Spectral Characteristics of Pedestrian Loads Under Different Crowd Densities Based on the Modified Bipedal Walking Model

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Walking speed is influenced by crowd density, which further affects the spectral characteristics of walking forces. Based on the fundamental relationship between walking speed and crowd density, and combined the modified bipedal walking model, this article studies the spectral characteristics of walking forces under different densities, and compared simulation results with relevant experimental statistics. Results show that the changing of crowd density has significant influence on first-order DLF, but little on high-order DLFs, which are consistent with experimental results.

Keywords: Crowd Density; Spectral Characteristic; Bipedal Walking Model.

1. Introduction

Walking speed is strongly influenced by crowd density [1], for example, pedestrians usually can walk in their free speed, but they have to adjust their speed to avoid mutual collisions in the case of high crowd density. Relevant experimental studies have confirmed that speed have significant influence on spectral characteristics of walking forces [2.3.4]. The newly proposed bipedal walking models can analyze the influence of walking speed on spectral characteristics through numerical simulation as well [5.6]. However, current numerical simulations analyze this influence from qualitative perspective merely, whether simulation results have the same spectral characteristics with experimental values needs further researches.

Aimed at above problems, and based on the fundamental relation between walking speed and crowd density in transportation engineering, further considered the imperfection of traditional bipedal walking model. Thus this article introduced the plantar excursion to correct the traditional bipedal walking model, and studied the spectral characteristics of walking forces under different densities with the modified bipedal walking model, further compared with
experimental values. Results show that modified bipedal walking model can simulate the spectral characteristics of walking forces that consistent with experimental values under different crowd densities.

2. Basic Relationship Between Walking Speed and Crowd Density

Walking speed is influenced by crowd density [1], meanwhile also affected by regional, cultural background and trip purpose, which have been studied in transportation engineering. A basic formula between walking speed and crowd density under comprehensive consideration of above factors has been proposed by certain researcher [1], see Eq.(1).

\[
V = V_M \left(1 - \exp \left[-\gamma \left(\rho - \rho_M \right)^3\right]\right)
\]

\[
V_M = 1.34 \alpha_G \alpha_T
\]

Where \(V\) is average speed under a certain crowd density; \(V_M\) denotes free walking speed; \(\rho\) is crowd density; \(\rho_M\) means density of crowd jams, and for western the value is 7.155 person/m², while 0.847 person/m² for eastern; \(\gamma\) means the sensitive index about travel purpose, while \(\alpha_G\) and \(\alpha_T\) are the correction factors of average free speed about region and travel purpose, and their values are shown in table 1. The fundamental relationship between walking speed and crowd density of eastern based on Eq.(1) are shown in figure 1. It can be seen from the figure that walking speed begin to decline significantly when crowd density reach 0.5 person/m², which indicated that it is necessary to consider the influence of crowd density on walking speed in high dense crowds.

<table>
<thead>
<tr>
<th>Sensitive index about travel purpose</th>
<th>Regional correction factor (\alpha_G)</th>
<th>Correction factor of trip purpose (\alpha_T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak Commute Fallow</td>
<td>Europ e Amer ican Asia</td>
<td>Peak Comm ute Fallow</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>0.273(\mu) 0.214(\mu) 0.245(\mu)</td>
<td>1.05 1.01 0.92 1.20 1.11 0.84</td>
</tr>
</tbody>
</table>

Figure 1. Fundamental relationship between walking speed and crowd density.
3. The Bipedal Walking Model

The bipedal walking model proposed in recent years is a novel load model considering dynamic characteristics of human body [5]. In the model, human body is simplified to equivalent centroid and massless bars with stiffness and damping. Elastic force and damping force are generated when bars contact with ground, which balance with gravity and inertial force to simulate human walking as shown in figure 2. The walking cycle of model including single and double support phase, which is converted through setting transition condition of walking gait, such as fixed impact angle $\theta_0$ [5].

