\theta}_d \) is the angle between \( V_t \) and TM', and \( \dot{\theta}_i \) is the angle between \( V_z \) and TM'.

3 DESIGN OF THE THREE-DIMENSIONAL ROBUST GUIDANCE LAW

In this section, the design of a guidance law with L₂-gain performance is considered. (8) can be rewritten as

\[
\dot{q}_l = \frac{2\dot{R}}{R} q - \frac{\dot{R}}{R} \dot{q} + q_d \dot{q}_i \cos q_d e_r
\]

\[
u = u^* - \Omega
\]  
(9)

\[
\Omega = \frac{R}{R} (q_d \dot{q}_i \cos q_d e_r - q_d \dot{q}_i \sin q_d e_n + q_d \dot{q}_i \sin q_d e_d)
\]

Where the input \( u \) contains guidance input \( u^* \) and coupling \( \Omega \). Define new states

\[
x_1 = \dot{q}_l, \quad x_2 = q_1, \quad a = 2\dot{R}/R = \dot{a} + \dot{b} \Delta_1, \quad
\]

\[
b = 1/R = \ddot{b} + \dot{b} \Delta_2, \quad \text{where} \quad \dot{a} = 2\dot{R}/\dot{R}, \quad \ddot{b} = 1/\dot{R}, \quad \dot{R}
\]

and \( \dot{R} \) are the measure value of \( R \) and \( \dot{R} \), thus \( \dot{b} \Delta_1 \) and \( \ddot{b} \Delta_2 \) stand for the measure error of \( \dot{a} \) and \( \dot{b} \) in turn. Then (9) can be rewritten as

\[
x_1 = x_2
\]

\[
x_2 = (\ddot{a} + \ddot{b} \Delta_1) x_2 + \frac{1}{2} (\ddot{a} + \ddot{b} \Delta_1) u^*
\]  
(10)

\[
= \ddot{a} x_2 + \frac{1}{2} \ddot{a} u^* + \ddot{b} w
\]

Where \( w = \Delta x_2 + \Delta u^* \) is the disturbance from the uncertainty of system model parameters.

Consider the nonlinear form of system as following:

\[
\begin{bmatrix}
\mathbf{x} = f(\mathbf{x}) + g_1(\mathbf{x}) \mathbf{w} + g_2(\mathbf{x}) \mathbf{u}^* \\
\mathbf{z} = r(t) \mathbf{x}
\end{bmatrix}
\]  
(11)
Where the $x_2$ is regarded as guidance performance, $r(t)$ is a weighting factor concerning the tradeoff between acceleration command and performance. The nonlinear control problem is to find the control $u^*$ such that the following $L_2$-gain condition of the system is less than or equal to $\epsilon$:

a) When $w=0$, the closed-loop system is globally stable.

b) Under zero initial conditions $x(0)=x_0$ and a positive constant $\epsilon$, for all $T>0$, and for all $\forall w \in L_2[0,T]$, the closed-open system satisfies the following inequality:

$$\int_0^T z^2 \, dt \leq \epsilon^2 \quad (12)$$

i.e., the guidance law $u^*$ keeps the influence under the action of the disturbance $w$ to be small and upper bounded.

Choose the guidance law as

$$u^* = -2x_2 + \frac{2b}{a}v \quad (13)$$

Introduce (13) to (10), then

$$x_2 = \tilde{b}(w + v) \quad (14)$$

Where $v$ is an auxiliary control input which will be determined later.

By using Lyapunov stability theory, set a positive definite Lyapunov function as

$$V = k_3 \left( x_1 - x_{1f} \right)^2 + \frac{x_1^2}{2b^2} \quad (15)$$

Where, $x_{1f}$ is vector of desired line of sight, $k_3>0$.

Taking into account (12), the time derivative of (15) is given by

$$V = k_3 \left( x_1 - x_{1f} \right) x_2 + \frac{1}{b} x_2 v + \frac{1}{2b^2 \epsilon^2} x_2^2 + \frac{\epsilon^2 - 1}{2} \left[ \frac{1}{b} x_2 - \epsilon w \right]^2 - \tilde{b} x_2^2$$

$$= -\frac{z^2}{2} + \frac{\epsilon^2(t)}{2} x_1^2 + k_3 \left( x_1 - x_{1f} \right) x_2 + \frac{1}{b} x_2 v$$

$$+ \frac{1}{2b^2 \epsilon^2} x_2^2 + \frac{\epsilon^2 - 1}{2} \left[ \frac{1}{b} x_2 - \epsilon w \right]^2 - \tilde{b} \left( x_1 - x_{1f} \right)^2$$

$$\leq -\frac{z^2}{2} - \frac{k_3}{2} x_2^2 - \frac{\tilde{b} \epsilon^2}{b} x_2^2 + \frac{\epsilon^2 - 1}{2} \left( \frac{1}{b} x_2 - \epsilon w \right)^2 - \tilde{b} \left( x_1 - x_{1f} \right)^2$$

$$+ k_3 \left( x_1 - x_{1f} \right) x_2 + \frac{1}{2b^2 \epsilon^2} x_2^2 + \frac{k_3}{2} x_2^2 + \frac{1}{b} x_2 v$$

And let

$$v = -\tilde{b} \left( \frac{r^2(t)}{2} x_2 + k_3 \left( x_1 - x_{1f} \right) + \frac{1}{2b^2 \epsilon^2} x_2^2 + \frac{k_3}{2} x_2^2 \right) \quad (17)$$

