The Solution of VRPPDTW Model with Service Cost
Based on Genetic Algorithm

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Abstract. This paper combines with the traditional pick-up and delivery vehicle routing problem, in addition to consider the fixed cost and the variable cost and the penalty cost of violating the time window in the process of delivery, it add the service cost of the loading and unloading cargo. By calculating the proportion of the service cost and other parts of the cost to determine the weight of each part of the cost in the total cost. A model of vehicle routing problem with pick-up and delivery and time window (VRPPDTW) which includes the service cost is established. It designs rules to determine the adaptive crossover probability then basing on the genetic algorithm to solve the problem. Using the operator of select crossover and mutate to ensure the diversity of the population and using the elite strategy to ensure the stability of the population. At last, a numerical example is given to verify the model and the calculation method. As a result the genetic algorithm using adaptive rules to determine crossover probability is more suitable than SGA to solve this model.

Introduction

VRP is a well-known combinatorial optimization problem in logistics distribution. It can be defined as a directed graph \( G=(V,E) \). \( V=\{0,1,\ldots,N\} \) represents the vertex set using 0 to sign the logistics center the others are customer points. \( E=\{(i,j):i,j\in V,i\neq j\} \) represents the arc set \([1]\). It can be described as the following: Vehicles come from one or more of logistics center in order to achieve the shortest travel distance or the least cost or the least time after the completion of loading and unloading tasks for all the customer points\([2]\). VRP with time windows (VRPTW) is based on the VRP problem with the customer’s access time window constraints \([3]\). In the soft time window constraints the customer points have the opportunity to be included in the set of feasible solutions only cause penalty cost of violating the time window. The vehicle comes early to the customer point caused the waiting time and late to the customer point caused the delay time \([4]\).

At present the reverse logistics system involves two-way flow of cargo. The pick-up and delivery can be carried out at the same time in the customer demand point is a two-way logistics distribution routing problem. The pick-up and delivery constraints and the time window constraints make the problem more difficult than the traditional VRP \([5]\)

Literature References

Jesica de Armasn and Belén Melián-Batista \([6]\) studied VRP with additional constraints. David S.W. Lai, Ozgun Caliskan Demirag \([7]\) studied heterogeneous VRP. Some scholars combine the VRP problem and the bin packing problem (BPP) to extend the VRP problem to 2D or 3D \([8]\). Zhixing Luo, Hu Qin \([9]\) point out the effect of the consistency of the service level and the service level on the cost is studied by assuming the maximum number of customers are available in any planning area. Multiple roles, multiple attributes and multiple objectives make the problem more complicated. We must do more detailed research on the VRP in the perspective of economy, service, environment and so on \([10]\).

Chao Wang, John W. Sutherland \([11]\) in solving vehicle routing problem with simultaneous pickup-delivery using a parallel simulated annealing algorithm (P-SA). The algorithm in parallel
process produced an initial solution begin on the basis for improving program. The results show that feasible solutions quality is good. Avci, Seyda Topaloglu\cite{12} using HLS algorithm generates a short-term taboo list based on an adaptive cooling strategy in solving heterogeneous vehicle routing problem with simultaneous pickup-delivery. As a result the short-term memory access operation can avoid the cycle in the search process by using a taboo list. In this paper we study pick-up and delivery vehicle routing problem with soft time windows. We divide the distribution costs exactly by introducing the service cost in the model. Then select suitable weights to balance the cost of each part. The genetic algorithm is proposed according to the characteristic of the model, the speed to solve the problem and the quality of the feasible solution. Finally, the validity of the model or the algorithm is verified by a numerical experiments.

Establish VRPPDTW Model

Weight Determination for Each Part Cost

The service cost is added in this paper therefore the cost is changed from three parts to four parts then we need to make adjustments in the weight ratio. Literature shows that the vehicle’s service cost is a function of the demand at the customer point \( i \). The pick-up or delivery demand is expressed as \( Q \), service cost is expressed as \( S \), so \( S = D(0, Q) \). In the case of processing setting the weight ratio of the service cost and the other costs is 0.1:0.9. This paper also set the weight ratio of service cost and the other expenses cost is 0.1:0.9. The other costs on behalf of the distribution cost and vehicles penalty cost for coming early or delay to customer point. Dr. Lang Maoxiang set the weight on the delivery cost and the penalty cost for the vehicle early or late is 0.1:0.3:0.6. Later, the weight ratio of these three parts is adopted by many articles published in the authority literature. We are based on the concept to set the weight ratio. So the service cost ,the delivery cost and the penalty cost for the vehicle early or late is 0.1:0.09:0.27:0.54.It means that the first part of the weight is 0.1 and the sum of other three parts is 0.09+0.27+0.54=0.9. A VRPPDTW model that contains the service cost is established in order to find a way to minimize the cost of distribution along a suitable route.

