Linear Instability Analysis of Charged Jet in Liquid-liquid Systems

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By discussing the perturbed pressure field and the perturbed electric field, the viscous dispersion equation of charged jet in water-biodiesel system was derived. Through numerical calculation, the temporal instability characteristics and the effects of parameters were discussed. The results show the effect of jet velocity and charged voltage of charged jet in liquid-liquid system on instability is same as that in liquid-gas system. With the increasing of jet velocity or charged voltage, the maximum of wave growth rate, the corresponding optimal wave number and the maximum of wave number turn bigger, and water droplet size gets smaller. Biodiesel viscosity promotes the jet breakup obviously, while water viscosity always hinders the jet breakup. The maximum growth rate and optimum wave number increase obviously with decreasing the water surface tension, while the biodiesel surface tension has the reverse effect on the maximum growth rate and optimum wave number. By comparison, the effect of water viscosity and biodiesel surface tension on instability is much less than that of biodiesel viscosity and water surface tension. Therefore, increasing jet velocity, charged voltage and biodiesel viscosity or decreasing water surface tension are good methods to obtain small water droplet in biodiesel.

Keywords: Instability; Charged jet; Liquid-liquid system; Viscosity; Interface wave.

1. Introduction

Emulsified fuel obtained by mixing fuel and water in a certain proportion has the characteristics of higher combustion efficiency, lower fuel consumption, lower content of nitrogen oxides in exhaust [1,2,3], therefore, mixing biodiesel and water to form emulsified biodiesel is to solve the problem of high nitrogen oxides content in exhaust of biodiesel. However, fuel emulsification process often use a great number of surfactant and that always bring out additional polluting gases when burning.

Droplets formed by electric atomization are monodisperse and have narrow particle size spectrum. Droplets diameter can be small to tens of nanometers in...
certain contain conditions in liquid-gas system [4,5]. The droplets have the same kind of charge and are not easy to coalesce because of coulomb repulsion, so we promote a new way to prepare emulsified biodiesel in this paper.

According to Rayleigh’s investigation, the main reason of jet deformation and breakup in liquid-gas system is surface wave which is caused by small disturbance. Some of these waves develop with time or space and lead to jet breakup to form droplets. Fang Li and Zhao-hui Wang established the dispersion equation respectively, and their studies showed jet velocity and charged voltage were vital factor in charged jet breakup. Both of them ignored the influence of viscosity, while the viscosity played very important role in jet deformation and breakup [6,7]. Ioana Laura Omocea’s study showed that the normalized breakup length, which was the ratio of the real breakup length and jet diameter, has a linear dependence with Reynolds and Capillary [8]. From the results, we can know the surface tension also have effect on jet breakup. Therefore, the effect of jet velocity, charged voltage, viscosity and surface tension on the instability of charged jet in water-biodiesel system will be discussed in this paper.

2. Dispersion Equation

2.1 The Perturbed Pressure Field

The surface of perturbed jet can be described as follows:

\[ r = a + \eta = a + \eta_0 e^{i(kz + n\theta)} \] (1)

Where \( r \) is the radius of perturbed jet, \( a \) is the radius of unperturbed jet, \( \eta \) is radial perturbations that is much less than the radius of unperturbed jet, \( \eta_0 \) is the initial amplitude of surface wave at the nozzle’s outlet, and it depends on the shape of nozzle, \( \beta \) is wave growth rate, \( k = 2\pi / \lambda \) is wave number, \( \lambda \) is wavelength, \( n \) is wave order, \( \theta \) is phase difference at the two sides of surface wave of jet.
In the liquid-liquid system, water is atomized into biodiesel. Biodiesel container and water jet are arranged coaxially, and then, the electric field can be described as shown in Fig.1. The radius of biodiesel container is \( R \), and \( R \) is much larger than the radius of unperturbed jet. If water jet is not perturbed, its radial velocity, circumferential velocity and axial velocity are expressed as follows: \( v_r = 0 \), \( v_\theta = 0 \), \( v_z = U \). When water jet is perturbed, its radial velocity, circumferential velocity and axial velocity are then written as \( v_r + v'_r \), \( v_\theta + v'_\theta \), \( v_z + v'_z \), where \( v_r \), \( v_\theta \), \( v_z \) are radial unperturbed velocity, circumferential unperturbed velocity and axial unperturbed velocity. All of the perturbation velocity are described as \( \vec{v}' = \hat{v}(r)e^{i(\omega t + k_r r + \theta)} \), where \( \hat{v}(r) \) is the initial amplitude of perturbed velocity. In perturbed jet, the pressure can be written as \( p + p' \), where \( p \) is pressure in unperturbed jet, \( p' \) is perturbed pressure that have the form: \( p' = \hat{p}(r)e^{i(\omega t + k_r r + \theta)} \). \( \hat{p}(r) \) is the initial amplitude of perturbed pressure.

