Theoretical Research on the Crane Operation Impact Caused by the Rail Joint Error

Ge-ning XU and Liang-bin HAN

School of Mechanical Engineering,
Taiyuan University of Science and Technology, Taiyuan, 030024

Keywords: The rail joint, The natural frequency, The operation impact.

Abstract. Aiming at the problem of the vertical impact dynamic effect caused by crane rail joint, the wheel - rail rigid body dynamics model is established, analyzes the influence degree of the crane operation impact caused by rail defects. According to the principle of maximum kinetic energy, the whole system is equivalent to a single degree of freedom system. On the basis of the response of the equivalent system to crane rail joint, the differential equation of the wheel - rail system is established, and the operation impact expression of the equivalent system is obtained. Which is compared and analyzed with the operation impact coefficient formula of the ISO8686-1:2012 "Cranes - Design principles for loads and load combinations - Part 1: General". Which provides a theoretical basis for the applicability of the operational impact coefficient formula of the ISO8686-1:2012"Cranes - Design principles for loads and load combinations - Part 1: General" and the GB/T3811-2008 "Design rules for cranes".

Introduction

The dynamic interaction between crane and rail is one of the most basic problems in the wheel - rail contact system. It directly restricts the speed increase and the weight increase of the crane, and also affects the safe operation of the crane [1]. The impact is caused by the crane through the rail joint, which results mainly in the destruction of the rail bolt hole crack and breakage of rail joints [2], thus reducing its service performance. In fact, the interaction force between the wheel and rail in addition to the low frequency response caused by the vibration, at the same time there is a very high frequency force [3]. The harm of excessive wheel - rail impact on the crane operation is great, especially in today's higher demands on the speed and heavy load, avoiding or reducing the adverse effects of wheel - rail dynamic impact, is China's crane industry facing the urgent problem needed to solve.

In recent years, the research on the dynamics of vehicle and track system (coupled) is a hot topic at home and abroad [4-6]. However, the research on the wheel - rail system is less in the field of crane. According to the principle of maximum kinetic energy, the whole system is equivalent to a single degree of freedom system. Due to the response of the equivalent system to the crane rail joint, the differential equation of the system is established, and the operation impact expression of the system is obtained. Which is analyzed and verified with the operational impact coefficient formula of the ISO8686-1:2012 "Cranes - Design principles for loads and load combinations - Part 1: General".

Dynamic Model

The motion of the center of the wheel is simplified when the wheel passes over the joint, as shown in figure 1. Where r is the radius of the wheel, $h_j$ is the height of the joint. The rail installation error requirement in the GB/T3811-2008 "Design rules for cranes" is known that joint height error is much smaller than the wheel radius, that is $h_j \leq r$, $\xi$ is the horizontal distance of the wheel center from the left to the right.
From their geometric relationships, the following formulas are known:

\[ e_s^2 + (r - h_s)^2 = r^2 \]  
\[ e_s = \sqrt{2rh_s - h_s^2} \]

(1)  
(2)

Because joint height error is much smaller than the wheel radius, \( h_s^2 \) is two order infinitesimal. The motion formula of the wheel center is:

\[ e_s \approx \sqrt{2rh_s}, \quad (h_s \leq r) \]

(3)

**A Mathematical Model of the Wheel over a Joint**

In Figure 2, the mathematical model of the cosine of the wheel center is established.

The assumed cosine function model is \( h(t) = A + B \cos(\Omega t) \), where \( t_s = \pi / v \), \( \Omega t_s = \pi \), \( h(0) = 0 \), \( h(t_s) = h_s \). The following formula can be obtained by the cosine function model:

\[ h(0) = A + B \cos(\Omega \times 0) \]
\[ h(t_s) = A + B \cos(\Omega t_s) \]

(4)  
(5)

So it can be calculated:

\[ A = \frac{h_s}{2}, \quad B = -\frac{h_s}{2} \]

(6)

\[ h(t) = \frac{h_s}{2}(1 - \cos \Omega t) \]

(7)

According to the two order derivative of the formula (7), the vertical acceleration of the roughness function when the wheel passes through the height of a joint is:

\[ \ddot{h}(t) = \frac{h_s}{2} \Omega^2 \cos \Omega t \]

(8)

