A New Numerical Model of Contact Pressure in Vane Seals

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ABSTRACT: The sealing contact pressure is critical in determining the film thickness of elastohydrodynamic lubrication, the friction force on the seal, and the sealing reliability of vane seals, so it is one of the most important parameters in evaluating the sealing performance of vane seals. A numerical model of contact pressure for vane seal is built based on elastic mechanic method, which is used to solve these problems of contact stress, strain and displacement on sealing surface. The interaction between adjacent units during deforming is fully considered for this model, which results in that this model is closer to the actual situation.

1 INTRODUCTION

The hydraulic rotary vane actuators (RVAs) are widely used in industry such as aircraft wing control surfaces, rotary vane steering gears on marines, anti-roll suspension systems on high performance cars, multi-directional tube bending equipment, and high performance robots[1-3].

A RVA is configured with a stator and a rotor and there are two or more vanes for the stator and the rotor (Fig.1). Those vanes create chambers between the rotor and the stator and chambers are filled with hydraulic fluid. With changing pressures in chambers, the rotor is forced to rotate and apply force to external equipment. The efficient operation of an RVA is dependent on the effective sealing of fluid chambers to reduce leakage in the RVA. The sealing contact pressure is critical in determining the film thickness of elastohydrodynamic lubrication (EHL), the friction force on the seal, and the sealing reliability of vane seals[4-6].

The present authors have contributed to fundamental sealing research with recent studies that addressed various topics, such as numerical modelling of the contact pressure in vane seals[7], the EHL film thickness of a non-rectangular section vane seal [8], Numerical study of the contact pressure of window-type vane seals[9], and so on. However, the numerical model of contact pressure for a sealing surface based on difference method did not take into account the interaction between adjacent units when the deformation was occurred for the sealing rubber. Thus, this method is only suitable for the sealing rubber with small deformation. This paper extends the research in numerical modeling of contact pressure in vane seals with the method of elastic mechanics, and presents a more accurate and wider adaptability numerical model.

2 NUMERICAL MODELING

2.1 Geometric model

As shown in Fig.2, h is the height in y direction for sealing rubber. w is the length in x direction. \( \delta_y \) is the pre-compression quantity, \( \delta_x \) is increment of around its both ends in x direction, R1 is the inner radius of stator.
2.2 Numerical modeling

For the plane strain problem of elastic mechanics, if the stress function is satisfied with compatibility equation:

\[
\frac{\partial^4 \Phi}{\partial x^4} + 2 \frac{\partial^4 \Phi}{\partial x^2 \partial y^2} + \frac{\partial^4 \Phi}{\partial y^4} = 0
\]  

(1)

Thus, the stress and displacement can be obtained by the following equation:

\[
\begin{align*}
\sigma_x &= \frac{\partial^2 \Phi}{\partial y^2} \\
\sigma_y &= \frac{\partial^2 \Phi}{\partial x^2} \\
\tau_{xy} &= -\frac{\partial^2 \Phi}{\partial x \partial y}
\end{align*}
\]  

(2)

Where, \( \sigma_x \) and \( \sigma_y \) are the stress in x and y directions respectively. \( \tau_{xy} \) is the shear stress. Meanwhile, the strain is solved by following:

\[
\begin{align*}
\varepsilon_x &= \frac{1 - \mu^2}{E} (\sigma_x - \frac{\mu}{1 - \mu} \sigma_y) \\
\varepsilon_y &= \frac{1 - \mu^2}{E} (\sigma_y - \frac{\mu}{1 - \mu} \sigma_x) \\
\gamma_{xy} &= \frac{2(1 + \mu) \tau_{xy}}{E}
\end{align*}
\]  

(3)

Where, \( \delta_x \) and \( \delta_y \) are the strain in x and y directions respectively. \( \gamma_{xy} \) is shear strain. \( E \) is Modulus of Elasticity. \( \mu \) is poisson ratio. The relationship between strain and displacement is as following:

\[
\begin{align*}
\varepsilon_x &= \frac{\partial u}{\partial x} \\
\varepsilon_y &= \frac{\partial v}{\partial y} \\
\gamma_{xy} &= \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}
\end{align*}
\]  

(4)

Where, \( u \) and \( v \) are displacement in x and y directions respectively.

