Analysis of Fault Evolution on Road Traffic Network Under Disruptive Traffic Accident

Wei Wan, Yuchen Huang*, Zhenhua Wang and Huanhuan Sun

ABSTRACT

This paper quantitatively analyzes statistical topology parameters of urban road network of Tongzhou based on complex network theory (CNT), and the value the degree, betweenness and strength are analyzed. Cascaded failure model is proposed based on the comprehensive consideration of the influence of road network topology and flow characteristics, and influence rule is analyzed with different disturbance values and different coupling strength. The results indicate that when the external perturbation is larger, the time of all nodes failure is earlier once the failure is triggered. Under attack, the threshold of the node-based largest betweenness damaged (1.6) is smaller than the node-based largest strength damaged (1.8). When the flow coupling coefficient is constant, no matter how the network topology coupling coefficient changes, the road network will have cascade failure. When the topology coupling coefficient is constant, the greater the flow coupling coefficient is, the more vulnerable the network to have cascading failures. The study is aimed at providing technical reference for control measures under sudden condition on road network.

KEY WORDS

Traffic Network; Coupled Map Lattices; Road Fault; Cascading Failure

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INTRODUCTION

With the rapid development of social economy, the number of traffic congestion incidents is also increasing due to natural disasters, bad weather, and terrorist activities. The sudden congestion phenomenon, occurring in some critical intersections or edges, will gradually spread to the adjacent intersections or edges, and even cause the whole road traffic network paralyzed. Therefore, studies on the cascade failure of road traffic network has significant implications on controlling road failures and taking emergency management for protecting road traffic network and improving operation stability.

In recent years, many scholars have carried out researches on the fault evolution of road traffic network in the world [1-5]. Peng et al.[6] provided an intelligent traffic flow congestion dispersal method through vehicle group autonomous cooperative scheduling. Yao et al.[7] proposed the SIS model of delay spread in the road traffic network by applying mean-field theory. And the analytical expression of critical value of delay spread is obtained in the steady state system of the SIS model. Zang et al. [8] studied the prediction model of vehicle queue length after traffic accidents on expressway based on the theory of vehicle flow fluctuation. Chowell et al. [9] used transim to analyze the linear correlation between degree and traffic flow. Yao et al. [10] discussed the whole process of the emergence, diffusion and extinction of congestion in the traffic network, and pointed out that congestion diffusion has distinct characteristics of media propagation in the RTN. Lu et al.[11] provided a traffic assignment algorithm for traffic congestion based on traffic control conditions. Zhang et al. [12] established a traffic network assignment model with steering delay and capacity constraints considering the characteristics of crowded traffic network.

However, the above published research mainly used queuing theory model and traffic wave model to analyze cascade failure of RNT, which is difficult to be applied to the dynamic analysis of complex road traffic network because of the large amount of calculation and complex modeling process. Present research conducts a cascade failure rule model to analyze road network considering the road network topology and traffic flow based on CML. A cascaded failure model is proposed based on the comprehensive consideration of the influence of road network topology and flow characteristics, and influence rule is simulated and analyzed with different disturbance values and different coupling strength.

METHODOLOGY

Basic Topological Property

CONNECTION INDICATOR-NODE DEGREE

The node degree (ki) refers to the number of edges connected to nodei, representing the location property of nodei. Where aij represents adjacency matrix of
the network. If there is a direct edge between node $i$ and node $j$, $a_{ij}$ will be 1; otherwise, $a_{ij}$ will be 0.

\[ k_i = \sum_{j \in N} a_{ij} \]  

(1)

**PATH INDICATOR-NODE BETWEENNESS**

Node betweenness counts the fraction of shortest paths passing through a given node and is an important evaluation index based on the path in the network. Where $B_i$ is node betweenness, $\sigma_{st}$ is the number of all shortest paths between nodes $s$ and $t$, and $\sigma_{st}(i)$ the number of shortest paths passing through node $i$ between node $s$ and node $t$.

\[ B_i = \sum_{s\neq t} \left( \frac{\sigma_{st}(i)}{\sigma_{st}} \right) \]  

(2)

**PASSENGER-FLOW INDICATOR-NODE STRENGTH**

Node strength denotes the sum of flow in edges connected with the given node. Where $s_i$ is the strength of node $i$, $n$ is the number of nodes, and $w_{ij}$ is the sectional passenger flow between two adjacent nodes $i$ and $j$.

\[ s_i = \frac{\sum_{j=1, j \neq i}^{n} w_{ij}}{n} \]  

(3)