Put \( \Omega t_s = \pi \), \( t_s = \pi / v \) and the formula (3) into the formula (8), the maximum vertical acceleration of the roughness function when the wheel at a speed \( v \) passes through a joint is:

\[ \ddot{h} = \frac{h_s}{2} \Omega^2 = \frac{h_s}{2} \left( \frac{\pi v}{\sqrt{2rh_s}} \right)^2 = \left( \frac{\pi}{2} \right)^2 \frac{v^2}{r} \]

(9)
Definition of Operational Impact Coefficient

The maximum vertical acceleration \( \ddot{z} \) is given by the following formula (10) and formula (11), when the mass \( m \) crosses a joint:

\[
\ddot{z} = \ddot{h} \xi_s (\alpha_s)
\]

(10)

\[
\alpha_s = \frac{\omega h_s}{\pi v} \sqrt{\frac{2r}{h_s}}
\]

(11)

In general, the vertical impact effect is generated when the crane is passing through the joint, and the definition of the operational impact coefficient is as follows:

\[
\phi_d = \frac{mg + m \ddot{z}}{mg} = 1 + \frac{h}{g} \xi
\]

(12)

By putting the formula (9) and the formula (10) into the formula (12), it can be known that the coefficient \( \phi_d \) is:

\[
\phi_d = 1 + \frac{h}{g} \xi_s (\alpha_s) = 1 + (\frac{\pi}{2})^2 \frac{v^2}{gr} \xi_s (\alpha_s)
\]

(13)

where: \( \xi \) is associated with \( \alpha \) coefficient.

Natural Frequency \( \omega \) of the System

Equivalent Simplification of the System

In the design, if the value \( y_d \) cannot be determined that can approximate \( y_d = [Y]S + [C]K \), where \([Y]\) for allowable rigidity of crane structure, the range of the value is \( 1/750 \sim 1/1000 \), \( S \) for span, \([C]\) for allowable rigidity of crane trolley, the range of the value also is \( 1/750 \sim 1/1000 \), \( K \) for trolley gauge of crane.

In the vibration of multi degree of freedom system, the first order fundamental frequency has practical significance for engineering applications \([8]\). In order to simplify the calculation, the three degree of freedom system is converted into an equivalent single degree of freedom system in Figure 3. The mass \( m_2+m_3 \) of the original system are transformed to the maximum displacement particle \( m_1 \), and the equivalent mass \( m_e \) of the single degree of freedom system is formed. Equivalent stiffness \( c_e \) is obtained from the series equivalent of \( c_1, c_2 \) and \( c_3 \). According to the equal deformation of the two systems under the same external force, the following formula (14) is shown:

\[
\frac{m_2g}{c_e} = \frac{m_1g}{c_1} + \frac{m_2g}{c_2} + \frac{m_3g}{c_3}
\]

(14)

The formula (15) could be obtained from formula (14):

\[
c_e = \frac{c_1c_2c_3}{c_1c_2 + c_2c_3 + c_1c_3}
\]

(15)

The following formula can be derived from the definition of stiffness.

\[
c_e = \frac{c_1c_2c_3}{c_1c_2 + c_2c_3 + c_1c_3} = \frac{m_2g}{y_e} = \frac{m_1g}{y_e}
\]

(16)

where: \( g \)——Acceleration of gravity;
\( y_e \)——Static displacement of equivalent single degree of freedom system under the action of equivalent loads \( m_2g \);
Under the rated lifting load \(m_1g\), the total static displacement of the crane structure, the wire rope winding and the trolley structure that is \(y_d = y_s + y_i + y_{s1}\) , where \(y_{s1} = m_{s1}g/c_1\), \(y_s = m_s g/c_2\), \(y_i = m_i g/c_3\).

The Equivalent Mass of the System

The equivalent single degree of freedom system and the original three degree of freedom system should have the same vibration mode and frequency.

Assuming \(m_i\) and \(m_1\) motion displacement and movement speed are the same, that is \(y_e = y_i\), \(\dot{y}_{max} = \dot{y}_{i, max}\), According to the principle of the maximum kinetic energy of the two systems, the following formula can be received:

\[
\frac{1}{2}m_{s1}\dot{y}_{s, max}^2 = \frac{1}{2}m_i\dot{y}_{i, max}^2 + \frac{1}{2}m_z\dot{y}_{z, max}^2 + \frac{1}{2}m_3\dot{y}_{3, max}^2
\]