Parameter Description

\( i \) is on behalf of the travel fixed costs and time required for loading or unloading of goods per ton. \( L_{ijk} \) on behalf of the actual load weight for vehicle \( K \) from point \( i \) to point \( j \) and \( [a, b] \) on behalf of the time window corresponding to the customer \( i \); \( w_1, w_2, w_3, w_4 \) on behalf of the weight of each part of the cost \( i \).

Model

\[
\begin{align*}
\min & \quad C \sum_{i=0}^{t} \sum_{j=0}^{t} \sum_{k=0}^{t} X_{ijk} + a \sum_{i=0}^{t} \sum_{j=0}^{t} \sum_{k=0}^{t} d_{ij} \cdot X_{ijk} + \\
& \quad w_1 \sum_{i=0}^{t} (d_i + p_i) + w_2 \sum_{i=0}^{t} \max(a_i - t_i, 0) + w_3 \sum_{i=0}^{t} \max(t_i - b_i, 0) + \sum_{i=0}^{t} \sum_{j=0}^{t} \sum_{k=0}^{t} X_{ijk} = 1 \\
& \quad \sum_{i=0}^{t} \sum_{j=0}^{t} X_{ijk} + \sum_{i=0}^{t} \sum_{j=0}^{t} X_{ijk} = 2 Y_{ijk} \\
& \quad \sum_{i=0}^{t} \sum_{j=0}^{t} X_{ijk} = \sum_{i=0}^{t} \sum_{j=0}^{t} X_{ijk} \\
& \quad 0 \leq L_{ijk} \sum_{i=0}^{t} \sum_{j=0}^{t} \sum_{k=0}^{t} X_{ijk}(d_i - p_i) \leq Q_{\max} 
\end{align*}
\]
\begin{align*}
0 & \leq d \sum_{i=0}^{n} \sum_{j=0}^{n} \sum_{k=0}^{n} X_{ijk} \leq Q_{\text{max}} \\
0 & \leq p \sum_{i=0}^{n} \sum_{j=0}^{n} \sum_{k=0}^{n} X_{ijk} \leq Q_{\text{max}} \\
\sum_{i=0}^{n} \sum_{j=0}^{n} \sum_{k=0}^{n} X_{ijk} d_{ij} & \leq d_{\text{max}} \\
t_i + t_i^* (d_i + p_j) + \max\{t_i - t_j, 0\} + t_j = t_j \ (i \neq j) \\
X_{ijk} & = \begin{cases} 1 & \text{the vehicle k comes from customer point i to customer point j} \\ 0 & \text{else if} \end{cases} \\
Y_{ijk} & = \begin{cases} 1 & \text{the customer i serviced by the vehicle k} \\ 0 & \text{else if} \end{cases}
\end{align*}

0 \leq i, \ j \leq I, \ 0 \leq k \leq K

Model Interpretation

(2): on behalf of the vehicle will back to the logistics center when it complete the service; (4)(7): on behalf of the actual carrying capacity must meet the constraint of the maximum loading capacity and the maximum travel distance constraint in the transport process; (8): on behalf of the total time of the vehicle reaches the customer point i, the vehicle’s loading and unloading time in the customer point i, the time window and the vehicle’s traveling time from the customer point i to the customer point j is equal to the time of the vehicle arrival to the customer point j;

Algorithm Design

Calculation Rules

**Crossover operation.** a. Crossover probability: Firstly, the order of the chromosome in the population is randomly disturbed. In addition the two adjacent chromosomes are matched into groups. At last, calculate the pc value of each group according to the rule

\[
PC = \begin{cases} 
\frac{k_1}{\text{max fits})^2} & \text{if } \frac{\text{fits(at(1))} + \text{fits(at(2))}}{\text{max fits}} \geq \text{ave fits} \\
\frac{\text{fits(at(1))} + \text{fits(at(2))}}{\text{max fits}} & \text{else if}
\end{cases}
\]

then determine the cross probability of each group.

b. Crossover operator: Arraying the genetic loci of the chromosome which will carry out the crossover operator randomly. The two adjacent chromosomes are matched into groups then the compare the two adjacent chromosomes in the same group with each other. At last according to the numerical value of gene adjust the sequence the two adjacent genes then exchange the two groups of chromosomes after the adjustment.

