Kinematics and Dynamic Modeling and Simulation Analysis of Three-wheeled Mobile Robot

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Abstract. Mobile robot is a typical nonlinear control system with nonholonomic constraints, so the system presents some complex features, and the system control becomes quite difficult. But nonholonomic makes robot structure has higher flexibility and reliability. Kinematics model and dynamic model are the foundations for system design and analysis of dynamic and kinematics performance, and are also the guarantee of controller practicability. After finishing nonholonomic characteristic analysis, coordinate transformation method and Lagrange equation method are used to establish kinematics and dynamic model of three-wheeled mobile robot with two driving wheels in this paper. The rationality and validity of the models are verified by simulation result of trajectory tracking.

Introduction

Mobile robot is an important branch of robotics, it is widely used in agricultural production, flexible manufacturing, unmanned exploration and other fields. Wheeled mobile mechanism is most-used and relatively simple. According to the quantity of wheels, wheeled robot can be divided into 1, 2, 3, 4 and multi-wheeled structure. 1 and 2 wheeled robots are difficult to solve the problems of stability and equilibrium, and more than 4 wheeled mechanism has problem of support degree redundancy. Considering the support redundancy degree, the best organization of walking mechanism is 3 groups. So three-wheeled mobile robot is analyzed in this paper.

Mobile robot control is most based on the kinematics model on the premise of low speed, low acceleration and low load. When the conditions cannot be meet, control precision is low. Therefore, dynamic modeling of mobile robot is also important. Wheeled mobile robot belongs to typical nonholonomic system, and the nonholonomic constraints increase the difficulty of dynamics modeling. The dynamic equation mainly is established by Lagrange equation and Newton-Euler method. Newton-Euler equation can directly incorporate nonholonomic constraints into the dynamic model, but the modeling process is complicated when robot has more components, so the Lagrange equation is adopted to establish the dynamic model of wheeled robot. The dynamic model of three-wheeled mobile robot with two independent driving wheels is developed in this paper. The simulation result verified the rationality and validity of the model which further provide a basis for the quality control of mobile robot [1,2].
Coordinate Establishment of Three-wheeled Robot

The robot consists of car body, two coaxial driving wheels and a driven wheel. Two driving wheels are driven by two independent motors. We use universal wheel as driven wheel to act the supporting role in movement. The structure diagram of the three-wheeled mobile robot studied in this paper is shown in Fig.1.

![Figure 1. Three-wheeled Robot Structure.](image)

Two coordinate systems will be set up: the global coordinate system and the robot local coordinate system, to describe the environmental factors (mainly environmental structure layout and obstacle distribution layout) of the mobile robot.

The global coordinate system is a two-dimensional coordinate (X-Y) with the origin at a certain point. It is used to describe the environment information, and identify the current position of the robot in the working environment. So it can be used for positioning the robot.

The origin of robot local coordinate system (X1-Y1) is the centroid of three-wheeled robot (assuming an overlap of driving shaft center and its centroid), and movement direction of robot is the transverse direction of the coordinate system. The robot can be abstracted as a point, and use this point position to describe the robot's position and orientation.

The parameters of three-wheeled robot shown in Fig.1 are: \((x, y)\) is Cartesian coordinates of the center of mass point \(O\) for nonholonomic mobile robot, namely, the position coordinates of nonholonomic mobile robots; \(\theta\) is angle relative to movement direction of mobile robot and X-axis, namely, the direction angle of mobile robot; \(\begin{bmatrix} v & w \end{bmatrix}^T\) are linear velocity and angular velocity of controlled variables for robot; \(v_L\) and \(v_R\) are linear velocity of robot’s left and right wheel; \(R\) is driving wheel radius of the robot; \(L\) is wheel track which is the distance between wheel centers of two driving wheels wheel; \(q = \begin{bmatrix} x & y & \theta \end{bmatrix}^T\) is robot pose \([3,4]\). The angle unit is radiant, and counterclockwise is negative. Length unit is m. The unit of time is second.

Nonholonomic Constraint Characteristics Analysis

Constraints in the movement process can be divided into holonomic constraints and nonholonomic constraints. Holonomic constraints limit the space location of moving object or limit space position and moving velocity of the object at the same time, but can be transformed
into space constraints by integration, so called geometric constraints; Nonholonomic constraint refers to the limit in space position and moving velocity of the object at the same time, cannot be transformed into space constraints by integration, so called non integrable constraints. Because of its non integrable constraints, it is difficult to stabilize, control and plan the nonholonomic constraint. The nonholonomic constraints are caused by pure roll and non sideslip restrictions of the wheels[5, 6]. Mobile robot is a typical nonlinear control system with nonholonomic constraints. The nonholonomic constraints make the system to present some complex features, such as hard to achieve linearization of input state, hard to achieve asymptotic stability by smooth nonlinear feedback, so it is difficult to be controlled, on the other hand, nonholonomic also makes the robot structure with higher flexibility and reliability.