The maximum velocity of a particle is proportional to the amplitude and is inversely proportional to the stiffness of the particle, so the formula (17) can be rewritten into:

\[
m_{s1}\left( \frac{1}{c_1} \right)^2 = m_i\left( \frac{1}{c_e} \right)^2 + m_z\left( \frac{1}{c_2} \right)^2 + m_3\left( \frac{1}{c_3} \right)^2
\]

The equivalent mass is obtained as the following formula:

\[
m_e = m_1 + m_2\left( \frac{c_e}{c_2} \right)^2 + m_3\left( \frac{c_e}{c_3} \right)^2 = m_1[1 + \frac{m_2}{m_1}\left( \frac{c_e}{c_2} \right)^2 + \frac{m_3}{m_1}\left( \frac{c_e}{c_3} \right)^2] = m_1(1 + \beta)
\]

\[
\beta = \frac{m_2}{m_1}\left( \frac{c_e}{c_2} \right)^2 + \frac{m_3}{m_1}\left( \frac{c_e}{c_3} \right)^2 = \frac{m_2}{m_1}\left( \frac{y_{d2}}{y_{d1}} \right)^2 + \frac{m_3}{m_1}\left( \frac{y_{d3}}{y_{d1}} \right)^2
\]

where: \(\beta\) is the influence coefficient of the equivalent mass.

Formula (16) and formula (19) can get the formula (20):

\[
y_e = y_d \frac{m_1}{m_e} = y_d (1 + \beta)
\]

According to the harmonic motion of the equivalent single degree of freedom system, the natural frequency expression is as follows:

\[
\omega = \sqrt{\frac{c_e}{m_e}} = \sqrt{\frac{m_1g}{m_e y_e}} = \sqrt{\frac{g}{y_d (1 + \beta)}}
\]

The Response of the Equivalent Single Degree of Freedom System at the Joint

As shown in Figure 4 with a roughness function \(h(t)\), the unit is \(m\) and coordinate \(z(t)\), the unit is \(m\). Describe the position of the spring supporting mass, the dynamic load expression of the spring is the formula (22), and the unit is \(N\):

\[
F(t) = k[h(t) - Z(t)]
\]

where: \(k\) is the equivalent stiffness of the system.

Response Analysis of Equivalent Single Degree of Freedom System over the Joint

When the mass \(m\) is over a joint of the track, the high and low misalignment error will provide a load excitation \(F(t) = k[\frac{h}{2}(1 - \cos \Omega t) - Z(t)]\) to the equivalent system.

In the study of wheel - rail dynamic action, the wheel - rail contact point is regarded as the absolute rigidity, and the contact deformation or the contact flexibility between the wheel and rail is neglected. The vertical movement of the wheel - rail system in the contact point is zero \([\cdot]\), that is \(z(t)=0\). The differential equation of motion of the system is:
\[ m\ddot{x} + kx = k \cdot h(t) \quad , \quad m\ddot{x} + kx = k \cdot \frac{h_s}{2} (1 - \cos \Omega t) \] (23)

The right side of the formula (23) is a constant and a harmonic function. According to the superposition principle, the steady state solution is the sum of the steady state solutions of the two following equations:

\[ m\ddot{x} + kx = k \cdot \frac{h_s}{2} \] (24)

\[ m\ddot{x} + kx = -k \cdot \frac{h_s}{2} \cos \Omega t \] (25)

The solution of formula (24) is:

\[ x_s(t) = \frac{h_s}{2} \] (26)

The homogeneous solution of formula (25) can be expressed as:

\[ x_h(t) = C_1 \cos \omega t + C_2 \sin \omega t \] (27)

where: \( \omega = \left( \frac{k}{m} \right)^{1/2} \) is the natural frequency of the system.

The excitation force \( k \cdot h(t) \) is harmonic form, so special solution also is harmonic form, and has the same frequency and excitation frequency. The form of the hypothetical solution is:

\[ x_p(t) = -X \cos \Omega t \] (28)

In the formula (28), the amplitude \( X \) of the expression \( x_p(t) \) is a constant. Put the equation (28) into the equation (25), the following formula can be gained:

\[ mX\Omega^2 \cos \Omega t - kX \cos \Omega t = -k \cdot \frac{h_s}{2} \cos \Omega t \]

\[ X = \frac{k \cdot \frac{h_s}{2}}{k - m\Omega^2} = \frac{h_s}{2\left[1 - (\Omega/\omega)^2\right]} \] (29)

The full solution of the formula (25) is:

\[ x(t) = C_1 \cos \omega t + C_2 \sin \omega t - \frac{h_s}{2\left[1 - (\Omega/\omega)^2\right]} \cos \Omega t \] (30)