Because water is incompressible, neglecting high order terms, the continuity equation and the momentum equation of the perturbed velocity of jet are expressed as follows:

\[
\frac{v'_r + \frac{\partial v'_r}{\partial r} + \frac{\partial v'_\theta}{\partial \theta} + \frac{\partial v'_z}{\partial z}}{r} = 0 \tag{2}
\]

\[
\frac{\partial v'_r}{\partial t} + U \frac{\partial v'_r}{\partial z} = -\frac{1}{\rho} \frac{\partial p'}{\partial r} + \nu \left( \nabla^2 v'_r - \frac{v'_r}{r^2} - \frac{2}{r^2} \frac{\partial v'_\theta}{\partial \theta} \right) \tag{3}
\]

\[
\frac{\partial v'_\theta}{\partial t} + U \frac{\partial v'_\theta}{\partial z} = -\frac{1}{\rho r} \frac{\partial p'}{\partial \theta} + \nu \left( \nabla^2 v'_\theta - \frac{v'_\theta}{r^2} + \frac{2}{r^2} \frac{\partial v'_r}{\partial \theta} \right) \tag{4}
\]
\[
\frac{\partial v'_z}{\partial t} + U \frac{\partial v'_z}{\partial z} = - \frac{1}{\rho} \frac{\partial p'}{\partial z} + \nu \nabla^2 v'_z. \tag{5}
\]

And then, the perturbed pressure can be written as
\[
\frac{1}{r} \frac{\partial p'}{\partial r} + \frac{\partial^2 p'}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 p'}{\partial \theta^2} + \frac{\partial^2 p'}{\partial z^2} = 0. \tag{6}
\]

Substituting the perturbed pressure \( p' = \hat{p}(r)e^{i(kz + n\theta)} \) into the Eq.6, we obtain a modified Bessel equation for order \( n \) for the pressure perturbation amplitude can be derived as follows:
\[
r^2 \frac{d^2 \hat{p}}{dr^2} + r \frac{d\hat{p}}{dr} - \left(k^2 r^2 + n^2\right)\hat{p} = 0. \tag{7}
\]

The solution of above equation is
\[
\hat{p}(r) = C_1 I_n(kr) + C_2 K_n(kr). \tag{8}
\]

where \( I_n(kr), K_n(kr) \) are nth-order modified Bessel functions of the first and the second kinds respectively, and \( C_1, C_2 \) are constants to be solved.

When \( kr \) is close to zero, \( K_n(kr) \) is infinite, however, \( p(r) \) must have a limit when \( r \) is close to zero. And then, \( C_2 \) is zero and the perturbed amplitude can be written as follows:
\[
\hat{p}(r) = C_1 I_n(kr). \tag{9}
\]

Based on the Eq. 3, we obtain the following expression for radial perturbed velocity amplitude:
\[
\hat{v}(r) = - \frac{C_1 k I'_n(kr)}{\rho(\beta + iUk)}. \tag{10}
\]

According to fluid dynamical boundary condition, the radial velocity of fluid micelle is equal to the radial velocity of the interface, therefore, the perturbed pressure is expressed as follows:
\[
p' = - \frac{\eta_0 \rho(\beta + iUk)^2 I_n(kr)}{k I'_n(ka)} e^{\beta k(z + n\theta)}. \tag{11}
\]
1.1. The Perturbed Electric Field

When the water jet is unperturbed, water jet can be considered as an equipotential body, and its potential can be expressed as $\phi(r, \theta, z) = \phi_0$. At the interface of water and biodiesel, interface potential of biodiesel is written as $\phi_b(a, \theta, z) = \phi_0$ , $\phi_b(R, \theta, z) = 0$. Then, we obtain the potential and electric filed intensity in biodiesel phase as follows:

$$\phi_b(r, \theta, z) = \phi_b(r) = \phi_0 \frac{\ln(r/R)}{\ln(a/R)}.$$  \hspace{1cm} (12)

$$\vec{E}_b(r, \theta, z) = \vec{E}_b(r) = -\frac{\phi_0}{r \ln(a/R)} \vec{e}_r.$$  \hspace{1cm} (13)

Where $\vec{e}_r$ is radial unit vector.