First: \(c_1 \) \(31|32|33|34|35|36|37|38| \ c_1 \) \(11|12|13|14|15|16|17|18\)
Second: \(c_1 \) \(37|32|38|35|36|34|33|31| \ c_1 \) \(15|14|18|12|16|11|17|13\)
Third: \(c_1 \) \(14|15|38|35|36|34|33|31| \ c_1 \) \(37|32|18|12|16|11|17|13\)

**Mutation Operation.** In this paper mutation strategy is to determine the probability of the mutation. First of all it specifies the pm value according to the experience and gives a definition of \(k_1\), if the evolution of the population stop then make \(k_1\) double. Secondly compare the pm value with the random number generated by the computer. If it is greater than the random number then set the first gene of the new chromosome as a random number. \(c_3 \) \(14|15|38|35|36|34|33|31|---\)
\(c_3 \) \(33|36|31|38|35|34|15|14\)
In the first place assuming that by the calculation \( pm > \text{rand} \) so it executes the mutation operation. Last but not the least set the number stored in first numerical gene whose label is \( |33| \) as a random number.

Table 1. Customer points demand information of tables.

<table>
<thead>
<tr>
<th>ID</th>
<th>x</th>
<th>y</th>
<th>d</th>
<th>p</th>
<th>UT</th>
<th>DT</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.8</td>
<td>5.5</td>
<td>0.8</td>
<td>0.5</td>
<td>6.0</td>
<td>14.6</td>
</tr>
<tr>
<td>2</td>
<td>15.2</td>
<td>10.9</td>
<td>0.6</td>
<td>1.6</td>
<td>2.4</td>
<td>13.8</td>
</tr>
<tr>
<td>3</td>
<td>18.6</td>
<td>12.9</td>
<td>0.4</td>
<td>0.7</td>
<td>0.1</td>
<td>12.3</td>
</tr>
<tr>
<td>4</td>
<td>11.9</td>
<td>8.2</td>
<td>1.6</td>
<td>0.3</td>
<td>0.6</td>
<td>14.1</td>
</tr>
<tr>
<td>5</td>
<td>10.2</td>
<td>9.5</td>
<td>0.8</td>
<td>0.6</td>
<td>1.7</td>
<td>13.8</td>
</tr>
<tr>
<td>6</td>
<td>5.3</td>
<td>9.6</td>
<td>0.6</td>
<td>1.4</td>
<td>0.3</td>
<td>10.4</td>
</tr>
<tr>
<td>7</td>
<td>0.6</td>
<td>9.9</td>
<td>1.9</td>
<td>1.7</td>
<td>5.9</td>
<td>12.6</td>
</tr>
<tr>
<td>8</td>
<td>6.1</td>
<td>15.0</td>
<td>1.3</td>
<td>1.0</td>
<td>1.7</td>
<td>8.5</td>
</tr>
<tr>
<td>9</td>
<td>7.6</td>
<td>19.2</td>
<td>1.8</td>
<td>0.9</td>
<td>0.6</td>
<td>10.3</td>
</tr>
<tr>
<td>10</td>
<td>16.0</td>
<td>15.7</td>
<td>1.8</td>
<td>1.9</td>
<td>6.1</td>
<td>13.9</td>
</tr>
<tr>
<td>11</td>
<td>15.3</td>
<td>15.2</td>
<td>0.4</td>
<td>1.0</td>
<td>4.1</td>
<td>11.8</td>
</tr>
<tr>
<td>12</td>
<td>1.6</td>
<td>14.7</td>
<td>1.6</td>
<td>1.8</td>
<td>4.0</td>
<td>12.4</td>
</tr>
<tr>
<td>13</td>
<td>9.0</td>
<td>9.2</td>
<td>1.1</td>
<td>1.4</td>
<td>0.1</td>
<td>7.1</td>
</tr>
<tr>
<td>14</td>
<td>5.4</td>
<td>13.3</td>
<td>1.6</td>
<td>0.4</td>
<td>0.1</td>
<td>9.8</td>
</tr>
<tr>
<td>15</td>
<td>7.8</td>
<td>10.0</td>
<td>1.0</td>
<td>1.8</td>
<td>8.1</td>
<td>14.7</td>
</tr>
<tr>
<td>16</td>
<td>18.6</td>
<td>7.8</td>
<td>0.8</td>
<td>1.8</td>
<td>6.7</td>
<td>12.5</td>
</tr>
<tr>
<td>17</td>
<td>14.5</td>
<td>5.3</td>
<td>1.4</td>
<td>0.8</td>
<td>0.5</td>
<td>9.4</td>
</tr>
<tr>
<td>18</td>
<td>15.0</td>
<td>18.7</td>
<td>1.2</td>
<td>0.2</td>
<td>0.0</td>
<td>13.0</td>
</tr>
<tr>
<td>19</td>
<td>9.8</td>
<td>5.0</td>
<td>0.4</td>
<td>0.5</td>
<td>1.1</td>
<td>10.9</td>
</tr>
<tr>
<td>20</td>
<td>1.4</td>
<td>6.9</td>
<td>1.4</td>
<td>1.0</td>
<td>2.9</td>
<td>7.5</td>
</tr>
</tbody>
</table>