The total solution of the formula (23) is:

\[ x(t) = \frac{h_s}{2} + C_1 \cos \omega t + C_2 \sin \omega t - \frac{h_s}{2\left[1 - (\Omega/\omega)^2\right]} \sin \Omega t \] (31)

Application of initial conditions \( x(t=0) = x_0 \) and \( \dot{x}(t=0) = \dot{x}_0 \):

\[ C_1 = x_0 + \frac{h_s}{2\left[1 - (\Omega/\omega)^2\right]} - \frac{\dot{x}_0}{\omega} \]

\[ C_2 = \frac{\dot{x}_0}{\omega} \] (32)

So the response of the system is:

\[ x(t) = \frac{h_s}{2} + \left(x_0 + \frac{h_s}{2\left[1 - (\Omega/\omega)^2\right]} \right) \cos \omega t + \frac{\dot{x}_0}{\omega} \sin \omega t - \frac{h_s}{2\left[1 - (\Omega/\omega)^2\right]} \cos \Omega t \] (33)

The vertical displacement acceleration of the mass \( m \) over a joint is:

\[ \ddot{x}(t) =-(x_0 + \frac{h_s}{2\left[1 - (\Omega/\omega)^2\right]} \right) \omega^2 \cos \omega t - \frac{\dot{x}_0}{\omega^2} \cos \omega t + \frac{h_s}{2\left[1 - (\Omega/\omega)^2\right]} \omega^2 \Omega^2 \cos \Omega t \] (34)

Mass \( m \) over a joint takes the vertical initial displacement \( x_0 = 0 \) and the initial velocity \( \dot{x}_0 = 0 \):
After the joint, the system will no longer bear the load excitation at the joint. Without considering the dissipation of energy, the system does not have damping free vibration, mechanical energy conservation. The size of the mechanical energy depends on the initial conditions and system parameters, i.e., the condition of the moment $t_s$, the kinetic energy and potential energy conversion. When the kinetic energy is completely converted to potential energy, the system speed is zero, the acceleration is maximum, i.e., the system acceleration of the moment $t_s + \Delta t$ is the maximum.

The speed of the moment $t_s$ is:

$$\dot{x}(t_s) = \frac{h}{2} \cdot \Omega \cdot \frac{\alpha_s^2 \cdot \Omega^2 \cdot \omega \cdot \sin \omega t}{1 - \alpha_s^2},$$

(39)

System kinetic energy at the moment $t_s$ is $E(t_s) = \frac{1}{2} m \cdot \dot{x}(t_s)$, and the potential energy of the moment $t_s + \Delta t$ is $U(t_s + \Delta t) = \frac{1}{2} k \cdot \Delta x^2$.

According to the principle of energy conservation, the following formula (40) can be obtained:

$$\frac{1}{2} m \cdot \ddot{x}^2(t_s) = \frac{1}{2} k \cdot \Delta x^2, \quad \Delta x = \sqrt{\frac{m}{k}} \cdot \dot{x}(t_s)$$

(40)

where: $\Delta x$ is a variation of the spring.

The kinetic energy is converted into elastic potential energy at the moment $t_s$, and the acceleration produced by the spring force $F = k \cdot \Delta x$ is:

$$\alpha_s = \frac{k \cdot \Delta x}{m} = \omega \cdot \dot{x}(t_s)$$

(41)

Put the formula (39) into the formula (41):

$$\alpha_s = \frac{h}{2} \cdot \frac{\alpha_s^2 \cdot \Omega^2 \cdot \omega \cdot \sin \omega t}{1 - \alpha_s^2},$$

(42)

The maximum acceleration of the system at the moment $t_s + \Delta t$ is:

$$\ddot{x}_{\text{max}} = \alpha_s + \ddot{x}(t_s) = \frac{h}{2} \cdot \Omega^2 \cdot \frac{\alpha_s^2 \cdot \omega \cdot \sin \pi \alpha_s + \pi \alpha_s}{1 - \alpha_s^2} \cdot (\cos \pi \alpha_s + \sin \pi \alpha_s + 1)$$

(43)

**Corresponding Relationship with International Standard**

When the wheel passes over a joint at the speed $v$, the maximum vertical acceleration of the roughness function is:

$$\ddot{h} = \frac{h}{2} \cdot \Omega^2 = \left(\frac{\pi}{2}\right)^2 \cdot \frac{v^2}{r}$$