Then (13) and (17) are rewritten as

$$u^* = -\left\{ 2 + \frac{\tilde{b} k_3}{a} + \frac{\tilde{b} \epsilon^2(t)}{2a} + \frac{1}{a \epsilon^2} \right\} x_2 \quad (18)$$

$$V \leq -\frac{k_3}{2} x_2^2 - \frac{\tilde{b}}{b^2} x_2^2 - \frac{\epsilon^2}{2} w^2 \leq \frac{\epsilon^2}{2} w^2 \quad (19)$$

When no disturbance $w$ is considered, setting

$$k_3 = k_4 \left| \frac{\tilde{b}}{b^3} \right|^3 = 2k_4 \tilde{R} |\tilde{R}|, k_4 \geq 1 \quad (20)$$

Then we have $V \leq 0$ which implies that $V$ is positive definite and $V$ is negative semi-definite. Therefore $x_2$ tend to zero and $x_1$ converges to $x_{1f}$ as $t \rightarrow \infty$.

When disturbance $w$ is considered, we have

$$-\frac{z^2}{2} + V \leq -\frac{k_3}{2} x_2^2 - \frac{\tilde{b}}{b^2} x_2^2 + \frac{\epsilon^2}{2} w^2 \leq \frac{\epsilon^2}{2} w^2 \quad (21)$$

Integrating (21) from 0 to $T$ with zero initial condition $x(0)=x_0$,

$$V(x_0) + \frac{1}{2} \int_0^T \left[ \epsilon^2 w^2 - z^2 \right] d\tau \geq V(x) \quad (22)$$

Using (12), then (22) implies that $V \leq 0$. We can conclude that the guidance law (18) is globally stable and satisfy the $L_2$ gain performance. Let

$$r(t) = \sqrt{2k_4 \tilde{R}} \tilde{R} \quad (23)$$

And take account of (20), then (18) is rewritten as

$$u^* = [(2 + k_2 + k_4) + \frac{\tilde{R}}{2R \epsilon^2}] x_2 + \frac{k_3 \left( x_1 - x_{1f} \right)}{R \tilde{R}} \quad (24)$$

Where $k_2>0$, $k_3>0$, $k_4>1$ are guidance parameters. Thus, we have the control input

$$u = [(2 + k_2 + k_4) + \frac{\tilde{R}}{2R \epsilon^2}] \left[ q_d e_d + q_n e_n \right] + \frac{k_3 \left( q_d - q_{\alpha d} \right) e_d + \left( q_d - q_{\alpha d} \right) e_n}{R \tilde{R}}$$

$$-\frac{\tilde{R}}{R} \left( q_d \dot{q}_d \cos q_d e_r - q_d \dot{q}_d \sin q_d e_r - q_d \dot{q}_d \sin q_d e_{\alpha} \right)$$

\(e_d, e_n\)
4 SIMULATION

In order to evaluate the guidance performance of the derived three dimensional robust guidance law, we present some simulation results. A traditional PD guidance with impact angle constraint is used in vertical plane and the bank plane for comparison:

\[ a_c = -k_1 \dot{R} \dot{q} - \frac{k_2 R (q - q_f)}{R} \]  \hspace{1cm} (26)

The flight angular limits are the angle of attack \( \alpha \in [-2^\circ, 8^\circ] \), the bank angle \( \gamma < 90^\circ \). We assume the initial velocity is 250 m/s, the initial locality of the missile is \((0, 10, 0)\) km and the locality of the target is \((40, 0.215, 0)\) km, the initial attitude angles are the bank angle \( \gamma = 0^\circ \), the pitch angle \( \varphi = 0^\circ \) and the yaw angle \( \psi = 90^\circ \). The anticipative impact angular constraints are \( q_i = 45^\circ \), \( q_d = 0^\circ \). We select the parameters of guidance law to be \( k_2 = 1 \), \( k_3 = 5 \), \( k_4 = 1 \), \( \varepsilon = 4 \). The simulation results are shown in Figures 2-5.

The final state simulation results are presented in Table 1. Any of the guidance laws was able to get close to the target with a large initial yaw angle. Compared with the traditional guidance law, the proposed robust guidance law resulted in a less miss distance and the impact angle constraints more accurate. According to Figure 2, the trajectory computed by the robust guidance laws was smoother and with a smaller turning curvature. Comparisons of angle of attack, angle of side slip and bank angle are shown in Figures 3-5 in turn, it can be seen that the angles of traditional guidance law were divergence at the terminal part, which demonstrated the robustness of the proposed robust guidance law.

Table 1. Comparison of final state simulation results.

<table>
<thead>
<tr>
<th>Flight time</th>
<th>Flight Miss distance</th>
<th>Flight path azimuth angle</th>
<th>Flight path angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Robust guidance law</td>
<td>230.53</td>
<td>0.14</td>
<td>0.035</td>
</tr>
<tr>
<td>Traditional guidance law</td>
<td>249.11</td>
<td>3.16</td>
<td>3.68</td>
</tr>
</tbody>
</table>

5 CONCLUSION

In this paper, we have proposed a 3D impact angle constrained guidance law by using robust control theory with \( L_2 \)-gain performance. Simulations are carried out to evaluate the performances of the proposed guidance law and are shown to work well compared with the traditional guidance law. 3D guidance to control both impact time and impact angle for BTT missile is another important area for the future extension of this work.
REFERENCES