**Vulnerability Assessment Model for Road Traffic Network Based on CML**

CML is a dynamical system, in which time and space are discretized variables and the state is continuous variable. The recent study of cascading failure in RTN that focuses on the topological network in CML. However, the cause for cascading failure is related not only to the network topology but also to the flow distribution. Therefore, this paper proposes a model that considered passenger flow to analyze the cascading failures of weighted RTN based on CML. The model provides a theoretical basis for the accurate vulnerability analysis of RTN. The following equation demonstrates the model:

\[ x_i(t+1) = (1 - \xi_1 - \xi_2) f(x_i(t)) + \xi_1 \sum_{j=1, j \neq i}^{N} a_{ij} f(x_j(t))/k(i) + \xi_2 \sum_{j=1, j \neq i}^{N} w_{ij} f(x_j(t))/s(i) \quad i=1,2,3...N \]  

(4)

Where $x_i(t)$ is the state of the intersection $I$ at the $t$-th time step, $k(i)$ is the degree of intersection $i$, $w_{ij}$ is the vehicle flow between nodes $I$ and $j$, and $s(i)$ is the sum of
flow in edges connected with intersection $i$. The connection information is given by the adjacency matrix $(a_{ij})_{N \times N}$ among $N$ intersections. If there is a direct connection between intersection $I$ and $j$, $a_{ij}$ and $a_{ji}$ will be $1$; otherwise, $a_{ij}$ and $a_{ji}$ will be $0$. $\zeta_1$ means topological structure coupling coefficients and $\zeta_2$ represents flow coupling coefficients; $\zeta_1$ and $\zeta_2$ have the constraints that $\zeta_1, \zeta_2 \in (0,1)$, $\zeta_1$ and $\zeta_2$ will be less than $1$. The parameters and coefficients in Eq.4 are used to ensure the state of each intersection being non-negative all the time. Function $f$ can be either a linear function or a non-linear function in CML. But the function $f$ demonstrates the local dynamic behaviors which is chosen in this work as the chaotic logistic map, $f(x) = 4x(1-x)$, to demonstrates the evolution law of volume or capacity constraints of intersections. If the initial states of all intersections in RTN are in the interval $(0,1)$ and there is no external perturbation, all intersections will keep a normal state forever, $(0 < x_i(t), t \leq l)$; vice versa, if the flow of intersection $i$ exceeds capacity constraints at the $l$-th time step, $(0 > x_i(t), t > l)$, the intersection $i$ will break down at the moment, and the state of failed intersections will be assumed $x_i(t) \equiv 0$ at any later time.

RTN encounters some sudden accidents, varying from a fundamental error to an anthropogenic attack, which leads to an intersection failure or shut down. An external perturbation $R \geq 1$ to the intersection $i$ at the $l$-th time step is added to show the effect of attacking. The modified model is demonstrated as follows:

$$x_i(t+1) = (1 - \zeta_1 - \zeta_2) f(x_i(t)) + \zeta_1 \sum_{j=i,j \neq i}^N a_{ij} f(x_j(t)) / k(i) + \zeta_2 \sum_{j=i,j \neq i}^N w_{ij} f(x_j(t)) s(i) + R$$

(5)

If the intersection $i$ at the $l$-th time step is failed, the state is $x_i(t) \equiv 0, t \geq l$. At the $(l+1)$-th time step, the states of those intersections directly connected with the intersection are affected, and the flow is recalculated (according to Eq. 5). At the moment, the states of some intersections are larger than $1$ and those intersections are failed. More serious is that the RTN occurs cascading failure.

For analyzing the vulnerability of RTN, the primary intersections are selected to under attacks of different evaluation indicators. If a intersection is attacked, the intersection and the edges connected with the intersection will be removed from RTN. Based on these attacking strategy, cascading failure proportion of the whole network is defined to evaluate the vulnerability of RTN. Where $I_t$ denotes failure proportion of the whole network, $N_t$ is the number of failed intersections in the network after attacking, and $N$ is the number of first intersections in the network.

$$I_t = N_t / N$$

(6)
CASE STUDY

Data Basis

In the paper, the scope of this paper is in Tongzhou: west from Wenyuhe West road, east to Beijing six ring, north from Luyuan North street, south to Yudaihe South street. To truly reflect the topological structure of the road traffic network, it is necessary to abstract the network reasonably. According to the map of Tongzhou city, the topology of Tongzhou RTN is constructed by space L (see Figure 1).

