An Efficient Zoning Method for Voltage Control in Distribution Networks Based on Sensitivity Coefficients

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Abstract. Zoning method based on sensitivity coefficients is very efficient. But only reactive power injection is taken into account in most researches. It’s more reasonable to consider active and reactive power injection influences simultaneously. Then, AP clustering method is adopted to select the zones and critical buses in each zone. The results gained by AP method are more appropriate in comparison with the results obtained by hierarchical clustering approach.

Introduction

The increased penetration of distribute energy resources (DERs) in power systems during the past years brought voltage security challenges. With more and more stringent time constraints required by real-time controls and higher dynamics of distribution networks compared to the transmission ones, optimal control of distribution networks gains more and more interests. In order to maintain the voltages of the distribution networks within the permissible range, the system operating conditions should be sent to the controller. However, it is impractical to monitor all bus voltages considering the system complexity and investment. So, the whole networks needs to be separated into several zones and critical buses of each zone should be chosen. The solution of these problems are usually obtained by means of sensitivity coefficients [1]-[6]. Obtaining sensitivity coefficients through updating Jacobian matrix derived from the load flow problem [1] is a typical approach adopted by many researches. However, the main disadvantage of this kind of methods is that the Jacobian matrix needs to be updated every time when the operation conditions of the network change. Lots of direct computation methods have been proposed in order to solve this problem [2][5]. However, these researches neglected network lines shunt parameters or mutual coupling between different phase conductors. Electrical distance was introduced based on the typological analysis of the networks and applied to determine zones and critical buses of the secondary voltage control as an effective method [3]. However, only the reactive power injection impacts were considered based on the assumption that the voltage variations are decoupled with the active power injections due to high ratio of line reactance versus resistance (X/R). Whereas such an assumption is no longer applicable to LV/MV distribution networks due to relatively small X/R ratio. So, the impact of active power injections are taken into account in this presented approach. Both active and reactive power injection influences are considered when calculating the attenuation matrix between all the nodes of the system. Then, the affinity propagation (AP) clustering method is adopted to identify the zones and the critical buses automatically based on the attenuation matrix.
Methodologies

Zoning Method for Voltage Control in Distribution Networks Based on Sensitivity Coefficients

The maximum attenuation of voltage variations between two nodes can reflect and quantify the magnitude of the coupling in terms of voltage between these two nodes of an electrical system. These attenuation can be obtained from the sensitivity matrix. The attenuation between buses i and j written as \( \alpha_{ij} \) was defined as Eq.1 in Ref. [3]. It is based on the assumption that the voltage variations are just coupled with the reactive power injections and decoupled with the active power injections due to high X/R ratio in transmission networks. However, this assumption is no longer reasonable in LV/MV distribution networks. So, the influences of active power injections should be considered. Then, Eq.1 should be updated to Eq.2.

\[
\alpha_{ij} = \frac{\partial V_i}{\partial q_{ij}} \left| \frac{\partial V_j}{\partial q_{ij}} \right|
\]

Typical optimal controls in active distribution networks (ADNs) are voltage and power flow controls. However, it is impossible to monitor all bus voltages in the networks at the same time. The representative critical buses which can reflect the whole region operating conditions as monitored buses should be identified. Inspired by the zoning and critical buses selection approach in the secondary voltage control of transmission systems, a systematic zoning method based on the AP clustering technique is investigated in the presented work to divide the buses into several groups based on \( \alpha_{ij} \).

An efficient sensitivity coefficients of node voltages computation method in unbalanced radial electrical distribution networks was presented in Ref. [4]. Suppose an N-buses electrical network including S slack buses and I buses with PQ injections. The relation between the bus voltages and power injections is as,

\[
\mathbf{V} = \mathbf{Y} \times \mathbf{P}
\]

where \( \mathbf{V} \) is the elements of the admittance matrix \( \mathbf{Y} \), \( \mathbf{P} \) and \( \mathbf{V}^* \) is the conjugate of \( \mathbf{V} \) and \( \mathbf{P} \) respectively; S and I denotes the set of slack buses and the buses with PQ injections respectively (\( S \cup I = \{1, 2, \ldots, N\} \) and \( S \cap I = \emptyset \)). According to the theorem in [4], the voltage sensitivity coefficients with respect to power injections can be calculated by,

\[
\frac{\partial V_i}{\partial P_i} = \frac{1}{V_i} Re \left( \mathbf{V}^* \frac{\partial \mathbf{V}}{\partial P_i} \right)
\]

where \( i \in S \cup I \) and \( l \in I \).

