Comprehensive Performance Evaluation of Omni-directional Mobile Robots Based on AHP-FCE

Zeyu Zhang, Jun Zhang, Wenchao Zheng, Yalin Wen and Xiang Yuan

ABSTRACT

In order to comprehensively analyze the comprehensive ability of omni-directional mobile robots in specific working conditions, the method of combining AHP and fuzzy mathematics was used to carry out comprehensive evaluation and analysis. A multi-level performance evaluation system was established and the weights of each index were calculated by setting and analyzing theoretically the working scene. The membership matrix was established and the comprehensive performance score was calculated from the bottom to top by using the multi-level comprehensive evaluation method, then the result was analyzed. Suggestions for improvement were provided according to the weight of influencing factors. The result shows that the applicability of the omni-directional mobile robot to accomplish the work is high. The evaluation method is consistent with the practice, and it is feasible.1

INTRODUCTION

With the progress of society and the rapid development of robot technology, mobile robots play an important role in various fields [1]. Because of their delicate structure, omni-directional mobile robots can pass through narrow channels and right angles easily, and also can carry out high-precision positioning and fine adjustment. Their excellent sporting characteristics and high precision make them capable of handling goods in tight spaces [2, 3]. However, omni-directional mobile

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robots are high-price, difficult to operate and have a relatively short life span, which makes enterprises think carefully before choosing these. Therefore, it is necessary to make a comprehensive and accurate evaluation of the overall performance of omni-directional mobile robots. At present, this system is not mature.

The overall performance is a complex, multi-objective and multi-level system problem. Presently, the evaluation methods are mainly divided into qualitative method (expert survey method) and quantitative method (analytic hierarchy process [4-6], grey theory [7], principal component analysis method [8], fuzzy evaluation method and its combined theory [9-12]).

The quantitative analysis results are simple and clear, and can better meet the needs of decision makers. AHP-FCE can better evaluate omni-directional mobile robot’s comprehensive performance. In this paper, the AHP-FCE method is used to comprehensively evaluate and analyze the omni-directional mobile robot for the specified working environment and requirements, and the conclusions are summarized.

**Comprehensive Performance Analysis of Omni-Directional Mobile Robots**

For the analysis of the comprehensive performance of omni-directional mobile robots, it is necessary to consider its performance, environment, cost and other factors [13], and choose a reasonable analysis method, mainly reflected in the following aspects:

1) Analysis of its own performance

   According to the material, size and structure of the car body, the weight of the car body can be budgeted, load capacity, minimum turning radius and stability can be calculated, and the performance and working ability can be estimated.

2) Analysis of the environment

   Analyzing the robots according to the objective environment, work intensity and daily maintenance of the robot.

   **Budget for economy**

   According to the factors such as the raw materials and the required processing technology, consider the cost of manufacturing, maintenance, etc., and estimate the monetary benefits it will bring, and analyze its economics. Estimate its service life based on its fatigue strength and working environment.

4) Choice of reasonable analytical methods

   Adopting a comprehensive and reasonable analysis method, considering various factors and combining with uncertain issues, so that systematic and comprehensive analysis results can be obtained.

**The Establishment of Comprehensive Performance Evaluation Model**

Through AHP-FCE, the analysis of omni-directional mobile robots becomes more hierarchical and quantified [14], in which the main steps involved are: establishing the hierarchical model, creating the remark set, constructing the
hierarchical judgement matrix, calculating the weight set, establishing the membership matrix and multi-level comprehensive evaluation.

 Establishment of Hierarchical Model. AHP can divide the complicated problems into hierarchical indexes and the hierarchical problems will be solved more easily. Firstly, the target $U_i$ is divided into $m$ sub-indexes $U_{ij}$: $i=1,2,…,m$, and then the corresponding level index set is obtained: $U=(U_1,U_2,…,U_m)$. Then each sub-index $U_{ij}$ is divided into $n$ secondary indexes $U_{ijn}$: $j=1,2,…,n$, and the corresponding secondary index set is obtained: $U_{ij}=(U_{i1},U_{i2},…,U_{in})$. Again, the secondary index set is divided continuously and a multi-level index system is finally obtained.

 As different indexes have different evaluation values, they are always divided into different grades so as to create a remark set: $V=(V_1,V_2,…,V_k)$, $V_k$ represents different evaluation grades. We generally divide the evaluation system into five intervals, that is, $V=(very \ low, \ low, \ medium, \ high, \ very \ high)$.

