The Fractal Nature of Chinese Calligraphy

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Abstract. This paper discusses the fractal nature of Chinese calligraphy. By comparing the Cantorian triadic bar with the Fuxi Sixty-four Hexagram Sequence Diagram that defines the cosmological and brush-and-ink-related technical significances of Chinese calligraphy, the author argues that the Fuxi Sixty-four Hexagram Sequence Diagram not only shares some similarities with the Cantorian triadic bar, but also defines the fractal nature of Chinese calligraphy. With the help of Technical Calligraphy Studies, Chinese calligraphy can be understood to be not only fractal but also algorithmic.

Introduction

I argue, in “Toward Fractal Calligraphy,” that Chinese calligraphy is fractal and based on algorithms [1]. It could be confusing, in particular, when the Yin and Yang features of ink traces in Chinese calligraphy are not comprehended both cosmologically as well as technically. When talking about the Mandelbrot set, Benoit Mandelbrot explains the powerful, visual scenery that nature provides us—“the same shapes are repeated everywhere, yet each repetition is somewhat different Compared to actual fractals, its structures are more numerous, its harmonies are richer, and its unexpectedness is more unexpected” [2]. Here, if we were to replace the Mandelbrot set with Chinese calligraphy, we would see that the fractal nature that Mandelbrot elaborates applies well to Chinese calligraphy—its basic strokes, brush techniques, and expected and unexpected calligraphic styles. This is one of the perspectives from which we may come to better understand the fractal nature of Chinese calligraphy.

In his fractal geometry classic, Benoit Mandelbrot defined the term fractal with a long list of definitions. The first two entries on the list read, “A fractal is by definition a set for which the hausdorff besicovitch dimension strictly exceeds the topological dimension. Every set with a noninteger D is a fractal. For example, the original Cantor set is a fractal” [3]. Considered the “essential skeleton or model” behind many fractals [2], the fractal sets that Cantor generated in the late 19th century, to be specific, the cantorian triadic bar (Fig. 1), are more or less an alternative version of the Fuxi Sixty-four Hexagram Sequence Diagram (i.e., Fuxi liushisigua cixu, as shown in Fig. 2, where all the Chinese characters actually should be removed in order to see the original diagram of black and white bars) of the Book of Changes (or I Ching). This ancient Chinese diagram was published at the latest by the Neo-Confucian master Zhu Xi (1130-1200) [4]. From a historical perspective, the cantorian triadic bar and the sixty-four hexagram diagram do not share a common origin. Mathematically, they are related: they are both fractal, and they are both created by continuous division, etc.

From the perspective of Chinese cosmology and philosophy, these continuous divisions represent the creation of Yin Yang dust—and they can continue to be divided endlessly. However, these iterations of Yin Yang dust can also be used to generate Yin Yang images. In particular, they can be seen as images of the black ink brush strokes in Chinese calligraphy written on white rice paper. All individual characters in Chinese calligraphy consist of the same, basic brush strokes repeated in many iterations. These iterations also follow specific rules or algorithms to form Chinese characters. Based on typical literary and brush stroke technical algorithms, these characters come together to form a piece of Chinese calligraphy.
Fractal Nature: Between Numbers and Cosmology

Both the Cantorian triadic bar and the Fuxi Sixty-four Hexagram Sequence Diagram have remarkable mathematical significance, which either define or support fractal geometry and Chinese calligraphy. On the one hand, the Cantorian ternary set is evidently mathematical, although it had been long considered a “mathematical [monster].” Mandelbrot even “had to erase every mention of Cantor” in order to get his work published [3]. With the middle 1/3 of the unit interval [0, 1] (shown as the top bar in Figure 1) being removed to generate the second two shorter or broken bars, the top two rows of solid and broken bars are very similar to the solid yang hexagram line and the broken yin hexagram line in the Book of Changes, specifically reminiscent of the upper portion of the trigram gen. This two lined “digram” represents the idea of suds (mo), stars (xing) and tininess (xiao), etc., which dates back to the first century B.C.E. [5]. This digram was also referred to as young yang (shaoyang) or the young son (shaonan) at an earlier age—at the latest around the fourth century B.C.E. [6]. The above procedure in Figure 1 continues to apply to each of the two remaining broken bars and so on. These continuous divisions eventually produce shorter and shorter segments until an infinite amount of “Cantor [dust]” is created [3]. Mandelbrot clearly defined such dust as “an informal equivalent to a set of topological dimension[s] \( D_T = 0 \), just as ‘curve’ and ‘surface’ denote sets of topological dimensions \( D_T = 1 \) and \( D_T = 2 \)” [3], but we are still able to see its cosmological nature in addition to its mathematical meaning. It is interesting to note that the upper half of the trigram gen (the digram shaoyang) is also related to the concept of tininess, although not dust-like, as well as young things in the universe.

On the other hand, the Fuxi Sixty-four Hexagram Sequence Diagram, which illustrates the cosmological nature and brush-and-ink-related technical significance of Chinese calligraphy, is indeed extraordinarily mathematical. Gottfried Wilhelm Leibniz took such mathematical significance as evidence for his statement that “the substance of the ancient theology of the Chinese is intact” because he believed that the moderns had missed the original meanings of their oldest books—in the best Hermetic tradition [7]. As he further commented, Fuxi or “Fo hi, the most ancient prince and philosopher of the Chinese, had understood the origin of things from unity and nothing, i.e., his mysterious figures reveal something of an analogy to Creation, containing the binary arithmetic (and yet hinting at greater things) that I rediscovered after so many thousands of years, where all numbers are written by only two notations, 0 and 1” [7]. With the dawn of the digital world, from which none of us is able to escape today, the mathematical significance of the binary Fuxi Sixty-four Hexagram Sequence Diagram cannot be overstated.

