Properties of Kinetic Energy Horizontal Gradient of Multilevel Barotropic Atmosphere

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ABSTRACT: Based on multilevel primary barotropic atmospheric equations of motion, the relations between the kinetic energy horizontal gradient at each level and corresponding geostrophic deviation of weather systems of different scales are obtained. The main conclusions are as follows. It is the physical essence of kinetic energy horizontal gradient that the values of kinetic energy gradient at each level are approximately proportional to the values of the corresponding geostrophic deviation in multilevel barotropic atmosphere. The analysis of the values of kinetic energy gradient makes it possible to diagnose and analyze atmosphere system of various scales in wind field terms. This is superior to meso scale systems.

1 INTRODUCTION

The motion of atmosphere contains not only energy transport, but also the transformation of kinds of energies. As one of the main energies in atmosphere, kinetic energy plays an important role in the motion of atmosphere. The distribution and evolution of it directly reflects the strength and evolution of weather systems. Geostrophic deviation is one of the key factors to the transformation of kinds of energies. The values of geostrophic deviation influence the development of the weather systems. This influence is more obvious in weather systems of meso and smaller scale. Thus, the diagnostic analysis of kinetic energy and geostrophic deviation has become an important part in dynamic diagnosis of atmosphere. Boualem Khouider et al. (2012) find that the transport of energy is from the weather systems of smaller scale to the larger ones in the study of the transfer of convective momentum. The horizontal divergence of actual wind is mainly determined by geostrophic deviation. Thus, geostrophic deviation has a decisive impact for the distribution and evolution of the vertical movement in atmosphere (Julien et al,
Besides, geostrophic deviation restricts the manufacturing and conversion of kinetic energy. In the prediction of rainstorm, the reason why the analyses of jet flow and its gale core always have satisfactory result is that these analyses reflect the distribution of kinetic energy and the horizontal gradient of it. From here it can be seen that there is close relationship between geostrophic deviation and kinetic energy horizontal gradient. Though the studies of the kinetic energy and geostrophic deviation have been a lot (Andersen et al., 2012; Sasaki and Hideharu, 2012), the physical relation between the two has not been extensive researched.

The relationship between the kinetic energy horizontal gradient and geostrophic deviation of weather systems of different scales based on primary barotropic atmospheric equations of motion has been discussed and the properties of kinetic energy horizontal gradient have been found (Yu and Zhang, 2012). It is indicated that the values of kinetic energy gradient are approximately proportional to the values of the geostrophic deviation. Because the motion in 500hPa can be approximately seen as barotropic, so the diagnosis of kinetic energy and its horizontal gradient in 500hPa has been made in the process of rainstorm on August 25, 2008 and the diagnosis is consistent with theoretical analysis.

As is known to all that the analysis of jet and its gale core in 700hPa and 850hPa is crucial to the prediction of the rainfall and the location of strong convection. But primary barotropic atmospheric equations of motion are not applicable to the lower atmosphere and the easiest way is to use multilevel primary barotropic atmospheric equations of motion. The relationships between the kinetic energy horizontal gradient at each level and corresponding geostrophic deviation are deduced in this paper.

2 ANALYSIS OF MULTILEVEL PRIMARY BAROTROPIC ATMOSPHERE

Assume that there are $K$ levels homogeneous fluid, the number of the level from top to bottom is $k = 1, 2, \ldots, K$. Suppose the corresponding density is $\rho_k$ and $\rho_k < \rho_{k+1}$. The height of upper interface in $k$ level is $z_k$. Its thickness is $h_k = z_k - z_{k+1}$, and $z_{K+1} = 0$ (without considering
terrain, \( h_k = z_k, \rho_0 = 0 \) (as shown in figure 1). It is also assumed that the velocity in each level is unchanged along the vertical direction. Thus, for \( k \) level (here \( k = 1, 2, \ldots, K \)), the multi-level primary barotropic atmospheric equations of motion can be written as follows (Zeng, 1979):

\[
\frac{\partial u_k}{\partial t} + u_k \frac{\partial u_k}{\partial x} + v_k \frac{\partial u_k}{\partial y} = -\frac{\partial F_k}{\partial x} + f v_k \tag{1.1}
\]

\[
\frac{\partial v_k}{\partial t} + u_k \frac{\partial v_k}{\partial x} + v_k \frac{\partial v_k}{\partial y} = -\frac{\partial F_k}{\partial y} - f u_k \tag{1.2}
\]

\[
\frac{\partial h_k}{\partial t} + u_k \frac{\partial h_k}{\partial x} + v_k \frac{\partial h_k}{\partial y} = -h_k \left( \frac{\partial u_k}{\partial x} + \frac{\partial v_k}{\partial y} \right) \tag{1.3}
\]

In the equations above, \( F_k = \sum_{j=1}^{k} \frac{\rho_j - \rho_{j+1}}{\rho_k} g z_j \) and \( z_j \) can be understood as deviation of the height of upper interface and the height when it is still for \( j \) level.

