Effect of the Multiple Cylinders Scattering of an Electromagnetic Plane Wave on the Formation of Whispering Gallery Modes

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Abstract. We report on the effect of scattering by multiple cylinders of an electromagnetic plane wave on the formation of whispering gallery modes (WGM) in the cylinder. It was found that WGM can arise thanks to the presence of additional cylinder scatterers, and at their special location, while WGM in a single cylinder can be formed only for specially chosen values of the cylinder radius and (or) wave length, and the accuracy of these matching should be much more than required in our model for the location of the additional cylinders. The analysis of the general solution of considered scattering problem has shown that the effect can be explained by the interference of waves scattered by additional cylinders (i) and incident on the main cylinder (ii).

Introduction

The term "whispering gallery modes" was introduced by Lord Rayleigh to explain the effect of sound propagation in a circular gallery [1, 2]. The name itself reflects the fact that sound in closed spaces can propagate along concave walls. Laser radiation attached to the perimeter of the miniature disk by multiple reflections from the walls can be treated as optical analogue of the "whispering gallery". If the reflection surface is smooth enough, the radiation propagates with minimal losses. However, since lateral surface is not smooth, and has a nonzero radius of curvature a part of the wave leaks out. As larger the radius of curvature, as more radiation stays inside. The possibility of creating electromagnetic cavities using WGM was pointed out for the first time by R. Richtmyer [3]. Spherical form is simplest for resonator using WGM. Theoretical study of the interaction of spherical particles with electromagnetic waves is studied more than a century and dates back to the theoretical work of MÖ [4], who considered the scattering of light by spherical particles, and Debye, who studied the scattering of the wave on a sphere in the form of a series of refracted and reflected waves of various order [5]. However, despite the problem (scattering of waves on an axisymmetric particle) is well known and studied long period, new important results were obtained in 2004: the authors of Ref. [6] found and studied a narrow, high-intensity beam of light (photonic nanojet) generated at the shadow-side surface of dielectric cylinders illuminated by a plane wave. Photonic nanojet could be formed also by spheres [7], a two-layer dielectric microsphere cylinders [8], and discs [9]. The influence of incident light polarization was investigated in [10]. Dielectric sphere with properly graded refractive index was used in [11] to increase the length of photonic nanojet. A nanojet generated by dielectric microcylinders with different metallic coatings has been analyzed numerically in [12]. Formation and transport of photonic nanojets for multiple cylinder scattering have been calculated in Ref. [13]. The renewed interest to the scattering of plane wave on the cylinder led to the need for a detailed study of options to get a WGM [14]. The authors found how the WGM form the focal spot outside of the cylinder, and also the contribution of the cylinder eigenmodes to the formation of the WGM. The
effect of multiple cylinders scattering of electromagnetic plane-wave on formation of high field intensity areas studied in [15].

In the present paper we continued the study of the formation of WGM in a cylinder. We found that for a fixed wavelength the location of two small cylinders in a special small region near the edge of the main cylinder \( A \) leads to the formation of WGM inside \( A \).

**Modeling**

Our model consists of three cylinders. One of them is basic, inside which we consider the formation of WGM. The two other has assistive function. Their positions vary relative to the basic cylinder, with the aim to find a location optimizing the characteristics of the WGM. Our model as well as distances and notations presented on Figure 1. Plane wave falls from the left side on the group of three cylinders. The direction of wave propagation is along the axis of symmetry of the cylinders which selected as the \( x \) axis. The origin of coordinates corresponds to the center of the main cylinder \( A \). The propagation and scattering of the electromagnetic plane wave was studied using MATLAB toolbox developed in Ref. [16]. TE-polarized plane wave (\( \lambda = 532 \text{ nm} \)) was used as the incident wave. The mesh grid size in space was equal to 0.04 \( \mu \text{m} \) (0.075 of wavelength). Electric permittivity \( \varepsilon =1.59 \) (kvarcevoe steklo). Cylinder’s \( A \) radius \( R_A=4 \lambda \). And for cylinders \( B \) and \( C \) the radii \( R_B = R_C = 0.25 R_A \) were used.

![Figure 1. Geometry to simulate scattering of plane wave by multiple cylinder.](image)

**Results**

The calculated distribution of the absolute value of the field is shown in Figure 2 (a). As can be seen, the scattering led to formation of the photonic jet. At the same time intensity of the WGM is at the background. Next we used the additional cylinders \( B \) and \( C \). The comparative picture for the case of one cylinder and of our model when \( L_x =-2.61, L_y =2.02 \) is shown in Figure 2 (b). As it is seen the presence of additional cylinders lead to the extremely increase of intensity of WGM.

In order to know how the positions of the cylinders \( B \) and \( C \) affect the maximal absolute value of the field inside the cylinders, we changed their positions moving the centers within ring defined in polar coordinates as interval of radii \( [R_A+R_B, R_A+1.6*R_B] \), and angle \( [\pi 3\pi 2] \). Step in the ring was \( 0.06R_B/10 \) along the radius, and \( (\pi 2)/75 \) for the angle. The resulting picture is depicted in Figure 3. Since the positions of the cylinders are symmetric to the \( x \)-axis, only positions of cylinder \( C \) were assumed. The positions of cylinders \( B \) and \( C \) used on Figure 2 are in the correspondence with data of Figure 3.