High voltage electrostatic field is irrotational, potential distribution meets with the Laplace equation. When the water jet is perturbed, velocity, pressure and radius of water jet changes with the perturbation, and the electric field of biodiesel phase is also affected by the perturbation. The perturbed potential is expressed as $\phi_b = \phi_b(r) e^{jkz + i(n \theta)}$, where $\phi_b(r)$ is the potential perturbation amplitude.

Substituting $\phi_b = \phi_b(r) e^{jkz + i(n \theta)}$ into the Laplace equation, the modified Bessel equation of order $n$ for the potential perturbation amplitude $\phi_b(r)$ can be derived as follows:

$$\frac{d^2 \phi_b}{dr^2} + \frac{1}{r} \frac{d \phi_b}{dr} - \left( k^2 + \frac{n^2}{r^2} \right) \frac{\phi_b}{r} = 0.$$  \hspace{1cm} (14)

The solution of Eq.14 and the perturbed electric filed intensity are

$$\hat{\phi}_b(r) = G_1 I_n(kr) + G_2 K_n(kr).$$  \hspace{1cm} (15)

$$\vec{E}_b = -\nabla \left\{ G_1 I_n(kr) + G_2 K_n(kr) e^{jkz + i(n \theta)} \right\}.$$  \hspace{1cm} (16)

and the total electric filed intensity is

$$\vec{E}_b = -\frac{\phi_0}{r \ln(r/R)} \vec{e}_r - \nabla \left\{ G_1 I_n(kr) + G_2 K_n(kr) e^{jkz + i(n \theta)} \right\}.$$  \hspace{1cm} (17)
So, the radial, circumferential and axial electric filed intensity can be described as follows:

\[ E_{br} = -\frac{\phi_0}{r \ln(a/R)} - \left[ kG_1 I'_n (kr) + kG_2 K'_n (kr) \right] e^{i(\ell z + n \theta)} \].

(18)

\[ E_{b\theta} = -\frac{1}{r} \frac{\partial \phi_0}{\partial \theta} = - \frac{i n}{r} \left[ G_1 I_n (kr) + G_2 K_n (kr) \right] e^{i(\ell z + n \theta)} \].

(19)

\[ E_{bz} = -\frac{\partial \phi_0}{\partial z} = -ik \left[ G_1 I_n (kr) + G_2 K_n (kr) \right] e^{i(\ell z + n \theta)} \].

(20)

\( \vec{n} \) is normal to the outer jet of liquid. It can be expressed as

\[ \vec{n} = \left( 1, -\frac{\partial \eta}{\partial \theta}, -\frac{\partial \eta}{\partial z} \right) \].

(21)

According to electrodynamics boundary conditions, the tangential electric field strength is equal on both sides of the interface, in the gas-liquid interface has \( \vec{n} \times \vec{E}_n = 0 \), therefore,

\[ G_1 I_n (ka) + G_2 K_n (ka) = -\frac{\phi_0 \eta_0}{a \ln(a/R)} \].

(22)

From the boundary condition, we can obtain the equation as follows:

\[ G_1 I_n (kR) + G_2 K_n (kR) = 0 \].

(23)

Union the Eq.22and Eq. 23, we can derive the \( G_1 \) and \( G_2 \).

\[ G_1 = -\frac{\phi_0}{a \ln(a/R)} \frac{K_n (kr) \eta_0}{I_n (ka) K_n (kr) - I_n (kr) K_n (ka)} \].

(24)

\[ G_2 = -\frac{\phi_0}{a \ln(a/R)} \frac{I_n (ka) \eta_0}{I_n (ka) K_n (kr) - I_n (kr) K_n (ka)} \].