![Figure 2. Bipedal walking model (from reference [5]).](image)

The motion equation of centroid as figure 2 can be established through Lagrange equation is shown as follows:

$$M \ddot{U} + C \dot{U} + KU = F(t)$$  \hspace{1cm} (3)

$$U = [z_1 \ z_2 \ u]$$  \hspace{1cm} (4)

$$M = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_1 + m_2 \end{bmatrix}$$  \hspace{1cm} (5)

$$C = \begin{bmatrix} c_1 & -c_1 & 0 \\ -c_1 & c_1 \sin^2\theta_0 + c_1 \sin\theta_0 \cos\theta_0 + c_1 \sin\theta_0 \cos\theta_0 & c_1 \sin\theta_0 \cos\theta_0 - c_1 \sin\theta_0 \cos\theta_0 \\ 0 & c_1 \sin\theta_0 \cos\theta_0 - c_1 \sin\theta_0 \cos\theta_0 & c_1 \sin^2\theta_0 + c_1 \sin\theta_0 \cos\theta_0 \end{bmatrix}$$  \hspace{1cm} (6)

$$K = \begin{bmatrix} k_l & -k_l & 0 \\ -k_l & k_l \left(1 - \frac{l_1}{l} \right) + k_l \left(1 - \frac{l_2}{l} \right) & 0 \\ 0 & 0 & k_l \left(1 - \frac{l_1}{l} \right) \left(1 - \frac{l_2}{l} \right) \left(1 - \sum_{i=1}^u \frac{l_i}{l} \right) - k_l \left(1 - \frac{l_1}{l} \right) \left(1 - \sum_{i=1}^{u-1} \frac{l_i}{l} \right) \end{bmatrix}$$  \hspace{1cm} (7)

$$F(t) = \begin{bmatrix} k_{l_1} l_1 m_1 g - k_{l_2} l_2 m_2 g \ F_{ext}(t) \end{bmatrix}^T$$  \hspace{1cm} (8)

In above equations, $z_1$ and $z_2$ are vertical displacement of centroid $m_1$ and $m_2$, and $u$ is horizontal displacement both of them; $c_l$, $c_1$ and $c_l$ are damping of body,
leading and trailing leg respectively; \( l_b \) and \( l_0 \) are length of body and legs; \( l_l \) and \( l_t \) are real-time length of leading and trailing leg; \( \theta_l \) and \( \theta_t \) are the angle of them with ground; \( d_i \) is length of step \( i \); \( F_{ctrl} \) is a control force to keep walking stable.

To verify validity of spectral characteristics of walking forces derived by bipedal walking model built in reference [5], and consider the influence that human body characteristic parameters on walking force [4]. This article took eastern and western human body parameters to simulate walking in different speed when impact angle is 68°, 69° and 70° respectively, then obtained spectrum of walking force through Fourier transform and compared with relevant experimental statistics. When comes to experimental statistics, the results obtained by Kerr [2], Zivanovic [3] and Chen [4] based on western and eastern human body characteristic parameters respectively are most representative, which shown in table 2. In table 2, \( \mu_k \) is the average value of first-order dynamic load factor(DLF) obtained by Kerr and Zivanovic, which expressed by Eq.(9); \( \mu_m \) and \( \mu_{75} \) are average value and value of 75% reliability of first-order DLF obtained by Chen, which expressed by Eq.(10) and Eq.(11).