The robot shown in Fig.1 has three wheels, while driven by two rear-wheels. The basic movements such as moving forward, backward, steering and so on can be controlled by setting the speed and direction data of the rear-wheels. Kinematic equation is following:

\[ v_L = w_L R, \quad v_R = w_R R \]  
\[ w = \frac{v_R - v_L}{L}, \quad v = \frac{v_R + v_L}{2} \]  

In (1) and (2), \( w \) is angular velocity of two driving wheel; Obviously, when the speed of the two driving wheels are same, \( w \) equals 0, that is the mobile robot walking along the straight line. When the speeds of two driving wheels are same while the direction is opposite, \( v \) equals 0, the robot will spin on the spot. Plug (1) into (2), we can get:

\[ w = \frac{w_R - w_L}{L} R, \quad v = \frac{v_R + v_L}{2} \]  

Caused by pure roll and non sideslip between driving wheels and ground, the nonholonomic constraints limit the movement direction of the platform, so the robot can only move vertically to the driving wheel shaft, which can be expressed in equation (4):

\[ y \cos \theta - \dot{x} \sin \theta = 0 \]  

Where \( \dot{x} = v \cos \theta, \dot{y} = v \sin \theta, \dot{\theta} = w \)  
Plug (3) into (4), can get:

\[ \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v \\ w \end{bmatrix} \]  

Equation (5) is written in matrix form, the constraint equation is:

\[ A(q) \dot{q} = 0 \]  

Where, \( A(q) = [-\sin \theta \quad \cos \theta \quad 0] \) and \( \dot{q} = [\dot{x} \quad \dot{y} \quad \dot{\theta}]^T \).  

With n-dimensional system state nonholonomic dynamic model of mobile robot can generally be described by the generalized mechanical systems with nonholonomic constraints

\[ M(q) \ddot{q} + C(q, \dot{q}) \dot{q} + G(q) + F(q, \dot{q}) + \tau_d(t, q, \dot{q}) = B(q) \tau - A^T(t) \lambda \]
Where, $q, \dot{q}, \ddot{q} \in \mathbb{R}^n$ is system state vectors, respectively denote the position vector, speed vector and acceleration vector; $M(q)$ is symmetric positive definite inertia matrix; $C(q, \dot{q})$ denotes the centripetal force and the Coriolis force related to the position and speed; $G(q)$ denotes the gravity about position; $F(q, \dot{q}) \in \mathbb{R}^{m \times n}$ respectively denote the friction torque vector related to the position and speed; $\gamma(t, q, \dot{q})$ is bounded unknown disturbance; $B(q) \in \mathbb{R}^{m \times r}$ is input transformation matrix; is the control input vector; $A(q) \in \mathbb{R}^{n \times m}$ is the constraint matrix; $\lambda \in \mathbb{R}^m$ represents the constraint force [7].

Mathematical Model of Three-wheeled Robot

Controller based on kinematics model is applicable under low performance requirement of the closed-loop system, but in reality, especially in complex operation, demands high accuracy. Now the mechanical systems are generally dynamic systems, which requires the dynamic model of the system. In the current research of mobile robot control, control law design is based on the combination of dynamic and kinematics model. The control input of dynamic system is torque, and control input of kinematics system is pose variation, so the two control input are not equivalent easily. Usually control input of dynamic system is integration of control input of kinematics system. So the dynamic and kinematics model of wheeled robot are demanded.

Kinematics Model of Three-wheeled Robot

Kinematics equation is a kind of presentation of the relation between the position and posture, which directly reveal the movement of system. Despite considering the reason why the system moves, it mostly researches the relationship between position and the derivative of time or other variables, which make the kinematics equation to be the foundation of the dynamic model. Before modeling and doing relative analysis, we do the following assumptions first:

1) The mobile robot moves on horizontal surface, and the car body is axial symmetry to the longitudinal axis;
2) Define the car body, all wheels and the system operation surface as rigid bodies;
3) The wheels keep point contacting on the running surface, the link line between the contact point and the two wheels center is consistently vertical to the surface;
4) The wheels running is pure roll and non sideslip.

In order to completely describe robot motion, (6) and (7) can be linked by selecting appropriate vector. We can choose a full rank matrix $S(q) = [S_1(q), \cdots, S_{n-m}(q)] \in \mathbb{R}^{m \times r}$ as a set of base of zero space, then get

$$A(q)S(q) = 0$$

(8)

According to (6) and (8), there are $n - m$ dimensional velocity vector as auxiliary input. And for time $t$, the nonholonomic constraints is:

$$\dot{q} = S(q)v(t)$$

(9)

Equation (9) is kinematics equation of three-wheeled robot [8].