In the RTN, nodes represent the intersections, and an edge exists between two intersections if they are adjacent. The length of the edge is the actual distance between intersections. The topological structure model is viewed as an undirected graph $G_{2017} = \langle V, E \rangle$. where $G_{2017}$ denotes the Tongzhou RTN in 2017, $V$ is a collection of intersections, and $E$ is the edges of elements of $V$. Besides, the adjacency matrix of networks is $(a_{ij})_{N \times N}$, representing the connection between intersections $i$ and $j$, which is defined as $a_{ij} = \{1, (i, j) \in E \}$ and $a_{ii}$ will be 0. Similarly, distance matrix can also be expressed as $D_{ij} = \{d_{ij}, (i, j) \in E \}$ and $D_{ii}$ will be 0, where $d_{ij}$ is the actual distance between intersections $i$ and $j$ (see Figure 2).

Figure 1. Topological structure of Tongzhou urban road network.

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Figure 2. Examples of adjacency matrix in Tongzhou RTN.
In the road traffic network, the traffic flow is also an important factor affecting the traffic operation state. If an emergency traffic accident occurs in the peak time, it is easy to cause the whole road traffic network paralyzed. The traffic flow of the main road intersections in Tongzhou is collected in weekday morning or evening two rush hours (7:30–9:30 am or 17:00-19:00) from June 11, 2017 to July 11, 2017 by manual observation. According to actual observation, the peak time of the whole road network is 7:00-9:00, and the traffic flow during 7:00-9:00 is chosen as peak hour traffic flow. Then, the vehicle running time is regarded as impedance parameter. And a gravity flow assignment model is used to calculate the origin and destination (OD) flow from survey data. Then, the sectional flow between two adjacent intersections is calculated by assigning the OD flows (see Figure 3).

Figure 3. Examples of Cross section flow in Tongzhou RTN.

**Topological Characteristics of Tongzhou RTN**

To verify whether Tongzhou RTN is a scale-free network in space L, the figure of distribution of cumulative degree is added in the new manuscript. Besides, the node degree and the distribution of cumulative degree of Tongzhou RTN are fitted in the double logarithmic coordinate system (see Figure 4). Its distribution of cumulative degree plot follows a power-law distribution $p(k) \sim 1.1361 * k^{-0.81}$. The power-law exponent is 0.81, which describes that RTN is scale-free network and has the firm heterogeneity. There are a few nodes called the "deadly" nodes, which play a dominant role in the overall network. Such heterogeneity makes the scale-free network more vulnerable to intentional attacks.

The node strength reflects the distribution of network traffic flow, and the node betweenness reflects the network structure. Therefore, the node betweenness and strength are selected to evaluate the key intersections of the networking this paper. The node betweenness and strength of each intersection in RTN are first calculated, and top 5 primary intersections are selected, as listed in Table I. Intersection 3 is the
node with the highest strength of 25005, which means that there are 25005 vehicles through the intersection. Intersection 19 possesses the largest node betweenness of 0.2409, indicating that 24% shortest paths within the overall network pass the intersection. It is also worth noting that the two rankings are quite different.

Figure 4. Fitting figure of probability of accumulative degree distribution.

<table>
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<tr>
<th>NO.</th>
<th>Node Ranking Based on Node Strength ($S_i$)</th>
<th>Node Ranking Based on Node Betweenness ($B_i$)</th>
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<td>$B_i$</td>
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<td>5</td>
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Vulnerability Assessment For Tongzhou Rtn

The complex weighted network is established based on the sectional flow between two adjacent intersections from June 11 to July 11, 2017. Considering that the interrupt phenomenon of RTN often happens from specific intersections in practical operation, we select malicious attacks in the attack simulation based on CML model.

PERTURBATION THRESHOLD FOR CASCADING FAILURE

Two intentional attacks, of which one is with the largest betweenness ($b=0.2409$), and one is with the largest strength ($s = 25005$). Given $\zeta_1$ and $\zeta_2$ is 0.25, different perturbed values are added to the selected primary intersections.
As shown in Figure 5, when failures start to be triggered in the network, the intersections in RTN are all failed faster with the external perturbation increases. When the intersection with the largest betweenness is attacked (see Figure 5(a)), the threshold of failure is $R_{Bb}=1.6$, which is smaller than that of the intersection with the largest strength ($R_{Bs}=1.8$, see Figure 5(b)), indicating that the topological structure is more vulnerable than the flow distribution structure.