<table>
<thead>
<tr>
<th>Dynamic load factor</th>
<th>Kerr-Zivanovic Average value</th>
<th>Standard deviation</th>
<th>Chen Average value</th>
<th>75% reliability value</th>
</tr>
</thead>
<tbody>
<tr>
<td>DLF1</td>
<td>( \mu_k ) ( = 0.166 )</td>
<td>0.03</td>
<td>( \mu_m )</td>
<td>0.0757</td>
</tr>
<tr>
<td>DLF2</td>
<td>( \mu_k ) 0.07</td>
<td>0.03</td>
<td>( \mu_m )</td>
<td>0.0409</td>
</tr>
<tr>
<td>DLF3</td>
<td>( \mu_k ) 0.05</td>
<td>0.02</td>
<td>( \mu_m )</td>
<td>0.0355</td>
</tr>
<tr>
<td>DLF4</td>
<td>( \mu_k ) 0.03</td>
<td>0.015</td>
<td>( \mu_m )</td>
<td>0.0261</td>
</tr>
<tr>
<td>DLF5</td>
<td>( \mu_k ) 0.03</td>
<td>0.015</td>
<td>( \mu_m )</td>
<td>0.0339</td>
</tr>
</tbody>
</table>

\[
\mu_k = -0.2649 f^3 + 1.3206 f^2 - 1.7597 f + 0.7613 \quad (9)
\]

\[
\mu_m = 0.2358 f - 0.2611 \quad (10)
\]

\[
\mu_{75} = 0.2358 f - 0.2010 \quad (11)
\]
Figure 3. Spectral characteristics of walking forces under different impact angles.

Comparison between simulation results and experiment results are shown in figure 3. As results show, the first-order DLF of simulation value decreases with increasing impact angle, meanwhile, it is greater than experimental results after stride frequency more than 2Hz and the deviation increases along with stride frequency under both parameters. Considering deviation between simulation results and experimental results, it is necessary to correct the bipedal walking model established in reference [5].

Relevant studies have shown that the center of plantar pressure (COP) will excursion from heel to toe, and centroid movement is the result of COP excursion together with rotation of supporting leg around COP during walking [7].

However, traditional bipedal walking model, namely the model established by reference [5], ignore the COP excursion, which makes bipedal model rely on larger rotation angle of supporting leg to get the same horizontal displacement as actual merely. Such a case, it would lead to decrease of impact angle and increase of first-order DLF, further aggravate the deviation between simulation
and experimental results in high stride frequency. Above all, it is necessary to introduce COP excursion to modify traditional bipedal walking model.

An empirical expression of COP excursion displacement under single support phase has been obtained through experimental observation in literature [8], such as Eq.(12). Where, $x_{cop}$ is displacement of COP; $d$ denotes total displacement of COP with value of 0.15cm; $\omega$ is frequency of single support phase. Meanwhile researches found $\omega$ increases with speed, and it highly correlated with natural frequency of the simplified human body system $\omega_n$[9]. Moreover, experimental observations indicated that COP excursion would stop when it migrates to the joint between metatarsus and phalanges [10], then walking phase transforms from single support phase to double support phase immediately.

$$x_{cop} = \frac{2d}{3\pi} \left( \sin \omega t - \frac{1}{3} \sin 2\omega t - \frac{3}{4} \omega t \right)$$

(12)

This article introduced COP excursion, expressed as Eq.(12), to modify traditional bipedal walking model, and defined $\omega = \lambda \omega_n$ to simulate COP excursion under different speed through changing parameter $\lambda$. Simultaneously, whether the COP migrates to the joint between metatarsus and phalanges can be severed as the transition condition that walking phase transform from single support phase to double support phase.

The extensive simulations have shown that the time history of single-step vertical force would have only one peak when speed is less than 1.1 m/s if parameter $\lambda$ is improper, and the deviation of peak value between left and right leg is significant when speed is greater than 1.6 m/s, which is inconsistent with actuality. In order to obtain walking force close to actuality, the value of parameter $\lambda$ and leg stiffness $k_2$ related to initial speed have been determined through gait simulation with speed from 1.1m/s to 1.6m/s. For example, $\lambda$ and $k_2$ calculated according to Eq.(13) and Eq.(14) respectively under western human body characteristics, while $\lambda$ and $k_2$ calculated according to Eq.(13) and Eq.(15) respectively under eastern human body characteristics can get reasonable results.

