Research on Bearing Fault Diagnosis Based on Slice Spectrum and Cepstrum

Long-bo LIU
Institute of Naval Equipment, Beijing 100191, China

Keywords: Slice spectrum, Cepstrum, High-order spectrum, Sideband, Bearing failure.

Abstract. In this paper, high-order spectral analysis and cepstrum analysis are combined to diagnosis of bearing failure. The high-order spectrum can effectively suppress the Gaussian noise and extract the weak fault signal, but it has the limitation when dealing with the added signal. The cepstrum, which less affected by the transmission path, can distinguish the periodic components in the sideband and simplify the clustered sideband into a single spectral line. In this paper, the two methods are combined to conduct the bispectrum analysis of the vibration signal, and then do the Inverse Fourier transform to the logarithm of the bispectrum model, that is, the cepstrum analysis of the third-order cumulant. The method proposed in this paper is verified by the vibration signal data of a bearing, and the results show that the method proposed in this paper can overcome the shortcomings of the traditional single method and make effective diagnosis of the bearing failure.

Introduction

The vibration signal measured by mechanical equipment is usually very complex, mainly the mechanical shaft rotation frequency and multiple harmonic components, mechanical bearing meshing frequency and the corresponding harmonic components and some hidden components and interference noise. When the mechanical equipment shaft or bearing fails, the fault vibration signal of these mechanical parts will change correspondingly with the increasing failure. Usually the sideband component of the corresponding fault characteristic frequency is generated on the spectrum, with accelerating failure, the corresponding amplitude characteristics of the sideband will gradually increase. At the same time, when the mechanical product fails, the resulting fault vibration signal is usually non-linear, which reflects the fault characteristics of the weak nonlinear signal is often submerged in the linear signal. Vibration signal often contains a large number of noise interference signal, and the signal to noise ratio of vibration signal is low, which makes it very difficult to extract the weak fault characteristics of the vibration signal.

Cepstrum analysis can deal well with the sideband. Transform the cluttered sideband on the spectrum into a single spectral line, which contributes to analyze the spectrum. At the same time, the cepstrum analysis is not affected by the path. The signal in the transmission process can form a different transfer function because of the different location of the sensor, which will lead to a different spectrum. After the cepstrum analysis of the signal, the influence of the transfer function can be avoided, so that the fault characteristics on the cepstrum will be almost the same [1,2,3]. But the noise has great influence on the cepstrum. When the amplitude of the frequency is not large or the signal contains a strong noise, the amplitude obtained in the cepstrum is not obvious.

Aiming at the characteristics of weak vibration signal of mechanical equipment, the classical vibration signal analysis method can’t effectively analyze these nonlinear vibration signals. The accuracy of the extraction of early weak fault characteristics is not high and can’t suppress the noise in the signal. But the weak fault characteristics extraction method based on higher order spectrum can solve these problems well. The high-order spectrum is a powerful tool to deal with non-Gaussian signals and identify non-linear system faults. It has high accuracy of recognition of early weak faults of mechanical systems and has a strong ability to suppress noise [4,5,6].

In this paper, high-order spectral analysis and cepstrum analysis are combined to diagnose the bearing failure. Through the high-order spectrum analysis to suppress the Gaussian noise, extract the
weak fault characteristics, and then through the cepstrum analysis of the sideband problem, more accurately find the fault frequency and diagnose the bearing failure. And then verified by the example of the oil pump bearing vibration signal.

**Failure Analysis Method**

**The High-order Spectrum**

In the high-order spectral vibration signal analysis method, bispectrum analysis theory is the most mature and the most widely used, the calculation is small, and facilitate the calculation of big data on the project. This paper mainly studies the bispectrum analysis method in high-order spectral analysis, and the most commonly used 1(1/2)-dimensional spectral analysis method in bispectrum analysis is studied [7,8].

**High-order Cumulant.** For a \( k \)-dimensional stationary random vectors with mean 0 \( X = [x_1, x_2, \ldots, x_k] \), assume its Joint probability density function is \( f(x_1, x_2, \ldots, x_k) \), then the cumulative generation function of this stable random vector can be defined as:

\[
\Psi_X(\omega) = \ln \Phi_X(\omega) = \ln E\left\{ e^{j\omega X} \right\}
\]

Find the \( r = r_1 + r_2 + \cdots + r_k \) -order partial derivative of the variable \( \omega \) for \( \Psi_X \), and make \( \omega_1 = \omega_2 = \cdots = \omega_k = 0 \), then the \( r \)-order cumulant \( c_{r_1 \ldots r_k} \) of \( X \) can be derived:

\[
c_{r_1 \ldots r_k} = \text{cum}\{x_1, x_2, \ldots, x_k\} = (-j)^r \frac{\partial^r \Psi_X(\omega_1, \ldots, \omega_k)}{\partial \omega_1^{r_1} \cdots \partial \omega_k^{r_k}} \bigg|_{\omega_1 = \cdots = \omega_k = 0}
\]

where \( \text{cum}\{\} \) strands for the cumulant, the formula above usually has \( r_1 = r_2 = \cdots = r_k = 1 \) in practical engineering applications.