 Construction of Judgement Matrix of Hierarchical Relation. The judgement matrix of hierarchical relation is of the comparisons based precedence relationships between different indexes at the same level according to experts’ investigations and relevant data. Judgement matrix is the core of AHP, and the judgement matrix of the index set $U_i$:

$$A = \begin{bmatrix} a_{11} & L & a_{1n} \\ M & O & M \\ a_{n1} & L & a_{nn} \end{bmatrix} \quad (1)$$

$A=(aij)_{n\times n}$ is a square matrix. Every time when two indexes $x_i$ and $x_j$ are taken, $aij$ is obtained by comparing them in their degree of importance, and then a judgement matrix is constructed. The comparative scaling technique$[15]$ adopted in this paper is as shown in Table 1:

 Calculation of Weight Set. This paper calculates the weights of Matrix $A$ according to its properties with the following methods:

 1) The Matrix $A$ is normalized in the column, and $E_i$ is obtained as below:

$$e_{ij} = \frac{a_{ij}}{\sum_{k=1}^{n} a_{kj}} \quad (2)$$

 2) Adding the matrices obtained in the row, and the vector $M$ is obtained as be

$$m_i = \sum_{j=1}^{n} e_{ij} \quad (3)$$

 3) Then carrying out normalization of the vector $M$ dimensionally, and the weight set is $W$ obtained as below:

$$W_i = \frac{m_i}{\sum_{i=1}^{n} m_i} \quad (4)$$

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TABLE I. THE VALUE STANDARDS OF JUDGEMENT MATRIX OF RELATIONS.

<table>
<thead>
<tr>
<th>$a_{ij}$</th>
<th>Comparison of the Two Indexes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>The two indexes are of same importance.</td>
</tr>
<tr>
<td>3</td>
<td>The index $i$ is a little more important than the index $j$.</td>
</tr>
<tr>
<td>5</td>
<td>The index $i$ is obviously more important than the index $j$.</td>
</tr>
<tr>
<td>7</td>
<td>The index $i$ is more important than the index $j$ greatly.</td>
</tr>
<tr>
<td>9</td>
<td>The index $i$ is more important than the index $j$ extremely.</td>
</tr>
<tr>
<td>2,4,6,8</td>
<td>The comparison-based degree of importance is situated between the two neighboring situation above mentioned.</td>
</tr>
</tbody>
</table>

The reciprocal of each value above mentioned

Similarly, other weight sets are obtained.

Construction of Membership Matrix. The membership degree method plays an important role in fuzzy evaluation. The construction approach of a membership matrix is as follows:

$$\mathbf{R}_i = \begin{bmatrix} r_{i1} & L & r_{ik} \\ M & O & M \\ r_{ni} & L & r_{nk} \end{bmatrix}$$ (5)

among which, $rij$ represents the membership degree of the index $i$ to the grade of $j$. In different conditions, the membership functions built are different greatly and the indexes involved are always difficult to quantify while the membership degree obtained through expert investigation is more authoritative.

Multi-level Comprehensive Evaluation. Multi-level comprehensive evaluation is a bottom-up computational method, with which different judgement sets are obtained subsequently and then the targets are obtained finally.

**Comprehensive Evaluation of Secondary Indexes**

According to the weight set $W$ of different secondary indexes and the membership matrix $R_i$ obtained, the judgement set $B_i$ of the secondary indexes can be calculated with the following method:

$$B_i = W_i^T \circ R_i = (W_{i1}, W_{i2}, L, W_{in}) \circ \begin{bmatrix} r_{i1} & L & r_{ik} \\ M & O & M \\ r_{ni} & L & r_{nk} \end{bmatrix} = (b_1, b_2, L, b_k)$$ (6)

Among which, $i=1,2,\ldots, m$, “$\circ$” is a fuzzy relation operator, the algorithm of which is as below:

$$b_{ij} = \max (w_{ij} \cap r_{je}) j = 1,2, L, n$$

$$e = 1,2, L, k$$

Carrying out normalization of Bidimensionally.
Comprehensive Evaluation of Level Indexes

The membership matrix $R$ of the level indexes can be obtained by combining the membership $B$ of different secondary indexes:

$$R = \begin{bmatrix} B_1 \\ B_2 \\ M \\ B_m \end{bmatrix} = \begin{bmatrix} W_1^T \circ R_1 \\ W_2^T \circ R_2 \\ M \\ W_m^T \circ R_m \end{bmatrix} = [r_{ij}]_{m \times k} \quad (8)$$

The judgement set of the level indexes can be obtained by combining $W$ and $R$:

$$B = W^T \circ R = (W_1, W_2, L, W_M) \circ \begin{bmatrix} B_1 \\ B_2 \\ M \\ B_m \end{bmatrix} = (b_1, b_2, L, b_k) \quad (9)$$

Carrying out normalization of $B$ dimensionally.