$$\lambda = 0.5v_i + 0.25$$

(13)

$$k_2 = 16v_i - 6.6$$

(14)

$$k_2 = 20v_i - 12$$

(15)

The force vector $F(t)$ in motion equations of single support phase changes from Eq.(8) to Eq.(16) after modification, while the motion equations of double
support phase are still the same with traditional bipedal walking model due to the COP excursion can be ignored.

\[
F(t) = \begin{bmatrix}
k l_b - m g \\
- k l_b - m g - c_j \dot{x}_{cp} \sin \theta_j \cos \theta_j \\
\dot{c}_j \dot{x}_{cp} \cos^2 \theta_j + F_{cp}(t)
\end{bmatrix}
\]  

(16)

4. The Spectral Characteristics of Modified Bipedal Walking Model

In the case of western human body parameters, namely upper trunk quality \( m_1 \) is 62Kg, lower trunk quality \( m_2 \) is 13Kg, and leg length \( l_0 \) is 1m, obtained the spectral characteristics of walking forces under initial walking speed is 1.1m/s to 1.6m/s respectively based on modified bipedal walking model, the results as shown in table 3. By the way, the value of rest model parameters are according to literature [5], namely body length \( l_b \) is 0.4m, trunk stiffness \( k_1 \) is 62KN/m, trunk damping \( c_1 \) is 14.6KN/m, and damping ratio of leg \( \xi \) is 0.08.

To prevent influence of model adaptive process, the DLFs of walking force are calculated through Fourier transform based on the walking force history of stable stage after 25s. The average walking speed and frequency are calculated by step length \( d_i \) and step number \( i \), and the relationship between walking speed and frequency, such as Eq.(17), can be obtained by fitting. Finally, combining Eq.(1), the crowd density corresponding to speed can be obtained.

Table 3. Simulation results of European commuting period.

<table>
<thead>
<tr>
<th>Density (p/m²)</th>
<th>Initial speed (m/s)</th>
<th>Average speed (m/s)</th>
<th>Frequency (Hz)</th>
<th>DLF1</th>
<th>DLF2</th>
<th>DLF3</th>
<th>DLF4</th>
<th>DLF5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4765</td>
<td>1.1</td>
<td>1.180</td>
<td>0.740</td>
<td>0.076</td>
<td>0.054</td>
<td>0.062</td>
<td>0.044</td>
<td>0.023</td>
</tr>
<tr>
<td>1.2854</td>
<td>1.2</td>
<td>1.349</td>
<td>0.722</td>
<td>0.110</td>
<td>0.054</td>
<td>0.059</td>
<td>0.044</td>
<td>0.024</td>
</tr>
<tr>
<td>1.1048</td>
<td>1.3</td>
<td>1.523</td>
<td>0.708</td>
<td>0.154</td>
<td>0.056</td>
<td>0.059</td>
<td>0.045</td>
<td>0.025</td>
</tr>
<tr>
<td>0.9420</td>
<td>1.4</td>
<td>1.700</td>
<td>0.695</td>
<td>0.210</td>
<td>0.059</td>
<td>0.060</td>
<td>0.047</td>
<td>0.026</td>
</tr>
<tr>
<td>0.7889</td>
<td>1.5</td>
<td>1.877</td>
<td>0.685</td>
<td>0.280</td>
<td>0.063</td>
<td>0.063</td>
<td>0.049</td>
<td>0.028</td>
</tr>
<tr>
<td>0.6384</td>
<td>1.6</td>
<td>2.060</td>
<td>0.674</td>
<td>0.366</td>
<td>0.068</td>
<td>0.068</td>
<td>0.053</td>
<td>0.029</td>
</tr>
</tbody>
</table>

\[
f = \frac{(V - 0.189)}{0.5822}
\]  

(17)

Comparing simulated DLFs under different speed with experimental results offered by Kerr-Zivanov and Chen, and the relationship between DLF and crowd density, as shown in figure 4, obtained through Eq.(1) and Eq.(17). In figure 4, first-order DLF decreases with the increase of crowd density, while two to five order DLFs less affected by density. What’s more, the simulated first-order DLF is bigger than result of Chen, while close to Kerr-Zivanovic’s.
Figure 4. Spectral characteristics of walking forces under western parameters.

