Auction-Based Fair Partner Selection for Decode-and-Forward Cooperative Relay Networks

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Abstract. In this paper, we propose a fair partner selection scheme for decode-and-forward cooperative relay networks based on auction theory. Modelling a single relay node as an auctioneer and multiple source nodes as bidders, the source nodes compete with each other by offering competitive prices to determine the recipient of the cooperative resources. To maximize the expected payoff, a closed-form Nash equilibrium strategy for each source node is derived over independent and identically distributed Rayleigh fading channels. Finally, numerical results and analysis verify the effectiveness of the proposed schemes.

1 Introduction

Cooperative relaying techniques have recently achieved a great deal of popularity as an efficient way to mitigate fading in wireless networks, especially in situations where physical multiple antennas are difficult to deploy at a single terminal. Various cooperative protocols have been proposed in [1, 2], such as amplify-and-forward (AF), decode-and-forward (DF), and estimate-and-forward (EF). From the perspective of practical implementation, the DF cooperative protocol is an appealing relay strategy [3].

In a cooperative relay network, partner selection is an important issue which affects the performance of the network significantly. Therefore, it is critical to design an effective partner selection scheme. The issues have been extensively studied in the literature [4-6] for DF relay networks. However, the vast majority of these works are based on the assumption that all terminals would be willing to participate in cooperative communication spontaneously. The assumption is unrealistic in autonomous, distributed, and flexible wireless networks, because cooperation consumes forwarders' extra resources such as power and bandwidth. Hence, it is necessary to design a mechanism to enforce cooperation for packet forwarding among greedy distributed terminals.

It is proved that the pricing-based mechanism [7-9] is an efficient method to stimulate cooperation among autonomous terminals, hence we introduce the approach. In pricing-based systems, a terminal is charged as source and reimbursed as relay. And the incentives to obtain virtual currency will make terminals willing to cooperate. In the pricing-based systems, there is an extensive literature on exploiting auction-based schemes for improving the efficiency of resource utilization. Mukherjee and Kwon in [10] developed a decentralized low-complexity cooperative partner selection scheme based on auction theory for dynamic ad hoc networks, which was proved to achieve outage performance close to a centralized partner selection scheme with complete channel state information (CSI). Works in [11] focused on the issues of resource allocation for distributed framework in cooperative networks, which were solved by mapping it into a multi-auctioneer multi-bidder power auction model. In [12], the authors studied a cooperation-based dynamic spectrum leasing mechanism via multi-winner auction of multiple bands. Based on a second-price auction mechanism, the primary users independently conduct auctions to determine winners who are then granted access to leased bands and prices for those bands. Zou et al. in [13] tackled the joint power and spectrum allocation problem under a cooperative cognitive radio framework, where primary users assist secondary transmissions and earn revenue, from selling the spectrum and cooperative power to secondary users. The trade between primary users and secondary users was modelled as an auction with two bundling commodities. However, the auction-based pricing method is still not involved to solve the distributed partner selection problems in the autonomous DF cooperative diversity system.

In this paper, we consider a DF cooperative relay network with a single relay node and multiple source nodes, in which the source nodes compete with each other to obtain the relay assistance. The problem is formulated as an auction game by making the relay node act as auctioneer and the source nodes act as bidders. In the game, the source nodes compete with each other by offering competitive prices. For independent and
identically distributed (i.i.d.) Rayleigh fading channels, the Nash equilibrium (NE) of each source node is derived in a closed form, based on which the expected payoff of the source node is also presented.

The rest of the paper is organized as follows. Section 2 presents the system model. Section 3 applies auction theory to analyse the partner selection problems, and derive the NE bidding strategy and expected payoff for each source node. Simulation results to demonstrate the effectiveness of the proposed scheme are presented in section 4. And it is followed by a conclusion in section 5.

2 System Model

As depicted in Figure 1, we consider a DF cooperative network consisting of one relay node and a set of source-destination pairs. Each pair includes a source node and a destination node.