The conditions of stress and deformation around sealing rubber are as following:

\[
\begin{align*}
u_{x=0} &= 0 \\
v_{y=0} &= 0 \\
\sigma_{y=\frac{h}{2}} &= p
\end{align*}
\]  

(5)

2.3 Numerical Solving

When \( \sigma_y \) is obtained by the function \( f(x) \):

\[
\sigma_y = f_0(x)
\]  

(6)

According to equation (2):

\[
\Phi(x, y) = \int f_0(x) dx dx
\]  

(7)

According to geometric relations, the relation curve of sealing rubber before uncompressed on seal surface is set as: \( y_1 = h \) And the circular curve for sealing rubber being pre-compressed is the same as stator or rotor, which is expressed by as following:

\[
y_3 = Ax^2 + Bx + C
\]  

(8)

Take three points of this circle \( (x_1, y_1), (x_2, y_2), (x_3, y_3) \). Meanwhile, the coordinates of these three points are satisfied with the equation (8):

\[
\begin{align*}
Ax_1^2 + Bx_1 + C &= y_1 \\
Ax_2^2 + Bx_2 + C &= y_2 \\
Ax_3^2 + Bx_3 + C &= y_3
\end{align*}
\]  

(9)

If it is stator, the coordinates of these three points are:

\[
\begin{align*}
(-\frac{w}{2}, \sqrt{R_i^2 - (\frac{w}{2})^2 - R_i + h - \delta_i}) \\
(0, h - \delta_i) \\
(\frac{w}{2}, \sqrt{R_i^2 - (\frac{w}{2})^2 - R_i + h - \delta_i})
\end{align*}
\]  

(10)

The equation (9) is written as matrix form:
\[
\begin{bmatrix}
\left(-\frac{w}{2}\right)^2 & -\frac{w}{2} & 1 \\
0 & 0 & 1 \\
\left(\frac{w}{2}\right)^2 & \frac{w}{2} & 1 \\
\end{bmatrix}
\begin{bmatrix}
A \\
B \\
C \\
\end{bmatrix} =
\begin{bmatrix}
\sqrt{R_i^2 - \left(-\frac{w}{2}\right)^2 - R_i + h - \delta_y} \\
\sqrt{R_i^2 - \left(-\frac{w}{2}\right)^2 - R_i + h - \delta_y} \\
\sqrt{R_i^2 - \left(-\frac{w}{2}\right)^2 - R_i + h - \delta_y} \\
\end{bmatrix}
\]
(11)

If it is rotor, the coordinates of these three points are:

\[
\left(-\frac{w}{2}, -\sqrt{R_i^2 - \left(-\frac{w}{2}\right)^2 + R_i + h - \delta_y}\right) \\
(0, h - \delta_y) \\
\left(\frac{w}{2}, \sqrt{R_i^2 - \left(-\frac{w}{2}\right)^2 + R_i + h - \delta_y}\right)
\]
(12)

The equation (9) is written as matrix form:

\[
\begin{bmatrix}
\left(-\frac{w}{2}\right)^2 & -\frac{w}{2} & 1 \\
0 & 0 & 1 \\
\left(\frac{w}{2}\right)^2 & \frac{w}{2} & 1 \\
\end{bmatrix}
\begin{bmatrix}
A \\
B \\
C \\
\end{bmatrix} =
\begin{bmatrix}
\sqrt{R_i^2 - \left(-\frac{w}{2}\right)^2 + R_i + h - \delta_y} \\
\sqrt{R_i^2 - \left(-\frac{w}{2}\right)^2 + R_i + h - \delta_y} \\
\sqrt{R_i^2 - \left(-\frac{w}{2}\right)^2 + R_i + h - \delta_y} \\
\end{bmatrix}
\]
(13)

The displacement \( v(x) \) for the sealing rubber in the y direction of sealing contact area is set as:

\[
v(x) = y_1 - y_3
\]
(14)

Where \( v(x) \) is the quadratic polynomial for \( x \). According to equations (2), (3) and (4), \( f(x) \) and \( f(y) \) are four times polynomials for \( x \) and \( y \) respectively. Thus, the equation (7) is changed as following:

\[
\Phi(x, y) = a_i x^4 + b_i x^3 + c_i x^2 + a_x y^4 + b_x y^3 + c_y y^2
\]
(15)

Then,

\[
\sigma_x = 12a_2 y^2 + 6b_2 y + 2c_2, \sigma_y = 12a_1 x^2 + 6b_1 x + 2c_1
\]
(16)

According to the equations (2), (3) and (4):

\[
\begin{aligned}
u &= \int \varepsilon_i dy \\
&= \frac{1 - \mu^2}{E} \left( \int \sigma_i dy - \frac{\mu}{1 - \mu} \int \sigma_y dy \right) \\
&= \frac{1 - \mu^2}{E} \left( (12a_2 y^2 + 6b_2 y + 2c_2) x - \frac{\mu}{1 - \mu} \left( 4a_i x^3 + 3b_i x^2 + 2c_i x \right) \right) + C_0
\end{aligned}
\]
(17)

\[
v = \int \varepsilon_y dy \\
= \frac{1 - \mu^2}{E} \left( \int \sigma_y dy - \frac{\mu}{1 - \mu} \int \sigma_i dy \right) \\
= \frac{1 - \mu^2}{E} \left( (12a_1 x^2 + 6b_1 x + 2c_1) y - \frac{\mu}{1 - \mu} \left( 4a_2 y^3 + 3b_2 y^2 + 2c_2 y \right) \right) + C_1
\]
(18)