Algorithm

Step1: A fitness function \( f(x) \) is defined on the domain space \( U \) and the corresponding parameters are stored in the data.m then visit the data stored in data.m;

Step2: By defining the size of the population the Chromosomes in \( U \) which can make up the initial population named \( chrom = \{ c_1, c_2, \ldots, c_{\text{nind}} \} \);

Step3: If it reaches the maximum \( gen \), the termination condition is satisfied. In the chrom, the best result is the chromosome which has the maximum fitness value.

Step4: Calculate the fitness value of each chromosome in the chrom and record the optimal individual, Capacity constraints are regarded as the conditions for judging whether a chromosome is qualified or not.

Step5: Select operation: Execute the roulette strategy with elite strategy to get the new chromosome named \( \text{chrom1} \);

Step6: Crossover operation: In \( \text{chrom1} \) If the \( pc >= \text{rand} \) it must take the cross operation for \( [\text{nind}/2] \) times Then the new chromosomes make up the new population \( \text{chrom2} \);

Step7: Mutation operation: In \( \text{chrom2} \), if \( pm >= \text{rand} \) the chromosome must take the mutation operation. The new chromosomes make up the new population \( \text{chrom3} \);

Step8: Take the population \( \text{chrom3} \) as a new one in place of \( \text{chrom1} \) and execute elite strategy. \( \text{gen} = \text{gen} + 1 \), return Step3;

Step9: Finding the best individual in each generation by looking up the index then generate an optimal set of individuals named \( \text{bestpop} \);

Step10: Take out the best individual named \( \text{minbestpop} \) from the optimal individual set named \( \text{bestpop} \) and decoding the optimal chromosome named \( \text{minbestpop} \) then carry out the remove duplicate path operation;
Numerical Experiment

It assumes the vehicle’s travel cost is 1 yuan / km, the wait the cost for the vehicle comes early is 3 yuan / hour, the opportunity loss cost for the vehicle comes late is 6 yuan / hour, the service cost for the vehicle loading and unloading at customer point is 1.1 yuan / ton, the use cost per vehicle is 20 yuan and the standard time of loading and unloading cargo is 1h/t.

The result of the genetic algorithm use the adaptive rule determines the crossover probability and the standard genetic algorithm show in the Table 2

Table 2. Distribution result statistics of tables.

<table>
<thead>
<tr>
<th>Route(AGA)</th>
<th>Route(SGA)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-21-2-6-14-5-20-1</td>
<td>1-3-17-4-1</td>
</tr>
<tr>
<td>1-15-7-16-13-1</td>
<td>1-18-5-6-14-1</td>
</tr>
<tr>
<td>1-3-4-17-18-1</td>
<td>1-15-19-20-1</td>
</tr>
<tr>
<td>1-12-11-19-1</td>
<td>1-13-10-11-12-16-1</td>
</tr>
<tr>
<td>1-8-10-9-1</td>
<td>1-2-21-7-8-9-1</td>
</tr>
<tr>
<td>Optimal cost</td>
<td>Optimal cost</td>
</tr>
<tr>
<td>275.9443</td>
<td>304.1886</td>
</tr>
<tr>
<td>Number of vehicles</td>
<td>Number of vehicles</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>Travel mileage</td>
<td>Travel mileage</td>
</tr>
<tr>
<td>148.6854</td>
<td>174.5041</td>
</tr>
<tr>
<td>Time using</td>
<td>Time using</td>
</tr>
<tr>
<td>15.719926s</td>
<td>15.983857s</td>
</tr>
</tbody>
</table>

Conclusion

The VRPPDTW model with service cost is studied in this paper. Considering the genetic algorithm has a strong robustness and global search capability which not depend on the gradient, so select the genetic algorithm to solve the problem in the paper. However, genetic algorithm has the disadvantage of premature convergence, so redesign the crossover operator to avoid the shortcoming that the individual will tend to the same state in a short period of time. In the end, the validity of the model and algorithm is verified by numerical experiments, the results show well.

In addition, the part of cost can be done a more detailed division. For example, considering the cost of environmental damage or the cost of transport damage per kilometer to the road and so on. The weight of each part can be determined by stochastic method and it is necessary for us do a further explore on how to allocate the weight better. It can also do more experiments to explore a more suitable algorithm for this model. Providing more detailed more practicable ways to simulate VRPPDTW and improving the capability of the algorithm for solving such problem which may be a research direction in the future.

Reference