Eq.4 is adopted to update the sensitivity coefficients of the critical buses in the presented work. The sensitivity coefficients of those other non-critical buses are assumed to be equal to the critical bus sensitivity coefficients.

AP Clustering Method

AP clustering method \(^{(7)}\) is a novel efficient clustering algorithm based on the concept of messages passing between data points. Unlike other clustering approaches such as fuzzy-c means algorithm, the hierarchical clustering method and k-means algorithm, AP clustering method (APCM) does not require to determine the number of clusters or the exemplars before running the algorithm. AP
mothed could be able to avoid many of the poor solutions caused by unsuitable initializations and hard decisions by considering all data points as potential exemplars and clustering gradually. It has fast processing speed and is suit to many tasks quite well [8-12].

There are three important parameters in AP method – responsibility $r(i,k)$, availability $a(i,k)$ and similarity $s(i,k)$ shown as Eq. (5)[7]. The parameters $a(i,k)$ and $r(i,k)$ are two kinds of messages transferred between data points: $a(i,k)$ reflects the current affinity that one data point has for choosing another data point as its exemplar at any point of in time; $r(i,k)$ passes the message that the accumulated evidence for how appropriate point $k$ is to serve as the exemplar for point $i$. The parameter $s(i,k)$ indicates the degree that the data point $k$ is appropriate to be the exemplar for the data point $I$ with the goal of minimizing squared error.

\[
\begin{align*}
\tau(i,k) & \leftarrow s(i,k) - \max_{k \neq i} \{a(i,k) + s(i,k)\} \\
a(i,k) & \leftarrow \min \{0, \tau(k,k) + \sum_{j \in \text{clusters}} \max \{0, \tau(i,k)\}\} \\
s(i,k) & = -\|x_i - x_k\|^2
\end{align*}
\]

A data point becomes an exemplar when the sum of its self-responsibility and availability is positive after several updating iterations. These exemplars are the centers in each group and the clustering number is identified simultaneously.

**Critical Buses Selection and Zoning Based on AP Method**

From the Eq. (5) we can find that similarity is a kind of distance. So, it could be applied to express the distance between two buses in distribution networks. After several repeating procedures the exemplars are gained as discussed in last section. These exemplars are the critical buses we are looking for and the number of zones is gained at the same time.

**Case Study**

The proposed zoning method is tested in a real Finnish distribution network [13] with two 20 kV feeders to evaluate the validity of the presented approach. The network topology is shown in Figure. I. There are eight distributed generation (DG) units in this Finnish distribution network including $3 \times 2$ MW WTs and $5 \times 500$ kW PVs and each DG is accompanied with an energy storage source (ESS) unit. The detailed information of these DG and ESS units is given in Table I. Zoning results based on hierarchical clustering method (HCM) is given for comparison. All simulations are completed in the same conditions: Windows7 64 bit OS; MATLAB with version R2016a; computer with 2.70GHz Intel Core Duo CPU and 16GB memory.

<table>
<thead>
<tr>
<th>DG No.</th>
<th>Location</th>
<th>Type</th>
<th>Capacity (MW)</th>
<th>ESS Capacity (MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DG 01</td>
<td>Bus05</td>
<td>PV</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>DG 02</td>
<td>Bus07</td>
<td>WT</td>
<td>2</td>
<td>0.4</td>
</tr>
<tr>
<td>DG 03</td>
<td>Bus20</td>
<td>WT</td>
<td>2</td>
<td>0.4</td>
</tr>
<tr>
<td>DG 04</td>
<td>Bus24</td>
<td>PV</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>DG 05</td>
<td>Bus31</td>
<td>PV</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>DG 06</td>
<td>Bus39</td>
<td>PV</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>DG 07</td>
<td>Bus36</td>
<td>WT</td>
<td>2</td>
<td>0.4</td>
</tr>
<tr>
<td>DG 08</td>
<td>Bus45</td>
<td>PV</td>
<td>0.5</td>
<td>0.1</td>
</tr>
</tbody>
</table>

**Clustering Procedures Based on AP Method**

Step1 Calculate the voltage sensitivity coefficients $\frac{\partial V}{\partial V}$ and $\frac{\partial V}{\partial Q}$ based on Eq.4.
Step2 Calculate the attenuation $\alpha_{ij}$ based on Eq. 2.
Step3 Construct the similarity matrix $S$ in AP method.

The element $s(i,j)$ of similarity matrix $S$ describes the similarity between point $i$ and point $j$, which is estimated by negative squared Euclidean distance in this work. Taking the attenuation
matrix $\alpha$ as the input matrix of AP method, which means to calculate the similarity matrix $S$ based on the attenuation matrix $\alpha$.