Eq.(1.1) and eq. (1.2) can also be written in another form(Ye and Li, 1965):

\[
\frac{\partial u_k}{\partial t} - (f + \zeta_k) v_k = -\frac{\partial P_k}{\partial x} \tag{2.1}
\]

\[
\frac{\partial v_k}{\partial t} + (f + \zeta_k) u_k = -\frac{\partial P_k}{\partial y} \tag{2.2}
\]

Here, \( P_k = F_k + \left( \frac{u_k^2 + v_k^2}{2} \right), \zeta_k = \frac{\partial v_k}{\partial x} - \frac{\partial u_k}{\partial y} \). Eq.(2.1) and eq. (2.2) are respectively similar with linearizationed eq.(1.1) and eq.(1.2). The difference is that \( f \) in the latter are substituted by absolute vorticity of the flow field, \( f + \zeta_k \), in the former. At the same time, \( F_k \) in the latter are substituted by \( P_k \). Ye and Chao (1998) point out that the term of time derivative is smaller than other terms for an order of magnitude. Therefore, \( \frac{\partial u_k}{\partial t} \) and \( \frac{\partial v_k}{\partial t} \) in equations (2) can be omitted. Thus, the relationship between \( u_k, v_k \) and \( P_k \) can be obtained. This relationship can reflect the evolvement law of meso and smaller scale systems (Ye and Li, 1965).

Multiply eq.(2.1) by \( u_k \) and eq.(2.2) by \( v_k \), then add them up. The sum of this and the result of eq.(1.3) multiplied by \( g \) can be obtained as:
Here, \( E_k = gh + (u_k^2 + v_k^2)/2 \). \( gh \) is the potential energy of unit density in \( k \) level, \( (u_k^2 + v_k^2)/2 \) is kinetic energy of unit density. \( E_k \) can be seen as the energy of unit density. For convenience, kinetic energy of unit density is hereinafter referred to as kinetic energy and kinetic energy horizontal gradient of unit density is referred to as kinetic energy horizontal gradient. Considering that \( gh = E_k - (u_k^2 + v_k^2)/2 \) and \( \zeta_k = \left( \frac{\partial}{\partial x} \right) u_k - \left( \frac{\partial}{\partial y} \right) v_k \), eq.(2.1), eq.(2.2) and eq.(3) constitute the closure equations.

3 RELATIONSHIP BETWEEN KINETIC ENERGY GRADIENT AND GEOSTROPHIC DEVIATION

Firstly, geostrophic velocity \( \bar{u}_k = -\left( \frac{\partial F_k}{\partial y} \right) / f \), \( \bar{v}_k = \left( \frac{\partial F_k}{\partial x} \right) / f \) and the corresponding geostrophic deviation \( u'_k = u_k - \bar{u}_k \), \( v'_k = v_k - \bar{v}_k \) are introduced in the multilevel primary barotropic atmospheric equations of motion. Besides, the dimensionless vorticity \( \hat{\zeta}_k \) is also used. Here, \( \left| \hat{\zeta}_k \right| \leq 1 \) and \( \zeta_k = \hat{\zeta}_k U_k / L_k \), where \( U_k \) is the scale of velocity of each level and \( L_k \) is the horizontal scale of the motion. Then, substitute \( \zeta_k \) in eq.(3) with \( \hat{\zeta} = \zeta_k L_k / U_k \). In \( k \) level, \( R_k = U_k / (L_k) \). Considering that the horizontal velocity is close to the horizontal scale of motion of each level, \( R_k \) can be substituted with \( R \). Finally, eq.(2.1) and eq.(2.2) can be written as follows:

\[
\begin{align*}
\frac{\partial E_k}{\partial t} + gh \left( \frac{\partial u_k}{\partial x} + \frac{\partial v_k}{\partial y} \right) &= -u_k \frac{\partial}{\partial x} (E_k + F_k) - v_k \frac{\partial}{\partial y} (E_k + F_k) \quad (3)
\end{align*}
\]

The followings are the discussion of the relationship between kinetic energy horizontal gradient and corresponding geostrophic deviation based on equations (4).