![System models for DF cooperative transmission.](image)

The transmission of each data block can be divided into two phases [2]. In phase 1, each source node broadcasts its information to its destination and the relay node with power $P_s$. In phase 2, the relay node first decodes the received information from each source node. Let $S = \{s_1, \ldots, s_N\}$ denote the subset of the source nodes whose information can be decoded successfully. And then, it selects one source node $s_i$ from $S$ and forwards the regenerated symbols to destination $d_i$ with power $P_s$. We assume that an ideal cyclic redundancy check (CRC) code has been applied such that the relay node can judge whether the transmitted symbol is correctly decoded or not.

Suppose that the channel between any two nodes of the network undergoes quasi-static Rayleigh fading such that the fading coefficient is constant for a given time frame. Let $h_{i,j}$ and $n_{j,k}$ denote the instantaneous channel fading coefficient and received noise between node $j$ and node $k$, respectively, where $j \in \{s, r\}$ and $k \in \{r, d\}$. $h_{i,j}$ is modelled as normalized complex Gaussian random variable such that $|h_{i,j}|$ is the Rayleigh-distributed fading magnitude with $E(|h_{i,j}|^2) = 1$. And $n_{j,k}$ is complex additive white Gaussian noise with variance $N_0$

Due to the fact that the performance of each source node is mainly determined by its received signal-to-noise ratio (SNR), we focus on the received SNR. Using maximal ratio combining (MRC), the effective received SNR of $s_i$ for DF cooperation scenario is [3]

$$\gamma_{s_i}^{DF} = \gamma_{s_i, d} + \gamma_{r, d},$$

where $\gamma_{s_i, d} = \mathcal{Y}_{s_i, d} | h_{s_i, d} |^2$, and $\mathcal{Y}_{s_i, d} = P_s / N_0$ represents the average received SNR of the channel over fading. While in the direct transmission case, the received SNR is $\gamma_{s_i}^{dir} = \gamma_{s_i, d}$. Thus, we can obtain [3]

$$\gamma_{s_i}^{DF} = \gamma_{s_i}^{dir} + \Delta \gamma_{s_i},$$

where $\Delta \gamma_{s_i} = \gamma_{s_i}^{dir}$ is the extra SNR increase obtained by $s_i$ compared with the direct transmission. And we take $\Delta \gamma_{s_i}$ as the performance criterion of $s_i$ to highlight the gains from cooperation and make a fair competition, we assume that the average received SNR $\mathcal{Y}_{s_i, d}$ is the same for each source node such that each extra SNR increase $\Delta \gamma_{s_i}$ is i.i.d.

3 Problem Formulations and Auction Game Model

3.1 Problem Formulation and Assumptions

Since the relay node $r$ can only forward one source node’s information in a given time frame, the following question needs to be answered: how should the relay node select a source node from $S$ to achieve cooperative transmission? We address this issue by designing an auction-based distributed partner selection framework, in which the relay node and the source nodes act as auctioneer and bidders, respectively. Due to the fact that the source nodes should compete with each other to determine the recipient of the cooperative resources, the proposed scheme is an example of so-called competitive fairness.

Before introducing the auction game model, some fundamental assumptions used in this paper are presented. We assume that the source nodes compete with each other by submitting competitive prices (virtual currency), and each source node has enough virtual currency to take part in the auction. All the source nodes are risk neutral, i.e., they seek to maximize their expected profits. Moreover, the number of the competing source nodes $M$ is a common knowledge.

Before submitting a bid, each source node evaluates its performance of cooperative transmission and makes a valuation. The valuation that the source node $s_i$ attaches to the relay node $r$ is proportional to its extra SNR increase $\Delta \gamma_{s_i}$. Thus, it can be written as

$$v_i = \lambda \cdot \Delta \gamma_{s_i} = \lambda \cdot \gamma_{s_i, d} - \gamma_{r, d},$$

(3)
where $\lambda$ is a constant. Obviously, $v_i$ is exponentially distributed with parameter $\frac{(\lambda \gamma_{r,d})^{-1}}{\lambda \gamma_{r,d}}$. Thus, its cumulative distribution function $F$ is

$$F(v_i) = 1 - \exp\left(-\frac{v_i}{\lambda \gamma_{r,d}}\right).$$

(4)

We assume that the instantaneous channel gain $\gamma_{r,d}$ is known to the source node $s_i$ but not to other source nodes. Consequently, each source node knows its valuation precisely and only that other source nodes' valuations are i.i.d. according to the cumulative distribution function $F$.