As shown in Figure 6, the relation between the scale and time step of cascading failures is approximately a normal distribution. Peak time is of great importance for taking effective measures. Under different intersections-based attacked, however, the scope of failure peak time having all nodes failed are different. Figure 6(a) presents that when the betweenness-based attack occurs and $R > R_{Bb}$, the failure peak time is $t_{pb}=12$. When the strength-based attack triggers failures (see Figure 6(b)), the peak time $t_{ps}=18$. This is consistent with the features of scale-free of Tongzhou RTN, indicating that the nodes with large betweenness play a key role in the network. The failures imposed on the key nodes of the network can spread quickly, and the changing of travel behavior in a small range cannot resist damage.

(a) The node-based largest betweenness is attacked. (b) The node-based largest strength is attacked.

Figure 5. The spreading process of cascading failure with different perturbations $R$.

(a) The node-based largest betweenness is attacked. (b) The node-based largest strength is attacked.

Figure 6. The relationship between failure scale and time step with different perturbations $R$. 
EFFECT OF TOPOLOGICAL COUPLING COEFFICIENT $\zeta_i$

The vulnerability of RTN under different topological coupling coefficients $\zeta_i$ (0.15 – 0.65) under perturbations of the largest betweenness and strength is studied, assuming $R=2>R_B^b$ and $R_B^b$, $\zeta_2=0.25$.

As shown in Figure 7, the spread range reaches 100% under the intersections with the largest strength or betweenness attacked. Moreover, when the values of $\zeta_2$ is constant, whether the intersections with the largest strength or betweenness are intentionally attacked, the spreading processes of failures are almost synchronized for different values of $\zeta_1$, (i.e., the peak times are $t_{pb}=12$ and $t_{ps}=[16,18]$, the failure peak proportions in a small range are $p_b=[0.125,0.165]$ and $p_s=[0.105,0.14]$ (see Figure 8). But the failures spread faster and the peak times are earlier while attacking the intersections possessing the largest betweenness than the one possessing the largest strength. In summary, RTN is vulnerable to the intersections with the largest betweenness.

![Figure 7. The spreading process of cascading failure with different topological coupling coefficients.](image)

(a) The node-based largest betweenness is attacked.  (b) The node-based largest strength is attacked.
The vulnerability of BRTN under different flow coupling coefficients $\zeta_2$ is studied. Firstly, a perturbation $R=2> R_{B}^{s}$ and $R_{B}^{b}$ is added to intersections with the largest betweenness or strength respectively, with $\zeta_1=0.25$ and $\zeta_2 = 0.15, 0.25, 0.35, 0.45, 0.55$, and $0.65$ respectively.

At $\zeta_2<\zeta_1$, when the intersections with the largest strength are intentionally attacked, there is no cascading failures occurred in road networks (see Figure 9). However, the spread range reaches 100% under the intersections with the largest betweenness attacked (see Figure 9(a)). At $\zeta_2\geq\zeta_1$, when the intersections with the largest betweenness or strength is attacked, hardly does $\zeta_2$ influence the spreading processes, cascading failures will occur in RTN. However, for the intersections with the largest betweenness, as the coupling coefficient increases, the cascading failure is more likely in RTN.

As shown in Figure 10, when $\zeta_2\geq\zeta_1$, a perturbation $R=2$ is added to the intersections with the largest betweenness, the peak times are $t_{pk}=11$, the peak proportion is $p_{pk}\in[0.13,0.14]$. If a perturbation $R=2$ is added to the intersections with the largest strength, the peak times are $t_{pk}=[18,19]$, the peak proportion is $p_{pk}=[0.105,0.125]$. The relevant operation management departments can effectively control cascading failures according to the peak time step.
DISCUSSIONS AND CONCLUSIONS

In this paper, the topology model of urban road network of Tongzhou is a scale-free network and has the firm heterogeneity. Then, the alicious attacks are selected in the attack simulation based on CML model. The dynamic simulation results indicate that once the failure is triggered in RTN of Tongzhou if the external perturbation is larger, the time of all nodes failure is earlier. Besides, when the values of $\zeta_2$ is constant, whether the intersections with the largest strength or betweenness are intentionally attacked, the spreading processes of failures are almost synchronized for different values of $\zeta_1$. When the values of $\zeta_1$ is constant, the failure is triggered in RTN of Tongzhou if the values of $\zeta_2$ is larger, the time of all nodes failure is earlier. There is a normal distribution relation between the scale and time.
step of cascading failures, which will help us find strategies to control the cascading failure spread and avoid further damage.

In this paper, the deficiency is that we only assume single node failed in the whole network, without considering the failure of multiple nodes. Future study will investigate cascading failures of road traffic network caused by multiple nodes congestion.

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