Based on Fuzzy Theory to Model and Analysis about Global Stability of Seawall Established on Deep Silt

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Abstract. This article is based on Swedish circle method and Fuzzy theory and make assumptions to cohesion of silt C and internal frictional angle of silt those are the main random variable effecting the stability of whole seawall, then give the model of fuzzy reliability as a reference to be consulted in engineering.

Introduction

In analysis methods of the stability of seawall, Slope safety factor evaluation method don’t think about uncertain factors and variability of parameters or just simple think about them. For this reason there always be the deviation between designed and actual is very large. So some scholars apply structural reliability to analysis methods of the stability, and by counting stability of structural reliability to measure the stability.

But in the actual projects, traditional reliability theory cannot be well in actual conditions because of the complexity of project, People’s cognitive limitations and some uncertain factors during construction. In recent years, the research and the gains with fuzzy theory used in analysis methods of stability of engineering structure turned out that giving the model of fuzzy reliability is more scientific and reasonable in analysis of stability. Fuzzy reliability and traditional reliability are not antithetical, traditional reliability is a special case of fuzzy reliability.[1]

Proposing fuzzy reliability

From fuzzy theory, we can know that fuzzy probability definition of fuzzy events A is

\[ P(A) = \int_{-\infty}^{+\infty} \mu_A(x) f(x) dx \]

\[ \mu_A(x) \] is membership function of fuzzy events A.

We have adopted the theory of paper 1: From structure in good condition to the structure failure is a program rather than either this or that. So the working status of structures is fuzzy. In actual projects, structural resistance R and load S all have uncertain factors. So they probably have fuzzy uncertainty. And we can write security situation \( Z=R-S>0 \) and damage \( Z=R-S<0 \) to fuzzy form. And this can be achieved by introducing tolerance such as the maximum structural resistance is \( d_R \) and the maximum load is \( d_S \), and for critical state, it become a fuzzy damage district \( \bar{Z} \) based on real number field \( \mathbb{R}^n \) whose number is random variables.

\[ \bar{Z} = \{ X \in \mathbb{R}^n \mid R(X) - S(X) \leq -d_R \} \cup \{ X \in \mathbb{R}^n \mid R(X) - S(X) \geq d_S \} \]

\[ Z = R(X) - S(X) - d_z \]

\[ = R(X) - S(X) - f(\mu_Z(X)) \]

\( d_z \) is damage tolerance. And it’s a function about \( \mu_Z \cdot \mu_Z(X) = 0 \), means the structure began to be destroy; \( \mu_Z(X) = 1 \),means the structure is complete broke; When \( \mu_Z(X) \) is between at 0 and 1, reflect the extent of the damage.
Giving Model of Seawall Program Fuzzy Reliability

Safety factor analysis methods of the stability of whole seawall has two main methods: immediate integration and swedish circle method. In this article, we based on Swedish circle method to build functions.

The details of swedish circle method and the arc calculation model theory of seawall sketch as follows:

![Figure 1. Sketch of Swedish circle method.](image1)

Circle method use moment of all forces for O to be as equilibrium condition, and we can get slip moment: $M_S = wd$, anti-slip moment: $M_R = R \int_0^c \sigma_{tg} \varphi \, de + c ACR$

All of anti-slip moment are as R, and all of slip moment are as S. We can get functions $Z$:

$$Z = R - S = \sum [(RW_i \cos \alpha_i \tan \varphi_i + C_i)] + TR - \sum RW_i \sin \alpha_i$$

$Z < 0$, means structure is damaged; $Z > 0$, means structure is reliable.

We assume that the distribution of R and S are normal distribution, and we can get the distribution of $Z$ is normal distribution too. Probability density function of $Z$ is

![Figure 2. The arc calculation model theory of seawall sketch.](image2)
\[ P(Z) = \frac{1}{\sqrt{2\pi}\sigma(Z)} \exp \left( -\frac{1}{2} \frac{Z - E(Z)}{\sigma(Z)}^2 \right) \]

\[ \mu_Z = \mu_R + \mu_S \]

\[ \sigma_Z^2 = \sigma_R^2 + \sigma_S^2 \]