(25)

### 2.2 Dispersion Equation of Charged Jets in Liquid-liquid System

Surface tension stress can be expressed as
\[ p = \sigma \left( \frac{1}{R_1} + \frac{1}{R_2} \right) = \sigma (\nabla \cdot \vec{n}) = \sigma \left( \frac{1}{r} - \frac{\partial^2 \eta}{r^2 \partial \theta^2} - \frac{\partial^2 \eta}{\partial z^2} \right). \] 

(26)

According to the hydrodynamic boundary conditions, the stresses on the interface of charged jet in liquid-liquid system meet the equation as follows:

\[
p + p' - \sigma \nabla \cdot \vec{n} - 2\mu \left[ \frac{\partial (v_x + v_z)}{\partial r} \right] = p_b - \sigma_b \nabla \cdot \vec{n} - 2\mu_b \left[ \frac{\partial (v_x + v_z)}{\partial r} \right] - \frac{\varepsilon_b}{2} \left\{ - \frac{\phi_b}{r \ln(a/R)} \left[ kG_1 (kr) + kG_2 K_0 (kr) \right] e^{i(\varepsilon - n_0)} \right\}^2.
\]

(27)

When the jet is not perturbed in liquid-liquid system, the stress equation can be written as

\[
p - \frac{\sigma}{a} - 2\mu \frac{\partial v_r}{\partial r} = p_b - \frac{\sigma_b}{a} - 2\mu_b \frac{\partial v_r}{\partial r} - \frac{\varepsilon_b}{2} \left\{ - \frac{\phi_b}{a \ln(a/R)} \right\}^2.
\]

(28)

After Eq. 28 minus Eq. 27, ignoring the square value of small perturbation of the electric field intensity and supposing \( \alpha k \approx \alpha \), we obtain the following dispersion equation of charged jets in liquid-liquid system.

\[
\beta^2 = \frac{\sigma}{\rho a^2} \frac{I_s(\alpha)}{I_s(\alpha)} \left[ (1 - n^2 - \alpha^2) - \frac{\sigma_s}{\rho a^2} \frac{I_s(\alpha)}{I_s(\alpha)} \left( (1 - n^2 - \alpha^2) + \frac{\rho \varepsilon_b}{2} \frac{\phi_b}{a \ln(a/R)} \right) \right] - 2\mu \frac{\alpha^2}{a^2} \frac{I_s(\alpha)}{I_s(\alpha)} \beta
\]

\[
+ 2\mu_a \frac{\alpha^2}{a^2} \frac{I_s(\alpha)}{I_s(\alpha)} \beta - \frac{\phi_b \varepsilon_b}{\rho a^2 \ln(a/R)} \left( \frac{K_1(\alpha)}{I_s(\alpha)} - \frac{K_2(\alpha)}{I_s(\alpha)} \right) \left( 1 + \alpha \frac{kG_1 (R/a) I_s(\alpha) - I_s(R/a) K_0 (\alpha)}{I_s(\alpha) K_1 (R/a) - I_s(R/a) K_0 (\alpha)} \right).
\]

(29)

2.3 Results

If the growth rate has a positive real part, the jet in liquid-liquid system is unstable; otherwise, the jet is stable. Using numerical software of Matlab, we can solve the Eq.29. The relevant physical parameters involved in the calculation are water density (1000kg/m³), water surface tension (0.073N/m), biodiesel surface tension (0.026N/m), the radius of jet (0.3mm), the permittivity of biodiesel (1.77×10⁻¹¹F/m), the jet velocity (0-1m/s) and the charged voltage (0-12kV).
When \( n \) is equal to 0, the interface wave we discussed is 0 order wave and its waveform is symmetrical. When \( n \) is equal to 1, the interface wave is one order wave and its waveform is unsymmetrical.

2.4 Effect of Jet Velocity

Fig.2 shows the relationship of wave growth rate and wave number in 0 order and 1 order with the jet velocity ranging from 0 to 1m/s. From Fig.2, we know the wave growth rate increases with the increasing of wave number to a value, and then it begins to decrease. The wave number corresponding to the maximum growth rate \( \beta_{\text{max}} \) is the optimum wave number \( \alpha_{\text{opt}} \). The wave with the optimum wave number will determine the breakup of the jet in liquid-liquid system, and its wave length can be expressed as \( \lambda_{\text{opt}} = \frac{2\pi a}{\alpha_{\text{opt}}} \).

According to fluid dynamic, small disturbance decays in low velocity flow because of liquid viscosity. Only the disturbance waves whose wave length is up to a critical value get stronger and result in jet breakup at last. The relationship curves of wave growth rate and wave number have maximum wave numbers, and the corresponding wave with minimum wave length that can develop in the interface of water and biodiesel also can be found out with the maximum wave number.