Step 4 Update responsibility $r(i, k)$ and availability $a(i, k)$.

Responsibility $r(i, k)$ and availability $a(i, k)$ describes two types passing messages between data points. They are updated based on Eq. 5 during the iteration.

Step 5 Repeat step 4 until the termination criterion is satisfied, such as the exemplar decisions keep constant for some prespecified iteration number or the maximum iteration number is reached.

Step 6 Making the assignments.

A data point becomes an exemplar when its self-responsibility plus self-availability becomes positive. For point $i$, its corresponding exemplar is defined as Eq. 6.

$$\text{exemplar}_i = \arg \max_k (a(i, k) + r(i, k))$$

Clustering Results Based on AP Method

The final critical buses selection and zoning results are obtained when clustering procedures mentioned above are completed. The results are presented in Table II in which the critical buses are in bold. The 45 buses are divided into 7 groups and bus 02, 07, 16, 22, 24, 26 and 36 are assigned as exemplars after 135 iterations. Due to all DG-connected buses are necessarily monitored, bus 05, 07, 20, 24, 31, 36, 39 and 45 should be chosen as the critical bus. So, combining the clustering exemplars and the DG-connected buses, bus 02, 05, 07, 16, 20, 22, 24, 26, 31, 36, 39 and 45 are selected as the critical buses (shown in red in Figure I).

Table 2. Zoning and critical bus selection based on AP method (in bold).

<table>
<thead>
<tr>
<th>No.</th>
<th>Buses</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>01, 02, 03, 04</td>
</tr>
<tr>
<td>2</td>
<td>05, 06, 07, 08, 09, 10, 11, 12, 13</td>
</tr>
<tr>
<td>3</td>
<td>14, 15, 16, 17, 18, 19, 20</td>
</tr>
<tr>
<td>4</td>
<td>21, 22, 23</td>
</tr>
<tr>
<td>5</td>
<td>24, 32</td>
</tr>
<tr>
<td>6</td>
<td>25, 26, 27, 28, 29, 30, 31</td>
</tr>
<tr>
<td>7</td>
<td>33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45</td>
</tr>
</tbody>
</table>
During the clustering procedure, another important parameter is the preference \((p)\), which is adjusted to identify different numbers of representative exemplar. Different preference value will result in different iterations, running time and data oscillations. The smaller preference value may cause oscillatory net similarity in iteration process. Likewise, larger preference can result in small net similarity and more cluster numbers with meaningless clustering work. In this case, the number of points is small enough that each data point may become the exemplar \(^9\). Therefore, the preference is set as a constant with \(p = \text{media} (S)/8\) in this presented work. The maximal iteration is normally 1,000.

**Clustering Results Based on Hierarchical Clustering Method**

Zoning is obtained based on hierarchical clustering method by choosing the threshold of 0.1 for comparison. The results are shown in Table III. There are seventeen buses needed to be controlled from the results gained based on hierarchical clustering method, while there are just twelve monitored buses gained by AP clustering method. The control process is much easier in the presented work. The zone 1, zone 2 and zone 3 in HCM are combined in APCM. We can find that this combination is more reasonable from Figure. I. There are no branches between these nodes. So, it’s more appropriate to zone them together. The zone 4 and zone 5 in HCM are combined in APCM. Nodes 08, 10 and 12 are connected to node 07 from Figure. I, so combining them together in one zone is more proper. Other zones are the similar situations. So, it’s not only not needing to pre-assign the number of groups but also more reasonable clustering results for APCM.

<table>
<thead>
<tr>
<th>No.</th>
<th>Buses</th>
<th>No.</th>
<th>Buses</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>01</td>
<td>9</td>
<td>24</td>
</tr>
<tr>
<td>2</td>
<td>02, 03</td>
<td>10</td>
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<tr>
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<tr>
<td>5</td>
<td>10, 11</td>
<td>13</td>
<td>33, 36, 37, 38, 40, 41, 42</td>
</tr>
<tr>
<td>6</td>
<td>14, 15, 16, 17, 18, 19, 30</td>
<td>15</td>
<td>34, 35, 45</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>22, 23</td>
<td></td>
<td>39</td>
</tr>
</tbody>
</table>

**Conclusion**

The influences of active and reactive power injections are considered to calculate the voltage sensitivity coefficients in this work. This approach is more reasonable than just considering the influence of reactive power injection in distribution network where the ratio of \(X/R\) is not high enough to be neglected. Then, AP clustering method is employed to decide the number of the zones and the monitored buses in each zone. The clustering results show that AP method acts better than hierarchical clustering method: the clustering number needn’t to be assigned before; the results can reflect the physical structure of the network better.

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**References**