Analysis theory of seawall project stability is very complex in calculation. And because what we assume is different from reality. So we use JC method to be as analysis theory of global stability of seawall established on deep silt. According to JC method random variable about influencing the global stability of seawall stability of seawall established on deep silt are cohesion C, internal friction angle \( \varphi \) and following N,T. So mean value and standard deviation of R and S are

\[ \mu_R = \gamma \cos \mu_c \tan \mu_\varphi + \mu_c + \mu_T \]

\[ \mu_S = \gamma \sin \mu_\alpha \]

\[ \sigma_R = \sqrt{\sigma_\gamma^2 \cos^2 \sigma_\alpha \tan^2 \sigma_\varphi + \sigma_\varphi^2 + \sigma_T^2 + \sigma_\alpha^2 \left( \frac{\partial \mu_R}{\partial \varphi} \right)^2 + \sigma_c^2 \left( \frac{\partial \mu_R}{\partial c} \right)^2} \]

\[ = \sqrt{\sigma_\gamma^2 \cos^2 \sigma_\alpha \tan^2 \sigma_\varphi + \sigma_\varphi^2 + \sigma_T^2 + \sigma_\alpha^2 \left( \frac{\gamma \cos \mu_c}{\cos^2 \mu_\alpha} \right)^2 + \sigma_c^2} \]

\[ \sigma_S = \sqrt{\sigma_\gamma^2 \sin^2 \sigma_\alpha} \]

Fuzzy probability about seawall crash

\[ P_f = \int_{-\infty}^{\infty} \mu_z(z) \, dz \]

\( \mu_z(z) \) is membership function.

Membership function \( \mu_z(z) \) reflect that what degree \( z \) belong to structural deterioration. There are some ways to ensure membership function, and their ships have many kinds. In this article we adopt common form.

\[ \mu_z(z) = \begin{cases} 1 & z \geq 0 \\ (z + a) / a & -a < z < 0 \\ 0 & z \leq 0 \end{cases} \]

The value of \( a \) adopt the value of \( 2 \sigma_z \), means the difference between maximum and minimum values of fuzzy resistance. So we can get fuzzy reliability.

\[ \tilde{\beta} = \phi^{-1}(1 - P_f) \]

\( \phi^{-1} \) is the inverse function of the distribution of standard teenage boy.

**Example**

There is a seawall that is built on deep silt. Original ground level is \(-1.0 \sim -3.2\)m. For easy figures random variable only have cohesion C and internal friction angle \( \varphi \), and their mean value are 12 and 10.3; standard deviation all be 1. The average value of sludge severe \( \gamma \) is 19KN/m³, it’s standard deviation is 0.1, \( \alpha \) is 10°, it’s standard deviation is 3.
Calculating

Table 1. The value of each average value and standard deviation.

<table>
<thead>
<tr>
<th>$\mu_R$</th>
<th>$\mu_S$</th>
<th>$\mu_Z$</th>
<th>$\sigma_R$</th>
<th>$\sigma_S$</th>
<th>$\sigma_Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.4</td>
<td>0.17</td>
<td>15.23</td>
<td>19.03</td>
<td>0.99</td>
<td>19.05</td>
</tr>
</tbody>
</table>

We can get membership function $\mu_{(z)}$

$$
\mu_{(z)} = \begin{cases} 
1 & z \geq 0 \\
1 + \frac{z}{38.1} & -38.1 < z < 0 \\
0 & z \leq 0 
\end{cases}
$$

Getting fuzzy reliability $\tilde{\beta} = 0.212$, If the numerical value of $\mu_{(z)}$ is 1, then we get the answer is 0.799. And the answer is same with what we get by traditional reliability theory. It verify the theory fuzzy reliability and traditional reliability are not antithetical, traditional reliability is a special case of fuzzy reliability.

We can see that the numerical value of $\tilde{\beta}$ is smaller than $\beta$. The reason is fuzzy reliability theory thinks about uncertain factors, and those factors always reduce the reliability of seawall.

**Summary**

Through combining fuzzy random reliability theory and Swedish circle method gives a model of fuzzy reliability about calculating seawall reliability to provide engineering as a reference.

Because practical engineering is very complex, so we made assumption is very differences with practical engineering and those difference need to be corrected with engineering experience; making sure of membership function has a lot of ways and the way we adopted lacks of experience.

**References**