3.1 For systems of synoptic scale

For systems of synoptic scale in the mid-high latitudes, \( R \ll 1 \) and \( \left| \hat{\zeta} \right| \leq 1 \). Compared with 1, the terms with \( \hat{\zeta}_k R_k \) can be omitted. Thus equations (4) can be written as the following vector form:
In the northern hemisphere \( f > 0 \), it can be written as:

\[
f(V^\times k) = \nabla \left( \frac{u_i^2 + v_j^2}{2} \right)
\]  

(5)

In eq.(5) and eq.(6), \( \frac{u_i^2 + v_j^2}{2} \) is the kinetic energy of \( k \) level, \( \nabla \left( \frac{u_i^2 + v_j^2}{2} \right) - i \frac{\partial}{\partial x} \left( \frac{u_i^2 + v_j^2}{2} \right) + j \frac{\partial}{\partial y} \left( \frac{u_i^2 + v_j^2}{2} \right) \) is the kinetic energy horizontal gradient and \( V^\prime = u^\prime i + v^\prime j \) is the corresponding geostrophic deviation, where \( i, j, k \) are unit vector and \( k \) points to the zenith. Eq.(6) indicates that values of kinetic energy gradient at each level are approximately proportional to the values of the corresponding geostrophic deviation. In the northern hemisphere, proportional coefficient is \( f \) and it has nothing to do with the motion itself. Although the motion of synoptic scale is quasi-geostrophic and the wind field is in geostrophic balance with pressure field, geostrophic deviation has effect on the development and evolution of motion. Thus the geostrophic deviation is still very significant.

3.2 For systems of meso-\( \alpha \)-scale

For systems of meso-\( \alpha \)-scale in mid-high latitudes, \( R_\alpha < 1 \) and \( R_\alpha \hat{\zeta} < << 1 \). That is to say, \( R_\alpha \approx 0.3 - 0.5 \) (Zhang et al, 2008). In this case, the terms with \( \hat{\zeta} R_\zeta \) can only be partly omitted in equations (4). Here, terms in the first bracket on the left side of equations (4) are omitted. Similar with the systems of synoptic scale, the following equqtions can be obtained:

\[
f(1 + \hat{\zeta}R_\zeta)(V^\times k) = \nabla \left( \frac{u_i^2 + v_j^2}{2} \right)
\]  

(7)

\[
f(1 + \hat{\zeta}R_\zeta)|V^\prime| = \left| \nabla \left( \frac{u_i^2 + v_j^2}{2} \right) \right|
\]  

(8)

Because \( \hat{\zeta} R_\zeta < 1 \), so \( |1 + \hat{\zeta}R_\zeta| = (1 + \hat{\zeta}R_\zeta) \). Eq.(8) indicates that values of kinetic energy gradient at each level are still approximately proportional to the values of the corresponding geostrophic deviation. In this case, proportional coefficient is \( f(1 + \hat{\zeta}R_\zeta) \) and it is related to not only \( f \) but also the motion itself. It also means that proportional coefficient in meso-\( \alpha \) scale is related to \( \hat{\zeta} \) and \( R_\zeta \). According to references (Zeng, 1979; Zhang et al, 2008), geostrophic deviation in the motion of meso-\( \alpha \)-scale is larger than that of synoptic scale and the quasi-geostrophic balance between wind field and pressure field is not existed. But they are still quasi-
balanced and this relationship can be written as balanced equations. The larger geostrophic deviation means the evolution of systems is faster and the analysis of kinetic energy horizontal gradient is more necessary.

3.3 For systems of meso-β scale

For systems of meso-β and the smaller scale, \( R_\beta \sim 1 \) or \( R_\beta > 1 \) (Zhang et al, 2008). In this case, the terms with \( \hat{\xi}_k R \) cannot be omitted in equations (4). The wind field and the pressure field are imbalanced and the geostrophic deviation \( u'_k \) and \( v'_k \) are larger than geostrophic velocity \( \tilde{u}_k \) and \( \tilde{v}_k \). Take this into account, \( \hat{\xi}_k R \tilde{u}_k \) and \( \hat{\xi}_k R \tilde{v}_k \) can be omitted compared with \( \hat{\xi}_k R u'_k \) and \( \hat{\xi}_k R v'_k \) in equations (4). Thus, eq.(7) and eq.(8) can also be obtained. In this case, values of kinetic energy gradient at each level are still approximately proportional to the values of the corresponding geostrophic deviation. The proportional coefficient in the northern hemisphere is \( f |1 + \hat{\xi}_k R| \), for the case \( (1 + \hat{\xi}_k R) < 0 \) may exist. It is closely related to the motion itself. For the systems of meso-β and the smaller scale, geostrophic deviation is largest and the evolution of systems is fastest. Obviously, the analysis of kinetic energy horizontal gradient is most necessary.

4 SUMMARY

In this paper, the relations between the kinetic energy horizontal gradient at each level and corresponding geostrophic deviation of weather systems of different scales are obtained. The main conclusions are as follows. It is the physical essence of kinetic energy horizontal gradient that the values of kinetic energy gradient at each level are approximately proportional to the values of the corresponding geostrophic deviation in multilevel barotropic atmosphere. In mid-high latitudes, the proportional coefficient is for motions of synoptic scale, which is not relevant to the motion itself. For motions of meso-α and meso-β scale, the proportional coefficient is closely related to motion. The analysis of the values of kinetic energy gradient makes it possible to diagnose and analyze atmosphere system of various scales in wind field terms. This is superior to meso scale systems.

REFERENCES


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