Generally, the eigenmodes of the cylinder are calculated for specific values of the relation radius of the cylinder/wavelength. As example we can consider simple expression \( 2\pi R n/\lambda =T_{ml} \), where \( T_{ml} \) is the \( l \) \( th \) root of the \( m \) \( th \) order Bessel function. I.e. if the wavelength is known, then to provide propagation of \( (ml) \) mode we have to choose the radius of the cylinder from the above expression. Moreover, as larger contribution of this mode to the field intensity, as more accurate the radius should be defined [14]. For example, for mode with \( m=15 \) increasing the intensity 8 times, the accuracy of matching radius is \( 10^{-4} \lambda \) [15]. As follows from Figure 3, to reach the same increase of intensity it is enough the accuracy of location within a few percent of the wavelength.
Let's note two obvious but important points following from Figure 2. First, the resonance increase in the intensity happening inside the cylinder A. That is, we are dealing with WGM, because the specific feature of WGMs is that the high intensity field in these modes is concentrated near the cavity walls. Second, the absence of areas with high field intensity inside a single scattering cylinder A indicates the presence of the additional cylinders B and C as reason of the effect.

To explain the effect we will analyze the general solution of the scattering problem on many cylinders. Our task in this analysis is reduced to assessment of contributions to the field inside the cylinder A of the components associated with the cylinders B and C. With this purpose we consider the expression corresponding to the solutions of Maxwell's equations for our model, given in [13]:

\[ H_\varepsilon = ik_i A_i \sum_{n=\infty}^{+\infty} e^{i \theta} J_n (k_i R_{ip}) A_{in} \]

(1)

where \( k_m = \frac{2\pi}{\lambda} \), \( a_i = R_A \), \( J_n \) – Bessel function of first kind, \( R_{ip} \) – distance between point P inside cylinder and center of cylinder with number l, and

\[ A_{in} = \left( j_n (k_i a_i) m^2 \right)^{-1} \left\{ \varepsilon_i e^{i \phi} j_n (k_m a_i) - a_i H_n (k_m a_i) - j_n (k_m a_i) \sum_{j=1}^{N} \sum_{s=0}^{N} (-i)^{s-n} H_{s-n} (k_m R_{ij}) e^{i (s-n) \theta} a_{js} \right\} \]

(2)

The expansion coefficients \( (a_{jm}, b_{jm}) \) are related to the single cylinder scattering coefficients \( (a_{jm}^0, b_{jm}^0) \) [17] and can be obtained by solving the following equation system [13]:

\[ \sum_{l=1}^{N} \sum_{s=0}^{N} \left( \delta_{jl} \delta_{ms} + (1-\delta_{jl}) G_{ls}^{jm} a_{jm}^0 \right) a_{ls} = \eta \gamma \exp \left( -ik_m x_j \cos (\phi) - y_j \sin (\phi) \right) \]

(3)

where \( G_{ls}^{jm} = (-i)^{s-n} H_{s-n} (k_m R_{ij}) e^{i (s-n) \theta} \), \( \eta \gamma = \exp \left( -ik_m x_j \cos (\phi) - y_j \sin (\phi) \right) \).
We are interested in the field inside of cylinder A, i.e. \( l=1 \) in (1). Contribution to the field from the other two cylinders \( (l=2,3) \) is contained in the coefficients \( a_{ln} \) and \( A_{ln} \). It can be shown that the dependence of the field versus parameters of the model has a wave character. Indeed, the arguments of the Bessel and Hankel functions are much more than 1 for the parameters of our model (for example: \( k_m a_l = 2\pi / \lambda R_A = 2\pi / 4 \lambda = 8\pi / 4 \lambda R_A < 2\pi / 8\pi (R_A + R_B) = 8\pi \)). Therefore, we can use their asymptotic expressions for large values of arguments [18]. Besides it, in (1) we use only second term of expression (2) for \( A_{ln} \). Our choose is due this term contains the contribution from the other cylinders (unlike the first term) and has a more simple kind (unlike the third term). Then we have for the field, up to multipliers which do not depend on the index of summation

\[
H'_z \propto \sum_{\gamma} \exp\left(\gamma R_p / 2\right) \exp\left(\pm i(k_i R_p - \gamma / 2 - \pi / 4)\right) \exp\left(-i k_m R_i \right) \sum_{s} b_{sn} \exp(-i s \gamma_i) (4)
\]

Thus:

\[
H'_z \propto \exp\left(\pm k_i R_p - k_m R_i \right) \sum_{\gamma} b_{sn} \exp\left(n \gamma_p - s \gamma_i \right) (5)
\]

Hear we have deal with sum of waves with different phases and amplitudes. A well known gain condition (due to the interference) leads to the equality

\[
n_1 \gamma_p - s_1 \gamma_i = 2\pi p_1 (6)
\]

where \( n_1, s_1, p_1 \) – integer numbers. A similar condition can be applied to the exponent before sum in (5):

\[
\pm k_i R_p - k_m R_i = 2\pi p_2 (7)
\]

where \( p_2 \) – integer numbers. Condition (7) leads to estimation of the angles, which will determine the directions where resonances expected:

\[
\gamma_p = (s_1 \gamma_i + 2\pi n_1) / n_1 (8)
\]

Since the coefficients \( a_{ln} \) varies inversely with \( R_i \) and become rapidly oscillating as \( R_i \) increase, we put \( p_2=0 \) in (8) since this is the case for the possible minimal values of \( R_i \). Then \( R_i = k_i R_j / k_m \leq k_i / k_m R_A \). And finally we come to the inequality

\[
R_i < n R_A (9)
\]

Figure 3 confirms the estimations (6) and (8).

Summary
In conclusion, we have simulated scattering of plane wave by group of three cylinders. It was found that WGMs inside the basic cylinder can be formed in the presence of additional cylinders B and C, without special requirement for wavelength and radius of the cylinders. But cylinders B and C should be occupied special positions nearby to edge of cylinder A. At that accuracy of the locations is much less then accuracy required for setting resonant radius of the cylinder, at which WGM can be observed. The analysis of the general solution for our model has shown that the effect can be explained by the interference of waves scattered by extra cylinders. Thus, our work describes a new way of formation of WGM in the cylinder.

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References