Meanwhile, the equations (17) and (18) must be satisfied with the compatibility function:

\[
a_1 + a_2 = 0 \quad \text{and when } x=0, y=0, \text{there are:}
\]

\[
u_{y=0} = C_1 = 0, \\
\sigma_{x=0} = C_0 = 0.
\]
(19)

When \( y=h \), there is:

\[
v_{y=h} = v(x) = -Ax^2 - Bx + h - C
\]

\[
= \frac{1 - \mu^2}{E} \left( (12a_i x^2 + 6b_i x + 2c_i) x - \frac{\mu}{1 - \mu} \left( 4a_1 x^3 + 3b_1 x^2 + 2c_1 x \right) \right)
\]

\[
- \frac{\mu + \mu^2}{E} \left( 4a_2 h^3 + 3b_2 h^2 + 2c_2 h \right) + C_i
\]
(20)

Thus,

\[
\begin{aligned}
\frac{12h(1 - \mu^2)}{E} a_i &= -A \\
\frac{6h(1 - \mu^2)}{E} b_i &= -B \\
\frac{2h(1 - \mu^2)}{E} c_i - \frac{\mu + \mu^2}{E} \left( 4a_2 h^3 + 3b_2 h^2 + 2c_2 h \right) + C_i &= h - C
\end{aligned}
\]
(21)

When \( x = \pm w/2 \), there is:

\[
\sigma_x = 12a_2 y^2 + 6b_2 y + 2c_2 = p
\]
(22)

According to the saint venant’s principle, the \( \sigma_x \) is equivalent to uniform pressure \( p \):

\[
2c_2 = p
\]
(23)

Thus.
The stresses in x and y directions are set as \( \sigma_x \) and \( \sigma_y \) for any point on seal rubber which initial coordinate is \((x_0, y_0)\). Therefore, the stress is obtained as following:

\[
\sigma(x_0, y_0) = \sqrt{\sigma_x^2 + \sigma_y^2}
\]  

(25)

The seal rubber becomes deformed after being compressed, which results in shift migration for each point. The offsets for each points in x and y directions are set as \( u(x_0, y_0) \) and \( v(x_0, y_0) \). Therefore, the final coordinate for each point in seal rubber is as following:

\[
\begin{align*}
x &= x_0 + u(x_0, y_0) \\
y &= y_0 + v(x_0, y_0)
\end{align*}
\]  

(26)

The stress distribution along y direction in the seal surface is equal to the contact pressure of vane seal, which is the \( \sigma_y \) when \( y_0=h \). Thus, the contact pressure of vane seal is obtained by following:

\[
P_c = \sigma_y(x_0, h), x = x_0 + u(x_0, h)
\]  

(27)

2.4 Practical calculation

Now take the related parameters of vane seal for a RVA: \( h=5\text{mm}, w=4\text{mm}, R_1=64\text{mm}, R_2=44\text{mm}, \) pre-compression \( \delta_0=0.5\text{mm}, \) sealed elastic modulus \( E=21.56\text{Mpa}, \) poisson ratio \( \mu=0.495, \) oil pressure \( p=0\text{MPa}. \)

The stress distribution of vane seal is obtained by simulation based on Matlab:

Figure 3 and figure 4 are the stress nephograms of stator and rotor vane seals respectively when the pre-compression is 10\% and oil pressure is 0. It can be found that the sizes of stator and rotor vane seals in x direction are changed when they are compressed in the y direction. It means that the contact width for seal surface is enlarged from 4mm to 4.4mm.

Figure 5 shows the contact pressure distribution curves of stator and rotor seals. It can be seen that the stress of stator vane seal is the smallest in the middle while it is opposite for the rotor vane seal. Meanwhile the stresses are similar for the stator and rotor under the condition of \( x=0 \).

3 CONCLUSIONS AND DISCUSSION

The sealing contact pressure is one of the most important parameters in evaluating the sealing performance of vane seals. A new numerical model of contact pressure on vane seal surface is built based on elastic mechanics method. The interaction between adjacent units during deforming is fully considered for this model, which results in that this model is closer to the actual situation. It means that it provide a method to solve these complicated problems with large deformation.

Moreover, the effect of related design parameters with sealing performance on contact pressure, including vane seal structure, pre-compression, rubber material performance, sealing pressure, will be studied.
4 ACKNOWLEDGMENT

The authors gratefully acknowledge the support of the National Natural Science Foundation of China (Grant No.51375352).

REFERENCES