Further, in the case of eastern human body parameters, namely upper trunk quality $m_1$ is 53.73Kg, lower trunk quality $m_2$ is 11.27Kg, leg length $l_0$ is 0.91m, and other parameters are same with literature [5], obtained the spectral characteristics of walking forces under initial walking speed is 1.1m/s, 1.2m/s, 1.3m/s, 1.4m/s, 1.5m/s and 1.6m/s respectively based on modified bipedal walking model, the results as shown in table 4. The relationship between walking speed and frequency, such as Eq.(18), can be obtained by fitting.
Table 4. Simulation results of Asian commuting period.

<table>
<thead>
<tr>
<th>Density p/m²</th>
<th>Initial speed m/s</th>
<th>Average speed m/s</th>
<th>Frequency /Hz</th>
<th>DLF1</th>
<th>DLF2</th>
<th>DLF3</th>
<th>DLF4</th>
<th>DLF5</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.8282</td>
<td>1.1</td>
<td>0.782</td>
<td>1.181</td>
<td>0.042</td>
<td>0.066</td>
<td>0.072</td>
<td>0.044</td>
<td>0.018</td>
</tr>
<tr>
<td>1.5365</td>
<td>1.2</td>
<td>0.889</td>
<td>1.389</td>
<td>0.072</td>
<td>0.047</td>
<td>0.066</td>
<td>0.044</td>
<td>0.020</td>
</tr>
<tr>
<td>1.2712</td>
<td>1.3</td>
<td>1.001</td>
<td>1.603</td>
<td>0.113</td>
<td>0.045</td>
<td>0.063</td>
<td>0.044</td>
<td>0.022</td>
</tr>
<tr>
<td>1.0291</td>
<td>1.4</td>
<td>1.112</td>
<td>1.821</td>
<td>0.165</td>
<td>0.050</td>
<td>0.061</td>
<td>0.046</td>
<td>0.024</td>
</tr>
<tr>
<td>0.7936</td>
<td>1.5</td>
<td>1.223</td>
<td>2.043</td>
<td>0.230</td>
<td>0.056</td>
<td>0.063</td>
<td>0.048</td>
<td>0.026</td>
</tr>
<tr>
<td>0.5061</td>
<td>1.6</td>
<td>1.332</td>
<td>2.262</td>
<td>0.314</td>
<td>0.064</td>
<td>0.066</td>
<td>0.051</td>
<td>0.028</td>
</tr>
</tbody>
</table>

Comparing simulated DLFs under different speed with experimental results offered by Chen, and the relationship between DLF and crowd density, as shown in figure 5, obtained through Eq.(1) and Eq.(18). As figure 5 shows, the simulated first-order DLF is close to the average result provided by Chen, while high-order DLFs under different human body parameters are basically invariable, which indicated that human body characteristic parameters mainly affect the first order component of walking force.

To sum up, the bipedal walking model considering COP excursion can simulate walking force consistent with experimental results under different crowd density in various human body parameters.

\[ f = (V - 0.189) / 0.5822 \]  (18)
5. Conclusion

Based on fundamental relationship between walking speed and crowd density in transportation engineering, and considered imperfection of traditional bipedal walking model. This article introduced plantar excursion to build the modified bipedal walking model, further studied spectral characteristics of walking forces under different densities and compared with relevant experimental statistics. Results show that: (1) through gait simulation based on modified bipedal walking model we can obtain walking force spectral characteristics consistent with experimental results under different crowd densities in the case of eastern or western human body parameters; (2) crowd density has significant influence on first order DLF, but little on high order DLFs.
References