(44)

Assuming
\[\xi_s(\alpha_s) = \frac{\alpha_s^2}{1 - \alpha_s^2} (\cos \pi \alpha_s + \sin \pi \alpha_s + 1) \quad (45)\]

Put the formula (44) and (45) into the formula (43), the maximum acceleration of the system is:

\[\ddot{x}_{\text{max}} = \frac{h_x}{2} \Omega^2 \cdot \frac{\alpha_s^2}{1 - \alpha_s^2} (\cos \omega t_s + \sin \omega t_s + 1) = h_x \xi_s(\alpha_s) \quad (46)\]

Because of \[\Omega t_s = \pi, t_s = \frac{e_s}{v} \text{ and } e_s \approx \sqrt{2rh}, (h_t \leq r);\]

\[\alpha_s = \frac{\omega}{\Omega} = \frac{\omega h_x}{\pi v} \sqrt{\frac{2r}{h_x}} \quad (47)\]

The vertical impact effect is generated when the crane is passing through the joint, and the operational impact coefficient \(\phi_4\) is:

\[\phi_4 = \frac{mg + mx}{mg} = 1 + \frac{\dot{x}}{g} = 1 + \frac{h_x}{g} \xi_s \quad (48)\]

Put the formula (44) and (46) into the formula (48), the coefficient \(\phi_4\) is:

\[\phi_4 = 1 + \frac{h_x}{g} \xi_s(\alpha_s) = 1 + \left(\frac{\pi}{2}\right)^2 \frac{v^2}{gr} \xi_s(\alpha_s) \quad (49)\]

**Result Analysis**

Select the required parameters. When the mass \(m\) passes through a joint, the change trend of the operational impact is analyzed with the operation speed and the error of high and low dislocation.

**Using the Method of This Paper to Analyze the Response at the Joint**

Figure 5 and figure 6 are the relationship diagrams of the operational impact coefficient \(\phi_4\) and the running speed \(v\) and the track height error \(h_x\), when the mass \(m\) crosses a rail joint. Selection of relevant parameters: According to the GB/T3811-2008 "Design rules for cranes"[9], crane wheel running speed range is (0.16~4) m/s; Crane wheel diameter range is (200~1000) mm. On the basis
of to the GB3811-1983 " Design rules for cranes"[10] and related literature [11], when the crane trolley is in the middle of the span lifting the rated load, suspension length of wire rope winding is equivalent to the rated lifting height, universal bridge crane natural circular frequency range is 2Hz \( f \leq 4\text{Hz} \).

In the range of the corresponding parameters, when the running velocity and the joint error increase, operational impact coefficient increases. When the running velocity \( v=4.0\text{m/s} \) and the joint error \( h_s=0.01\text{m} \), the operational impact coefficient is 1.8173.

According to \( \Omega = \frac{\pi v}{\sqrt{2r}h_s} \), \( \alpha_s = \frac{\omega}{\Omega} = \frac{\omega}{\pi} \sqrt{\frac{2r}{h_s}} \), \( \omega = 2\pi f \), \( r=0.5\text{m}, f=4\text{Hz} \) should be determined. So it could be obtained the three dimensional diagram of the operation impact coefficient and the operation speed and the joint error (Figure 6).

Figure 7 and figure 8 are the three dimensional diagrams of the operation impact coefficient and the operation speed and the joint error based on MATLAB in the crane rail joint error 0.001m.

In the range of the corresponding parameters, when the running velocity \( v=4.0\text{m/s} \) and the joint error \( h_s=0.001\text{m} \), the maximum operational impact coefficient is 1.0711.

According to \( \Omega = \frac{\pi v}{\sqrt{2r}h_s} \), \( \alpha_s = \frac{\omega}{\Omega} = \frac{\omega}{\pi} \sqrt{\frac{2r}{h_s}} \), \( \omega = 2\pi f \), \( r=0.5\text{m}, f=4\text{Hz} \) should be determined. So it could be gotten the three dimensional diagram of the operation impact coefficient and the operation speed and the joint error (Figure 8).

**Using the Method of the International Standard to Analyze the Response at the Joint**

Figure 9 and figure 10 are the results of the analysis in the crane rail joint error 0.01m.