$S(q)$ is Jacobian matrix, which converts speed $v$ in robot coordinate system to the speed $\dot{q}$ in cartesian coordinate system. From $A(q) = [-\sin \theta, \cos \theta, 0]$ and (9), we can get
\[ S(q) = \begin{bmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{bmatrix} \]  

(10)

Because \( n = 3 \), \( m = 1 \), so \( v \) is a two-dimensional vector. Taking \( v = [v_o, \omega_o]^T \) of which \( v_o \) is linear velocity of point O and \( \omega_o \) is angular velocity of point O. Kinematics equation of three-wheeled robot can be written as (11):

\[
\begin{bmatrix}
\dot{x} \\
\dot{y} \\
\dot{\theta}
\end{bmatrix} = \begin{bmatrix} \cos \theta & 0 \\ \sin \theta & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} v_o \\ \omega_o \end{bmatrix}
\]

(11)

Dynamic Model of Three-wheeled Robot

Dynamics equation is used to describe the relationship between the outside force with the position, velocity and acceleration of the system. It describes the dynamic performance of the system, which makes it to be the essential model of the system. Therefore, dynamic equation is the foundation for system design and analysis of dynamic performance, and is also the guarantee of controller practicability. There are many methods to establish the dynamics equation of mechanical system, such as Lagrange method, Newton-Euler method and Kane method, etc. Lagrange method is widely used because it is simple, just calculates kinetic energy and potential energy. The parameters of dynamics (7) obtained by Lagrange are as follows:

\[
M(q) = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I \end{bmatrix}, \quad C(q, \dot{q}) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B(q) = \frac{1}{r} \begin{bmatrix} \cos \theta & \cos \theta \\ \sin \theta & \sin \theta \\ R & -R \end{bmatrix}
\]

(12)

\[ \tau = (\tau_x, \tau_y), \lambda = -m(\dot{x} \cos \theta + \dot{y} \sin \theta) \dot{\theta} \]

Assuming the robot moves in the horizontal plane, and there is no friction, then \( G(q) \) and \( F(q, \dot{q}) \) equal 0. Taking the derivative of (9) and substituting to (7), then multiplying by \( s^T \) on both sides, we get:

\[ S^T M \dot{\nu} + S^T (MS + CS) v + S^T \tau_d = S^T B \tau \]

(13)

Because of \( S \in N(A) \), the constraint matrix \( A^T(q) \lambda \) is get rid of in transformational process. For the convenience of discussion, Equation (14) can be replaced to:

\[ \overline{M}(q) \dot{\nu} + \overline{C}(q, \dot{q}) v + \overline{\tau}_d = \overline{\tau} \]

(14)

Where, \( \overline{M} = S^T MS = \begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix}, \overline{\tau} = S^T B \tau, \overline{C} = S^T (MS + CS) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \)
Dynamic Simulation of Wheeled Robot

For verifying the rationality of the dynamic model of mobile robot, circular trajectory simulation was completed. Three-wheeled robot platform LK1 which designed by project team is shown in Fig.2. The left and right wheel motors and reduction ratio are consistent, and inertias of left and right wheel are same.

The related parameters of LK1 used in the simulation are: mass \( m = 13.55 \text{ kg} \), wheel track \( L = 0.48 \text{ m} \), wheel radius \( R = 0.0625 \text{ m} \), original value \( q_0 = [0 \ 0]^T \), \( \dot{q}_0 = [0 \ 0]^T \). Setting target value \( q_d = [1.0 \ 1.0]^T \), reasonable selecting controller parameters, circular trajectory simulation result is achieved when \( v = 1 \text{ m/s} \), \( w = 1 \text{ rad/s} \), adding external incentive \( F = 2\sin(2t + \pi/3) \), simulation result is shown in Fig.3. Trajectory tracking result has a little deviation of the expected trajectory, but overall tracking effect is ideal.

The simulation result shows that the proposed kinematics and dynamics modeling methods of mobile robot accord with the actual application requirements. The modeling methods are simple, and are easy to implement. The models provide the basis of theoretical analysis for further research on autonomous navigation, path planning of differential mobile robots.

Conclusion

Kinematics equation is to describe the relations between the position and posture of robot. Dynamics equation is used to describe the relationship between the outside force, velocity and acceleration of robot. Considering wheeled mobile robot has the characteristic of nonholonomic constraints, the kinematics and dynamic model of the three-wheeled mobile robot are established by using coordinate transformation method and Lagrange equation for providing theoretical basis for the controller design in this paper. Simulate result verifies the rationality and validity of the models. So Kinematics and dynamic models provide the mathematical basis for mobile robot controller design.
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