### 3.2 Auction Game Model

There are two most prevalent auction forms, i.e., the first-price auction and second-price auction [14]. As the first-price auction is more familiar and even nature, it will be applied in this paper. In a first-price auction, the source node with the highest bid gets the relay node’s resources and pays the amount he bids. Each source node $s_i \in S$ submits a bid $b_i$ to the relay node, and given these bids, the payoff of $s_i$ is [14]

$$P_i = \begin{cases} v_i - b_i & \text{if } b_i > \max_{j \neq i} b_j \\ 0 & \text{if } b_i < \max_{j \neq i} b_j \end{cases}$$

(5)

If there is a tie, the relay node goes to each winning source node with equal probability.

We can see that $s_i$ faces a trade-off while fixing the bidding behaviour of others. An increase in the bid will increase the probability of winning while, at the same time reducing the gains from winning. Thus, each source node should determine the bidding strategy $\beta$ to maximize its expected payoff. The desirable outcome is called a NE, which is a bidding strategy $\beta^*$ such that no bidder wants to deviate unilaterally.

In the following we find the NE of each source node. As all the source nodes are symmetric, we have $\beta = \beta$ for each source node $s_i$. Notice that a source node with valuation 0 would never submit a positive bid since he would make a loss if it were to win the auction. Thus, $\beta(0) = 0$. We first suppose that $\beta$ is increasing and differentiable. Then, after finding a NE, we check that the bidding function has such properties. The expected payoff of a source node with valuation $v$ who bids $b$ is

$$(v - b)\Pr\{b \text{ is the highest bid}\}.$$  

(6)

Since the source nodes are independent with each other, the probability that the bids of all other $M - 1$ source nodes are at most $b$ is $[F(\beta^*(b))]^{M-1}$. So the expected payoff can be written as

$$(v - b)\left[F(\beta^*(b))\right]^{M-1}.$$  

(7)

To maximize the expected payoff, the derivative of (7) with respect to $b$ is zero:

$$\frac{(v-b)(M-1)[F(\beta^*(b))]^{M-2}F'(\beta^*(b))}{\beta^*(\beta^*(b))} - \frac{1}{F(\beta^*(b))^{M-1}} = 0.$$  

(8)

At a symmetric NE, $b = \beta(v)$, and thus we have

$$\beta'(v)F(v)^{M-1} + (M-1)\beta(v)F(v)^{M-2}F'(v) = (M-1)vF(v)^{M-2}F'(v).$$  

(9)

To simplify representation, we write $F(x)^N$ to denote $[F(x)]^N$. Integrating both sides, since $\beta(0) = 0$, we can get

$$\beta(v)F(v)^{M-1} = \int_0^v (M-1)xF(x)^{M-2}F'(x)dx$$

$$= vF(v)^{M-1} - \int_0^v F(x)^{M-1}dx,$$

(10)

where the second equality is obtained as a result of integration by parts. Thus, the NE is

$$\beta'(v) = v - \int_0^v F(x)/F(v)dx.$$  

(11)

Now, the symmetric NE of each source node is derived. Next, we will check its properties. The derivative of $\beta'(v)$ with respect to $v$ is

$$\beta''(v) = \frac{(M-1)F'(v)\int_0^v F(x)^{M-1}dx}{F(v)^N} > 0.$$  

(12)

Thus, the bidding function is indeed differentiable and increasing in $v$, which confirms our hypothesis. Moreover, it is obvious that, $\beta'(v)$ is increasing in $M$, and if $M$ is large enough, $\beta'(v)$ approaches $v$.

Based on the NE, the expected payoff of the source node is

$$m = [v - \beta'(v)]F(v)^{M-1}$$

$$= \int_0^v F(x)^{M-1}dx.$$  

(13)

Obviously, $m$ is increasing in $v$ and decreasing in $M$.

