Reliability Analysis of Equipment System Based on State Transition Method

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ABSTRACT: The repairable system with dynamic random fault are often analyzed with static approximation processing method, which will result in the great differences between calculated value of the reliability index and the actual situation. In this paper, aiming at the dynamic analysis problem, the state transition equation in which the reliability parameters of system component are changed with time will be established applying the markov process theory. The corresponding mathematical model was established according to the principle and operation characteristics of a certain equipment system, and the method was analyzed using the example. The analysis results show that the model can reasonably analyze and calculate the changes of unreliability of a certain equipment system along with time. The markov state transition algorithm is effective and practical method, and it will promote the application and development of the dynamic reliability analysis technology in equipment system.

KEYWORDS: State transfer method; Dynamic reliability analysis; Unreliability.

1 INTRODUCTION

The traditional reliability modeling methods mainly include failure mode and effect analysis (FMEA), reliability block diagram (BD), fault tree analysis (FTA), and event tree (ET) and so on, which can't meet the needs of analyzing the reliability of complicated dynamic system. The state transition methods are mainly diveded into explicit state transition method and implicit state transition method. The explicit state transition method mainly includes markov state transition method and event sequence diagram (ESD), and the implicit state transition method mainly includes continuous event tree, dynamic logic analysis method, dynamic event tree method and discrete event simulation method, etc. [1] This paper will research the application of markov state transfer method in a certain equipment system, which can provide a new analysis technique for the reliability of equipment system.

2 ESTABLISHING EQUIPMENT SYSTEM

A certain equipment system consists of two pump, two check valves and all kinds of accessories, and it can supply the water for cooling the equipment in the primary circuit. The schematic diagram of a equipment system can be shown in figure 1.In the model of the equipment system, pump 1 and pump 2 are parts of the same type, and check valve 1 and check valve 2 are also parts of the same type.

![Figure 1. The basic schematic diagram of a certain equipment system.](image)

3 MARKOV STATE TRANSITION METHOD

Markov analysis method is a kind of statistics analysis method which is used to analyze the quantitative relation in the development and change process of the objective object with the random mathematical model based on the theory of probability and random process [2]. Markov process is a stochastic process with no aftereffect. The parameter space and state space of markov process can be a discrete set and also can be a consecutive set, and the markov process with discrete time and state can usually be known as a markov chain. The probability of a transfer between various states can
be easily calculated with markov process, and markov process is used to study system of "system" and state of "transfer". The possibility switching from one state to another state can be called a state transition probability.

Assumes that the system is composed of n parts, and the life of the ith component obeys the distribution: \(1 - e^{-\lambda t} , t \geq 0\), the distribution of repair time after failure is as follows: \(1 - e^{-\mu t} , t \geq 0\), and among them: \(\lambda_i, \mu_i > 0\), \(i = 1,2,\cdots,n\).

Assume that all random variables are independent and the life distribution of fault component is thesame as the new. In order to distinguish the different situation, system can be defined as follows:

State 0: all components are normal.

State i: the ith component is in failure and the rest components are normal, \(i = 1,2,\cdots,n\).

\(X(t)\) indicates the state at the moment of t, even if all components are normal at the moment of t
\[X(t) = \begin{cases} \text{all components are normal at the moment of } t & \text{if } \lambda_i, \mu_i \geq 0 \text{ for } i = 1,2,\cdots,n \text{ and among them} \\ \text{the ith component is in failure and the rest components are normal at the moment of } t & i = 1,2,\cdots,n \end{cases} \]

It can be proved that \{\{X(t),t \geq 0\}\} is the markov process in which the state space is \(E=\{0,1,\cdots,n\}\). The state transition diagram of system within the delta t is shown in figure 1 as follows.

![Figure 2. Markov state transition diagram.](image)

The transfer rate matrix can be stated as follows due to the markov state transfer diagram.
\[
A = \begin{bmatrix} -\Lambda & \lambda_1 & \lambda_2 & \cdots & \lambda_n \\ \mu_1 & -\mu_1 & 0 & \cdots & 0 \\ \mu_2 & 0 & -\mu_2 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \mu_n & 0 & 0 & \cdots & -\mu_n \end{bmatrix}, \text{ and } \Lambda = \sum_{i=1}^{n} \lambda_i
\]

In order to calculate the instantaneous system availability and fault frequency of system, the following differential equations should be solved:
\[
\begin{align*}
\frac{d}{dt} \begin{pmatrix} P_0(t) \\ P_1(t) \\ P_2(t) \\ \vdots \\ P_n(t) \end{pmatrix} &= \begin{pmatrix} \sum_{i=1}^{n} \mu_i P_i(t) \\ \lambda_1 P_0(t) - \mu_1 P_1(t) \\ \lambda_2 P_0(t) - \mu_2 P_2(t) \\ \vdots \\ \lambda_n P_0(t) - \mu_n P_n(t) \end{pmatrix} \\
\text{initial condition} &\begin{pmatrix} P_0(0) \\ P_1(0) \\ P_2(0) \\ \vdots \\ P_n(0) \end{pmatrix}
\end{align*}
\]

(1)