![Figure 9](image1.png)  
**Figure 9. Relationship of \( \phi_s, v \) and \( h_s \).**

![Figure 10](image2.png)  
**Figure 10. Maximum relationship of \( \phi_s, v \) and \( h_s \).**

![Figure 11](image3.png)  
**Figure 11. Relationship of \( \phi_s, v \) and \( h_s \).**

![Figure 12](image4.png)  
**Figure 12. Maximum relationship of \( \phi_s, v \) and \( h_s \).**

![Figure 13](image5.png)  
**Figure 13. The relation of \( \phi_s, v (h_s=0.01\text{m}) \).**

![Figure 14](image6.png)  
**Figure 14. The relation of \( \phi_s, v (h_s=0.001\text{m}) \).**

In the range of the corresponding parameters, when the running velocity \( v=4.0\text{m/s} \) and the joint error \( h_s=0.01\text{m} \), the maximum operational impact coefficient is 1.6370. According to \( \Omega = \frac{\pi v}{\sqrt{2r}h_s} \),

\[
\alpha_s = \frac{\omega}{\Omega} = \frac{\omega}{\pi} \sqrt{\frac{2r}{h_s}} \quad \omega = 2\pi f \quad r=0.5\text{m}, f=4\text{Hz}
\]

should be determined. So it would be gained the three
dimensional diagram of the operation impact coefficient and the operation velocity and the joint error (Figure 10).

Figure 11 and figure 12 are the results of the analysis in the crane rail joint error 0.001m. In the range of the corresponding parameters, when the running velocity \( v = 4.0 \text{m/s} \) and the joint error \( h = 0.001 \text{m} \), the maximum operational impact coefficient is 1.0644. According to the formula:

\[
\alpha = \frac{\omega}{\Omega} = \frac{\omega h}{2 \pi r} \sqrt{\frac{r}{h}}, \quad \omega = 2\pi f, \quad r = 0.5 \text{m}, \quad f = 4 \text{Hz}
\]

should be determined. So it is derived the three dimensional diagram of the operation impact coefficient and the operation velocity and the joint error (Figure 12).

**Comparative Analysis**

Figure 13 and figure 14 are the change trend of the operation impact coefficient, when \( f = 4 \text{Hz}, \ r = 0.5 \text{m} \), the running velocity range is \([0.16, 4.0]\) m/s. The solid line (•) is the formula (49) of this paper method; the dotted line (--) is the international standard formula (c.11) given by ISO8686-1:2012; the star dotted line is the empirical formula (9) given by GB3811-2008. From Figure 13 which is shown, in the velocity range \([0.16, 2.0]\) m/s, the coefficient \( \phi \) increased significantly with the increasing velocity; in the velocity range \([2.0, 4.0]\) m/s, the coefficient \( \phi \) of the formula (49) is slightly smaller. When the running velocity is \( 2 \text{m/s} \), the maximum value of \( \phi \) is 1.8667; the coefficient \( \phi \) of the formula (c.11) is slightly larger. When the running velocity is \( 2 \text{m/s} \), the maximum value of \( \phi \) is 1.6370; When the velocity range is \([2.0, 4.0]\) m/s, it could be gained the solid line (•) always on the dotted line (--) above in Figure 13.

Figure 14 which are derived, in the velocity range \([0.16, 0.63]\) m/s, the coefficient \( \phi \) increased significantly with the increasing velocity; in the velocity range \([0.63, 4.0]\) m/s, the coefficient \( \phi \) of the formula (49) is slightly smaller. When the running velocity is \( 0.63 \text{m/s} \), the maximum value of \( \phi \) is 1.0867; the coefficient \( \phi \) of the formula (c.11) is slightly larger. When the running velocity is \( 4 \text{m/s} \), the maximum value of \( \phi \) is 1.0644; When the velocity range is \([0.30, 4.0]\) m/s, it can be derived the solid line (•) always on the dotted line (--) above in Figure 13.

**Conclusions**

(1) The crane rail joint error is 0.01m and the velocity range is \([0.89, 4.0]\) m/s. When the mass \( m \) passes through a joint, the operational impact coefficient \( \phi \) is given by this paper method formula (49) relative to the ISO8686-1:2012 formula (c.11) is more secure.

(2) The crane rail joint error is 0.001 m and the operating speed range is \([0.63, 4.0]\) m/s. When the mass \( m \) crosses a joint, the operation impact coefficient \( \phi \) given by this paper formula (49) is more secure than the ISO8686-1:2012 formula (c.11) also is more economic than the GB3811-2008 formula (9).

**References**