Substituting (4) into (11) and (13), the closed-form NE and expected payoff for the source node $s_i$ can be derived respectively as follows:
by adopting NE strategy. We have

$$\beta'(v_i) = v_i - \int_0^{v_i} \left[ 1 - \exp \left( -x / \lambda \mathcal{F}_{i,i} \right) \right]^{M-1} dx / F(v_i)^{M-1}$$

$$= v_i - \int_0^{v_i} \sum_{j=0}^{M-1} \left[ \exp \left( -x / \lambda \mathcal{F}_{i,i} \right) \right] F(v_j) dx / F(v_i)^{M-1}$$

$$= v_i - \int_0^{v_i} \sum_{j=0}^{M-1} \left[ \exp \left( -x / \lambda \mathcal{F}_{i,i} \right) \right] F(v_j) dx / F(v_i)^{M-1}$$

$$= v_i - \int_0^{v_i} \sum_{j=0}^{M-1} \left[ \exp \left( -x / \lambda \mathcal{F}_{i,i} \right) \right] F(v_j) dx / F(v_i)^{M-1}$$

and

$$m_i = \int_0^{v_i} \left[ 1 - \exp \left( -x / \lambda \mathcal{F}_{i,i} \right) \right]^{M-1} dx / F(v_i)^{M-1}$$

$$= v_i + \lambda \mathcal{F}_{i,i} \sum_{j=0}^{M-1} \left[ \exp \left( -x / \lambda \mathcal{F}_{i,i} \right) \right] F(v_j) / F(v_i)^{M-1}$$

$$= v_i + \lambda \mathcal{F}_{i,i} \sum_{j=0}^{M-1} \left[ \exp \left( -x / \lambda \mathcal{F}_{i,i} \right) \right] F(v_j) / F(v_i)^{M-1}$$

where $C'_{i,i} = (M-1)!/[j!(M-1-j)!]$.}

4 Simulation Results

In this section, computer simulations are carried out to validate the derived analytical expressions. The parameters used in the simulations include $\lambda = 1$ and $\mathcal{F}_{i,i} = 10\text{dB}$.

The NE bidding strategies of $s_i$ versus its valuation are illustrated in Figure 2. It can be seen that the bid $\beta'(v_i)$ is increasing in $v_i$. Thus, the higher valuation a source attaches to the relay for cooperative transmission, the higher bid it will submit. Moreover, it is clear that $\beta'(v_i)$ is affected by the sources’ number $M$. For fixed $v_i$, as $M$ increases, $\beta'(v_i)$ also increases and increasingly approaches $v_i$.

Figure 2. NE bidding strategy versus valuation.

Figure 3 depicts the source node $s_i$’s expected payoff $m_i$ versus its valuation $v_i$ by adopting NE strategy. We can see that all the analytical results are consistent with Monte Carlo simulation results which are obtained by averaging the total payoffs over $10^5$ different instantaneous channel gains. And it is observed that $m_i$ is increasing in $v_i$, i.e., the source node with higher valuation usually gains higher payoff. Furthermore, the effect of the competing source nodes’ number $M$ on the expected payoff is also investigated. It is clear that, for fixed $v_i$, as $M$ increases, $m_i$ decreases and vice versa. Thus, the more source nodes whose information can be decoded successfully in the network, the less expected payoff a source node will achieve. This is consistent with our intuition: in the resource constrained scenario, it is more difficult for each competitor to win as the number of competitors increases, and thus the less expected profit each competitor get.

Conclusion

In this paper, the partner selection problems for distributed cooperative wireless networks have been investigated, we first proposed an auction-based pricing scheme to stimulate cooperation, and then solved the problem of partner selection for DF cooperative relay networks based on NE strategies. The following question has been answered: How should a relay node fairly allocate its resources to one of the multiple competitive source nodes? The issue has been formulated as an auction game which was named as the first-price auction. In the game, the relay node and the source nodes act as auctioneer and bidders, respectively. For i.i.d. Rayleigh fading channels, a closed-form NE of each source node has been derived. Then, based on the strategies, the expected payoffs for each source node have been given. Numerical results and analysis verified the effectiveness of the proposed scheme.
References