For a \( k \)-order stochastic steady signal \( x(t) \) with mean 0, assume \( x_1 = x(t) \), \( x_2 = x(t + \tau_1) \), \( \cdots \), \( x_k = x(t + \tau_{k-1}) \), where \( \tau \) is defined as the lag in time. Then the \( k \)-order cumulant of this stochastic steady signal can be defined as:

\[
c_{k\tau_1 \tau_2 \cdots \tau_{k-1}}(x(t), x(t + \tau_1), \cdots, x(t + \tau_{k-1})) = \text{cum}\{x(t), x(t + \tau_1), \cdots, x(t + \tau_{k-1})\}
\]

For the Gaussian signal \( x(t) \), which mean is 0, and variance is \( \sigma^2 \), namely \( x(t) \sim N(0, \sigma^2) \). Thus, its probability density function is obtained as \( p(x) = (1/\sqrt{2\pi\sigma})e^{-(x^2/2\sigma^2)} \). Based on the above algorithm, we can calculate the corresponding cumulative function of the Gaussian signal as \( \Psi(\omega) = (-\sigma^2 \omega^2)/2 \).

Put the resulting cumulant generation function into the above formula, the \( k \)-order cumulant of \( x(t) \) follows:

\[
c_{k\tau} = \begin{cases} 0, & k = 1, 3, 4, 5, \cdots \\ \sigma^2, & k = 2 \end{cases}
\]

For any Gaussian signals with mean of 0, the value of the high-order cumulant generation function is zero. Therefore, the high-order cumulant can suppress the Gaussian noise contained in the vibration signal, and the accuracy of the vibration signal analysis can be improved.
Bispectrum

The third-order cumulant and bispectrum are the two most commonly used statistical methods in advanced statistics. For a signal $x(t)$ with mean of 0, its third-order cumulant is defined as:

$$c_{3x}(\tau_1, \tau_2) = E\{x(t)x(t+\tau_1)x(t+\tau_2)\}$$ (5)

Bispectrum is defined as a third-order cumulant of two-dimensional Fourier transform:

$$B_x(\omega_1, \omega_2) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} c_{3x}(\tau_1, \tau_2)e^{-j(\omega_1\tau_1+\omega_2\tau_2)}d\tau_1d\tau_2$$ (6)

The high-order statistics of the signals are generally multidimensional functions. The third-order cumulant $c_{3x}(\tau_1, \tau_2)$ is a two-dimensional function. The bispectrum $B_x(\omega_1, \omega_2)$ obtained by the two-dimensional Fourier transform is also a two-dimensional function. The computational workload is large, the computational resources are occupied much and the application is inconvenient. In engineering applications, two-dimensional functions are often transformed into one-dimensional functions for analysis. Such as extract a set of data in a row, a column, or a diagonal of a two-dimensional function, which is called Slice.

The 1(1/2)-dimensional spectrum of the signal is defined as the one-dimensional Fourier transform of the third-order cumulant diagonal slice. Then the so-called third-order cumulant diagonal slice is $c_{3x}(\tau_1, \tau_2)$, where $\tau_1 = \tau_2 = \tau$, i.e.:

$$c_{3x}(\tau_1, \tau_2) = c_{3x}(\tau, \tau) = E\{x(t)x(t+\tau)x(t+\tau)\}$$ (7)

And the 1(1/2)-dimensional spectrum of the signal is defined as:

$$S(\omega) = \sum_{\tau=-\infty}^{\infty} c(\tau) \cdot e^{-j\omega \tau}$$ (8)

Cepstrum

The cepstrum is a spectral reanalysis of the spectrum, which is very effective for spectral analysis with homogeneous or alien spectral waves and multi-component sidebands, and has a deconvolution effect that separates and extracts the source signal or transmission system characteristics. There are two types of cepstrum of mathematical description: one is the Real Cepstrum, the other is Complex Cepstrum.