Although Shao Yong (1011-1077), perhaps the author, the editor, or the earliest transmitter of the Fuxi Sixty-four Hexagram Sequence Diagram, attributed the diagram to the prehistoric and legendary king Fuxi, his explanation of the diagram remains the best interpretation of its cosmological and mathematical meanings. According to Shao Yong, after the supreme ultimate Taiji—illustrated as the bottom white bar in Figure 2 or the top white bar in Figure 3—had divided into Yin and Yang—the black and white bars immediately above the bottom white bar in Figure 2 or those below the top white bar in Figure 3, Yin and Yang communicated with each other and gave birth to the Four Images or the four seasons, illustrated as the two pairs of black and white bars immediately above or below.
the *Yin* and *Yang* bars in Figures 2 and 3, respectively. Further communication between *Yin* and *Yang* led to the birth of the four images in the Heaven and the four images on the Earth. Consequently, the Eight Trigrams were established. The Eight Trigrams communicated with each other, and thus the sixty-four hexagrams and the ten thousand things were born. “Therefore, one divides into two; two divides into four, four divides into eight; eight divides into sixteen; sixteen divides into thirty-two; thirty-two divides into sixty-four… Ten divides into one hundred; one hundred divides into one thousand; one thousand divides into ten thousands… When they are combined together, they become the One; when the One separates, it becomes the Ten Thousand” [8]. The last portion of Shao’s elaboration elucidates the fractal features, from the perspectives of divinatory numerology and cosmology, of the *Yin* and *Yang* hexagram lines in Figures 2 and 3. This is exactly how Chinese calligraphy has been traditionally perceived. Before any brush strokes are written, the empty, white piece of rice paper is the intact Taiji or even Wuji, the ultimateless ultimate. As soon as the first stroke is written, *Yin* and *Yang* come into existence, i.e., the black brush stroke and the negative space left on the white rice paper. As the artist continues to draw repetitive, basic brushstrokes in variations and using similar components and character arrangements, “its structures are more numerous, its harmonies are richer, and its unexpectness is more unexpected” [2]. “When they are combined together, they become the One; when the One separates, it becomes the Ten Thousand” (Hezhi si wei yi, yanzhi si wei wan) [8]. These are some of the fractal features of the divinatory numbers that define the divinatory lines seen in the *Fuxi Sixty-four Hexagram Sequence Diagram*. This is how brushstrokes in Chinese calligraphy are organized according to the *Yin* and *Yang* theory illustrated in the *Fuxi Sixty-four Hexagram Sequence Diagram* (Figure 3), and also one perspective from which to perceive the fractal nature of Chinese calligraphy.

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**Fractal Calligraphy: From the Perspective of Technical Calligraphy Studies**

Fractal Calligraphy is definitely not the most important part of Technical Calligraphy Studies [9], yet it is perhaps the most attractive portion of it. Technical Calligraphy Studies has been a focus of our work at The Dartmouth Institute for Calligraphy and Manuscript Culture in China, Dartmouth College during recent years [10][11]. Supported by a generous Neukom CompX grant from The Neukom Institute for Computational Science at Dartmouth, we digitized representative strokes and related components in clerical script dated back to the Western Han (206 B.C.E.-25 C.E.) dynasty. It is this Neukom digital project that brought my attention to those numerous, repeated, similar or even identical brush strokes in clerical script, as well as to my pioneering development of Chinese Fractal Calligraphy and Chinese Fractal Mountains-and-Waters Painting [12].

From the perspective of Technical Calligraphy Studies, the study of fractal calligraphy facilitates our research, teaching and experiential learning of Chinese calligraphy. As I demonstrated in Figure 4 of my article “Toward Fractal Calligraphy,” the contemporary forgery of the so-called Western Han dynasty *wei* characters in clerical script does not fit in the stylistic development of the character in...
the Sierpinski’s gasket [1][13]. Such stylistic awkwardness can also be further examined by considering the geographic context (Figure 4) [14][15]. As for experiential teaching and learning, the fractal implications of the binary *Yin* and *Yang* system of the *Fuxi Sixty-four Hexagram Sequence Diagram* make it possible for the student to learn and comprehend the cosmological and fractal nature of Chinese calligraphy from the unique perspective of Technical Calligraphy Studies, such as through interactive learning using Fuxi’s fractal diagram created with the floating commands: “float maxdist = 60” and “float adjustbrightness = 255*(maxdist-d)/maxdist” in Processing 3.2.1 (as shown in Figure 5).

The fractal nature of Chinese calligraphy is best illustrated in creative works of Fractal Calligraphy. For example, Figure 6a is the *yong* character from the *Orchid Pavilion* in a Barnsley’s fern [16]. If we zoom in and take a look at the Barnsley’s fern at its 12th, 8th, and 2nd iterations in Figures 6b-d, it is self-evident that with the help of Technical Calligraphy Studies, Chinese calligraphy is not only fractal but also just an algorithm.

![Interactive Fuxi’s Hexogram Diagram in Processing 3.2.1.](image)

**Figure 5.** Interactive Fuxi’s Hexgram Diagram in Processing 3.2.1.

![Yong in the Barnsley’s Fern (24 Iterations).](image)

**Figure 6.** *Yong* in the Barnsley’s Fern (24 Iterations).

![Yong in the Barnsley’s Fern with 12, 8 and 2 Iterations.](image)

**Figure 7.** *Yong* in the Barnsley’s Fern with 12, 8 and 2 Iterations.

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