Comprehensive Evaluation of Targets

The hierarchical model probably involves many intermediate layers. Therefore, according to the methods above mentioned, it is available to obtained the judgement

![Diagram of the Hierarchical Model of Comprehensive Performance of Omni-directional Robots.](image-url)
set $B$ of the targets by solving the different layers from the bottom to the top. To work out the final scores, the score set $G$ is generally constructed based on the intermediate value within the interval of the remark set $V$: $G=(0.1,0.3,0.5,0.7,0.9)$. Here is the computing method of the comprehensive evaluation of the target indexes $U$:

$$D = B \ast G^T = (b_1, b_2, \ldots, b_K) \ast (g_1, g_2, \ldots, g_k)^T$$

(10)

### Application of the Model in Omni-directional Robots

At present, an omni-directional robot is required for carrying goods indoor, the maximum load of which should be four tons with; in addition, the requirements for the robot’s movement performance are also higher. Now there is a product having been designed, but the producer prices of it are very high and the factors affecting the performance are still unknown, so it is very necessary to carry out evaluation of the product’s comprehensive performance before the production.

Establishment of the Hierarchical Model. According to the different performance of the omni-directional robot, the multi-level index model is established mainly by taking the motor ability, operational capacity and economy, as shown in Figure 1:

Creation of Remark Set. The different performance is divided into five grades and a remark set is further created as shown in Table 2:

Construction of Judgement Matrix of Relations. According to the hierarchical models in Figure 1 and the precedence relationships between different indexes at a same level, it is available to construct weight matrices of different indexes as below:

$$A = \begin{bmatrix} 1 & 6 & 5 \\ 1/6 & 1 & 2 \\ 1/5 & 1/2 & 1 \end{bmatrix}, B_1 = \begin{bmatrix} 1 & 1/8 & 1/6 & 1/5 \\ 8 & 1 & 1 & 2 \\ 6 & 1 & 1 & 3 \\ 5 & 1/2 & 1/3 & 1 \end{bmatrix}$$

**TABLE 2. GRADE STANDARDS.**

<table>
<thead>
<tr>
<th>$V_k$</th>
<th>Scoring</th>
<th>Grade</th>
<th>Description of the performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Low</td>
<td>0.0-0.2</td>
<td>Grade One</td>
<td>very poor.</td>
</tr>
<tr>
<td>Low</td>
<td>0.2-0.4</td>
<td>Grade Two</td>
<td>a little poor.</td>
</tr>
<tr>
<td>Medium</td>
<td>0.4-0.6</td>
<td>Grade Three</td>
<td>medium.</td>
</tr>
<tr>
<td>High</td>
<td>0.6-0.8</td>
<td>Grade Four</td>
<td>a little good.</td>
</tr>
<tr>
<td>Very High</td>
<td>0.8-1.0</td>
<td>Grade Five</td>
<td>very good.</td>
</tr>
</tbody>
</table>

$$B_2 = \begin{bmatrix} 1 & 1/6 & 1/9 \\ 6 & 1 & 1/2 \\ 9 & 2 & 1 \end{bmatrix}, B_3 = \begin{bmatrix} 1 & 5 & 4 & 2 \\ 1/5 & 1 & 1/2 & 1/3 \\ 1/4 & 2 & 1 & 1/2 \\ 1/2 & 3 & 2 & 1 \end{bmatrix}$$
Calculation of Weight Set. According to the judgement matrix of relation above mentioned, different weight sets can be obtained by applying the Formula (2), (3) and (4).

\[
W = (0.7189 \ 0.1684 \ 0.1127)^T \\
W_1 = (0.0491 \ 0.3759 \ 0.3912 \ 0.1838)^T \\
W_2 = (0.0614 \ 0.3337 \ 0.6049 \ )^T \\
W_3 = (0.5056 \ 0.0868 \ 0.1434 \ 0.2642)^T
\]

Construction of Membership Matrix. Considering the comprehensiveness, the expert investigation based evaluation results of the index set \(U_1=(U_{11}, U_{12}, U_{13}, U_{14})\) of the motor ability is given in Table 3:

That is, the membership matrix \(R_1, R_2\) and \(R_3\) can be worked out:

\[
R_1 = \begin{bmatrix}
0.05 & 0.09 & 0.11 & 0.28 & 0.47 \\
0.01 & 0.03 & 0.12 & 0.32 & 0.52 \\
0.02 & 0.07 & 0.21 & 0.26 & 0.44 \\
0.04 & 0.13 & 0.25 & 0.17 & 0.41 \\
\end{bmatrix}
, \quad
R_2 = \begin{bmatrix}
0.09 & 0.20 & 0.15 & 0.17 & 0.39 \\
0.16 & 0.08 & 0.28 & 0.11 & 0.37 \\
0.15 & 0.16 & 0.32 & 0.20 & 0.17 \\
\end{bmatrix}
R_3 = \begin{bmatrix}
0.24 & 0.32 & 0.11 & 0.21 & 0.12 \\
0.12 & 0.18 & 0.32 & 0.25 & 0.13 \\
0.31 & 0.24 & 0.21 & 0.20 & 0.04 \\
0.17 & 0.22 & 0.33 & 0.14 & 0.14 \\
\end{bmatrix}
\]

Multi-level Comprehensive Judgement. Different judgement sets are obtained from the bottom to top.

Comprehensive Judgement of Secondary Indexes

By applying Formula (6) and (7), the normalized judgement set of the secondary indexes \(B_i\) based on the weight sets \(W_i\) and the membership matrices \(R_i\) of these secondary indexes:

\[
B_1 = (0.045 \ 0.120 \ 0.194 \ 0.295 \ 0.346), \quad B_2 = (0.136 \ 0.136 \ 0.273 \ 0.171 \ 0.284), \quad B_3 = (0.204 \ 0.273 \ 0.225 \ 0.179 \ 0.119)
\]

Multi-level Comprehensive Judgement. Different judgement sets are obtained from the bottom to top.

Comprehensive Judgement of Secondary Indexes
By applying Formula (6) and (7), the normalized judgement set of the secondary indexes $B_1$ based on the weight sets $W_1$ and the membership matrices $R$ of the secondary indexes:

$B_1 = (0.045 \ 0.120 \ 0.194 \ 0.295 \ 0.346)$, $B_2 = (0.136 \ 0.136 \ 0.273 \ 0.171 \ 0.284)$, $B_3 = (0.204 \ 0.273 \ 0.225 \ 0.179 \ 0.119)$.

Comprehensive Judgement of the Level Indexes

The membership matrix of the level indexes can be obtained by combining different memberships of the secondary indexes based on Formula (8):

$$R = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} = \begin{bmatrix} 0.045 & 0.120 & 0.194 & 0.295 & 0.346 \\ 0.136 & 0.136 & 0.273 & 0.171 & 0.284 \\ 0.204 & 0.273 & 0.225 & 0.179 & 0.119 \end{bmatrix}$$

By combining the weight set of the level indexes and the memberships of the secondary indexes, the normalized judgement set of the level indexes can be worked out based on Formula (9):

$$B = W^T \circ R = (0.123 \ 0.123 \ 0.175 \ 0.266 \ 0.313)$$

Comprehensive Judgement of Targets

According to Formula (8), it can be discovered that the final score of the targets $U_i$ is the product of the judgement set $B$ and the score set $G$ of the targets, that is, the comprehensive score of the omni-directional mobile robot in the applicability to work is:

$$D = BG^T = 0.6046$$

According to the evaluation analysis above mentioned, the comprehensive score of the omni-directional mobile robot is $D=0.6046$. It can be discovered from the evaluation grade standards in Figure 2, the score of the omni-directional mobile robot in the comprehensive performance is high, so the factory is suggested to use it.

See from $W$, the biggest factor affecting the robot’s comprehensive performance is the motor ability while again from $W_1$, the factors such as flexibility and accuracy are the secondary. Therefore, to improve the comprehensive performance, it is suggested to improve the flexibility and accuracy.

CONCLUSIONS

By considering the characteristics of the omni-directional mobile robot’s performance, comprehensive evaluation of them are carried out by combining AHP and fuzzy mathematics and the following conclusions can be obtained:

Through the setting of the working scene and theoretical analysis, multi-level evaluation of performance indexes are carried out with AHP-FCE, which makes the
scoring more objective and comprehensive; the intuitive score results can more meet the requirements of the decision-makers; it is available to find out the emphasis for improvement according to the different scores.

Through the estimation of the omni-directional mobile robot’s comprehensive performance with this comprehensive performance evaluation, the foreseeability in the design and selection of the robot is obtained, which reduces the losses brought by the errors in predication. The evaluation system in this paper is also available to more evaluation problems.

For the evaluation system of indexes in this paper, a systematic evaluation method which can be also realized by MATLAB programming is established. As a tool of comprehensive evaluation, this method improves the decision-making efficiency to a great extent.

REFERENCES