Assume that the system is in normal state at the moment 0, this is necessary and sufficient conditions for the normal work of the series system, and it is the sufficient conditions for the normal work of the non-series system [3]. Assuming that the initial condition can be given as follows:
\[
\begin{pmatrix} P_0(0) \\ P_1(0) \\ P_2(0) \\ \vdots \\ P_n(0) \end{pmatrix} = (1,0,0,\cdots,0)
\]

Both ends of the formula (1) is carried out by Laplace transform:
\[
\begin{align*}
sP_0^*(s) - 1 &= -\lambda_0 P_0^*(s) + \sum_{i=1}^{n} \mu_i P_i^*(s) \\
sP_i^*(s) &= \lambda_i P_0^*(s) - \mu_i P_i^*(s)
\end{align*}
\]

(2)

The linear equations (2) can be solved as follows:
\[
\begin{align*}
P_0^*(s) &= \frac{1}{s + \sum_{i=1}^{n} \frac{\lambda_i}{\mu_i}} \\
P_i^*(s) &= \frac{\lambda_i}{s + \mu_i} P_0^*(s)
\end{align*}
\]

(3)

Then the inversion calculation of \(P_0^*(s)\) and \(P_i^*(s)\) are respectively carried out \(i = 1,2,\cdots,n\), by which \(P_0(t)\) and \(P_i(t)\) are solved \((i = 1,2,\cdots,n)\). Ultimately the system instantaneous availability and instantaneous frequency can be obtained through state combination.

4 APPLICATION EXAMPLE

It is assumed that the failure rate of water source is 0, and the system reliability block diagram can be simplified as follows:

![Figure 3. Equipment system reliability block diagram.](image)

In figure 3, signal 1 and signal 3 respectively represent pump 1 and pump 2, and signal 1 and signal 4 respectively represent check valve 1 and check valve 2. Therefore, the system state transition diagram in the delta t can be shown in figure 4.

![Figure 4. State transition diagram of the equipment system.](image)

In figure 4, signal 0 means that all components work normally, signal 1 means that the first component is out of order, signal 2 means that the second component is out of order, signal 3 means that the third component is out of order, and signal 4 means that the fourth component is out of order. It is assumed that the failure rate of pump 1, pump 2, check valve 1 and check valve 2 in the cooling system.
is respectively $\lambda_1, \lambda_2, \lambda_3$ and $\lambda_4$, and the maintenance rate is respectively $\mu_1, \mu_2, \mu_3$ and $\mu_4$. Because pump 1 and pump 3 are the same type components, and check valve 1 and check valve 2 are also the same type components, it is assumed that $\lambda_1 = \lambda_3, \lambda_2 = \lambda_4, \mu_1 = \mu_3, \mu_2 = \mu_4$. The transfer rate matrix can be obtained through markov state transition diagram shown in figure 4:

$$A = \begin{bmatrix}
-\Lambda & \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 \\
\mu_1 & -\mu_1 & 0 & 0 & 0 \\
\mu_2 & 0 & -\mu_2 & 0 & 0 \\
\mu_3 & 0 & 0 & -\mu_3 & 0 \\
\mu_4 & 0 & 0 & 0 & -\mu_4
\end{bmatrix} , \Lambda = \sum_{i=1}^{4} \lambda_i$$

The differential equations can be obtained according to the transfer rate matrix as follows:

$$\frac{dP_1(t)}{dt} = -(\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)P_1(t) + \mu_1P_1(t) + \mu_2P_2(t) + \mu_3P_3(t) + \mu_4P_4(t)$$

$$\frac{dP_2(t)}{dt} = -\mu_1P_1(t) + \lambda_2P_2(t)$$

$$\frac{dP_3(t)}{dt} = -\mu_2P_2(t) + \lambda_3P_3(t)$$

$$\frac{dP_4(t)}{dt} = -\mu_3P_3(t) + \lambda_4P_4(t)$$

$$\frac{dP_5(t)}{dt} = -\mu_4P_4(t) + \lambda_1P_5(t)$$

The recommended value of international atomic energy agency is used as the unit data of a certain equipment system [4], as shown in table 1 as follows.

Table 1. Operator data of a certain equipment system.

<table>
<thead>
<tr>
<th>Component name</th>
<th>Failure rate /a$^{-1}$</th>
<th>Mean time to repair /h</th>
</tr>
</thead>
<tbody>
<tr>
<td>The water source</td>
<td>0</td>
<td>—</td>
</tr>
<tr>
<td>pump 1, 2</td>
<td>121.010</td>
<td>4.0</td>
</tr>
<tr>
<td>The check valve 1, 2</td>
<td>2.475</td>
<td>3.0</td>
</tr>
</tbody>
</table>

By solving differential equations (4) with markov method above, the probability of a certain equipment system in different states and the change of the equipment system unavailability along with time can be obtained and respectively shown in figure 5 and figure 6.

5 CONCLUSION

The markov model has been established aiming at the difficulty of system dynamic reliability analysis, and the probability of a certain equipment system in different states and the change of the equipment system unavailability along with time can be obtained by analyzing the probability of equipment system with the markov state transition method. It is effective and feasible to analyze the reliability of dynamic system by adopting markov state transition method and it has a certain practical value.

REFERENCES