**Real Cepstrum.** Assume the signal unilateral power spectrum is $G_{xx}(q)$, the Real Cepstrum can be expressed as:

$$G_r(q) = |F\{\lg G_{xx}(f)\}|^2$$ (9)

where $F\{\}$ is the Fourier transform symbol. The above equation is the power spectrum cepstrum, where process the Fourier transform to the logarithmic power spectrum and then take the square of its modulus. While in Engineering the form of the square root of the Eq. 9 of is more commonly used. Namely:

$$G_r(q) = |F\{\lg G_{xx}(f)\}|$$ (10)

which is called Amplitude Cepstrum.

**Complex Cepstrum.** The above cepstrum definition has lost the phase information, and the actual work often requires to retain the phase. In order to restore the original signal, the definition of Complex Cepstrum is proposed as:
Assume $X(f)$ is the Fourier transform of signal $x(t)$, i.e.:

$$X(f) = X_r(f) + jX_i(f)$$

Thus, the Complex Cepstrum $C_r(q)$ follows:

$$C_r(q) = F^{-1}\{\log X(f)\}$$

The independent variable $q$ in the cepstrum is called the inverted frequency.

The small value of $q$ is called the low inverted frequency, which indicates the fast fluctuation and dense harmonic on the spectrum. The large value of $q$ is called the high inverted frequency, which indicates the slow fluctuation and sparse harmonic on the spectrum.

**Fault Diagnosis Method Based on High-order Spectrum and Cepstrum**

High-order spectral analysis and cepstrum analysis can be applied to the common fault diagnosis of bearings, but both have their own advantages and disadvantages. In practical engineering, only using one method to diagnose the bearing failure is not very good. The combination of high-order spectral analysis and cepstrum analysis, can diagnose the bearing failure better and find out the fault frequency.

For the limitations of the cepstrum in dealing with the noise signal, the high-order spectrum can make up for its shortcomings. The high-order spectrum can effectively suppress the Gaussian noise and extract the weak fault signal from the noise. In practical engineering, due to the impact of the site environment, the sensor layout will have a certain deviation, the location of the test point is not very good. Due to the different transmission paths, the derived spectrums are not the same when deal with the different sensors with the high-order spectrum analysis. Therefore, the cepstrum analysis method can avoid the interference problem brought by transmission paths. In this paper, the bispectrum of the vibration signal is analyzed firstly, and then the bispectrum is obtained and calculate its modulus. The inverse Fourier transform is subjected to the logarithm of the bispectrum’s modulus at last, that is, the third-order cumulant is subjected to cepstrum analysis as follows:

$$C(q) = F^{-1}\left\{\log \left| \sum_{\tau=-\infty}^{\infty} c(\tau) \cdot e^{-j2\pi ft} \right| \right\}$$

**Case Analysis**

In the following, analyze the signal data obtained by the reliability vibration test of the turbo oil pump for 1000h. Turbine oil pump in the steam pressure of 2.45Mpa, the speed of 10600rpm operating conditions, the vibration is the most intense. The vibration signal under this condition can most reflect the potential weak fault characteristics and the trends of fault characteristics of the oil pump. Vibration signal acquisition is continuous operation during the continuous operation of the oil pump, sampling frequency is 2560Hz, and 10min is a data record section. And according to the inspection data and bearing parameters, obtain the bearing failure frequency is 46Hz. As the demolition needs a large amount of work, cost massive manpower and material, and has long inspection period, generally do not use this method to determine the fault. Therefore, it is more practical to diagnose the fault by analyzing the collected vibration signal.

This paper selects the vibration signal of the bearing on the shaft of the oil pump to analyze. First of all, the vibration signal is subjected to FFT analysis, as shown in Figure 1. From the figure can be seen that the line is chaotic, there are noise and sideband. The fault signal cannot be observed.
Then, do the slice spectrum analysis to the original signal, as shown in Figure 2. The noise is effectively suppressed, the line is simple and clear. But because the corresponding amplitude of 46Hz is too small, it’s almost impossible to be observed in the figure. Only 3-times multiplier of 138Hz and other multipliers can be seen. The frequency of failure can’t be determined intuitively and effectively.

At last, based on the method described in this paper, we can carry out cepstrum analysis on the basis of slice spectrum analysis, and the obtained spectrum is shown in Figure 3. It can be seen that there is a clear line at 46.5Hz (1 / 0.02148) and its double frequency of 91.4Hz (1 / 0.01094), and the fault frequency is accurately determined as 46Hz.
Summary

When the mechanical equipment shaft or bearing fails, the vibration signal spectrum will produce the sideband components of the corresponding fault characteristics. At the same time the resulting fault vibration signal is usually non-linear, and a large number of noise interference signal, as well as low signal to noise ratio.

High-order spectral analysis can effectively suppress the Gaussian noise, cepstrum analysis can avoid the interference problem of transmission path. With combining the advantages of these two methods in this paper, the bearing failure frequency can be accurately determined.

References