Fig.2 shows with the increasing of jet velocity, the maximum growth rate and the corresponding optimum wave number increase, and the corresponding optimum wave length decreases. Because the wave with the maximum growth rate leads the jet breakup process, droplet diameter also gets smaller. The result is same as that is described in reference [9]. In the jet velocity range discussed, the maximum growth rate of 0 order wave is greater than that of 1 order wave and 0 order wave dominate the jet breakup.

2.5 Effect of Electric Filed Intensity

Fig.3 indicates the effect of electric field intensity on the development of instable interface wave when the jet velocity is 1m/s. The maximum growth rate for 0 order and 1 order wave increases substantially with the electric field intensity, so the electric field intensity is very important factor to breakup of jet in liquid-liquid system, same as the experimental result showed in reference [10]. When the charged voltage is 0, 3, 6kV, the maximum growth rate of 0 order wave is greater than that of 1 order wave, which means 0 order wave dominate the jet breakup in low charged voltage range. While the charged voltage is up to 9kV, the maximum growth rate of 1 order wave is close to that of 0 order wave and the curves of
1 order wave and 0 order wave almost coincide. Therefore, with the increasing of the electric field intensity, 1 order wave plays more and more important role in jet breakup and may lead to different atomization mode.

2.6 Effect of Liquid Viscosity

In general, higher liquid viscosity always lead to smaller droplet diameter in liquid-gas jet breakup, while in liquid-liquid system, the liquid viscosity of water and biodiesel have different impact on instability of jet. Fig. 4 and Fig. 5 show the influence of water viscosity and biodiesel viscosity on the growth rate without charged voltage respectively. The symbol of 100% in figure means the curve is obtained with the viscosity in 20 centigrade. It is well known that liquid viscosity turns lower when the temperature gets higher, so lower viscosity liquid can be got by heating. In Fig. 4 and Fig. 5, the liquids whose viscosity is reduced to 40%, 60% and 80% of viscosity in 20 centigrade are discussed.

Fig. 4 shows the maximum growth rate and the optimum wave number increases with the increasing of biodiesel viscosity, therefore, biodiesel viscosity promote the jet breakup. To get smaller droplet in liquid-liquid atomization, we should keep or increase the biodiesel viscosity as much as possible by water bath or other measurements. On the contrary, the maximum growth rate decreases with the increasing of water viscosity, but that is not obvious, showing in Fig. 5. Therefore, it is not necessary to reduce the water viscosity because that has no significant effect on jet breakup in liquid-liquid atomization.

Both Fig. 4 and Fig. 5 shown the maximum growth rate of 0 order wave is higher than that of 1 order wave and the 0 order wave dominates the jet breakup when the jet is not charged. The two figures also show these curves share the same end points with different viscosity both for 0 order wave and 1 order wave. That means the disturbance waves which can develop have the smallest wave length, and the viscosity just promote or slow down the deformation and breakup of jet, instead of deciding the fluid flow in liquid-liquid atomization.
2.7 Effect of Liquid Surface Tension

Fig. 6 and Fig. 7 illustrate the effect of water surface tension and biodiesel surface tension on the growth rate of interface wave. The signal of 100% in figures means the liquid share the usual surface tension, while the signal of 80% and 60% means the surface tension of the liquid has been reduce to 80% and 60% of usual value by adding surfactant.

The maximum growth rate and optimum wave number increase obviously by decreasing the water surface tension, according to Fig. 4, so adding surfactant to reduce water surface tension is a very useful method to obtain micro droplets of water in biodiesel. However, the maximum growth rate and optimum wave number decrease obviously by decreasing the biodiesel surface tension. Therefore, biodiesel surface tension has reverse effect on growth rate with water surface tension.
3. Conclusion

(1) Based on the surface wave theory and force characteristics analysis, the viscous dispersion equation in water-biodiesel system is derived.

(2) By numerical calculation, the effects of jet velocity, charged voltage, water viscosity, biodiesel viscosity, water surface tension and biodiesel surface tension on jet instability are studied.

(3) By increasing jet velocity or charged voltage, the maximum growth rate and optimum wave number increase obviously. Higher biodiesel viscosity is beneficial to jet breakup, while higher water viscosity has the converse effect, although the influence of water viscosity is not obvious. Reducing water surface tension is helpful to promote the jet deformation and breakup, but reducing biodiesel surface tension is not useful because it cuts down the maximum growth rate and optimum wave number.

(4) To obtain smaller water droplet in biodiesel by electric atomization, we can choose to increase jet velocity, charged voltage and biodiesel viscosity or to decrease water surface tension.
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References