Development of a Constrained Automated Geometry Reasoning System

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ABSTRACT

Dynamic geometry technology has been extensively used in geometry education. To aid the teaching and learning of geometry reasoning, many efficient automated geometry reasoning algorithms have been integrated into the dynamic geometry system. However, the reasoning functionalities in most current dynamic geometry system cannot fully meet the requirement of the geometry reasoning education. To tap the educational potential of automated geometry reasoning to the maximum, we proposed the concept and implementation mechanism of constrained automated geometry reasoning in this paper, and developed a constrained automated geometry reasoning system with a convenient dynamic geometry interface. Experimental results show that the proposed system could successfully solve 107 out of 132 geometry proof problems, which means it can meet the basic requirement of the geometry education.

INTRODUCTION

Geometry holds an important role in mathematical education, primarily through its focus on deductive reasoning [1,2]. To aid the teaching and learning of geometric proof, many new technologies have been used in geometry education, among which,
dynamic geometry software (DGS) is a paradigm [3-5]. DGS helps students to progress in their understanding of the dependence relationships among components of a figure and amongst families of figures, and so advance towards a progressive abstraction in their justifications [6, 7]. In recent years, with the development of automated geometry reasoning (AGR), the AGR functionalities have been incorporated into some DGS to assist the learning and teaching of geometry reasoning [8].

However, DGS with automated reasoning ability are not widely used in the learning and teaching of geometry reasoning in secondary school. The major reason is that the proofs produced by those systems are not human-readable [9]. In this paper, a so-called constrained automated geometry reasoning (CAGR) is proposed and realized based on the synthetic deduction approach, which is combined with an easy-to-use dynamic geometry interface. The meaning of “constrained” is that this kind of AGR is confined to some extent so that it can meet the practical requirement of learning and teaching of geometric reasoning in secondary school, which mainly manifests in the following aspects: (1) the proofs produced by CAGR are displayed in traditional readable style, so that it is acceptable for students; (2) the definitions, the axioms and the geometric rules used in CAGR are within the scope of geometry textbooks, i.e., the axiomatic geometry system taught in school, so that the proofs are comprehensible for the students; (3) the proof are short enough so that is would not dampen the learning interest of students.

DYNAMIC GEOMETRY INTERFACE FOR CAGR

We designed and implemented a dynamic geometry system for CAGR. As Fig. 1 shows, the interface mainly consists of three regions, i.e., drawing region, operating region and information region.

![Figure 1. The interface of CAGR.](image-url)
Our DGS possess the common dynamic geometry functions, such as drawing geometric shapes and function curves, animation, transformation, trace, locus, and measure. For example, a demonstration of locus is shown in Fig.1.

**IMPLEMENTATION OF GAGR**

As mentioned above, to be suitable for the geometric education, a CAGR system should be based on the synthetic deduction approach, which mainly involves with the forward reasoning and backward reasoning [10].

The schematic diagram of forward chaining based CAGR is demonstrated in Fig.2, which could be divided into two stages. At the first stage, the geometric information is generated from the geometric shapes as the initial facts, which are inputted to the second stage as the information list for further reasoning. At the second stage, the reasoning engine applies the inference rules to the initial facts to find new data recursively until a goal is reached or fix-point is reached if no goal is given (i.e., nothing more can be inferred from current information lists) [11].

For a reasoning engine, the predicates and the inference rules are the basic elements, which are also constrained to some extent in our program, embodying the “constrained feature” of the CAGR.

![Figure 2. General framework of forward-chain based AGR.](image)

**Constrained Predicates and Inference Rules**

According to the geometry predicates, the facts are sorted as a serial of information lists, such as the list of collinear points, list of midpoints, list of equal segments, list of congruent triangles, list of parallel segments, list of parallelogram, etc.

An inference rule is a syntactic rule that takes premises and returns a conclusion, which take the form of "If p then q", in the sense that if the premise p is true then so is the conclusion q. For example, a theorem about the median line of a triangle can be
described as: if $\text{midp} (D, A, B)$ and $\text{midp} (E, A, C)$, then $\text{para} (D, E, B, C)$. The more formal expression of it is as follows:

$$\text{para} (D, E, B, C): - \text{midp} (D, A, B), \text{midp} (E, A, C)$$

Based on the lemmas, axioms and theorems in the geometry textbooks, the constrained inference rule set can be easily obtained.

**Readable Proof**

When the reasoning engine is applied to a geometric shape, all the facts about which could be obtained, otherwise the deduction process would be terminated once the goal is found. After the deduction process is over, all the deduced facts, including the initial facts, will be shown in the tree structure in the information region of the system.

Fig. 3 gives an example, where quadrilateral ABCD is a parallelogram, segment AE is perpendicular to segment AD at the foot D, and segment CF is perpendicular to segment AC at the foot F. As no goal is given, all the facts about the geometric shape have been obtained, which are classified into a set of lists according to the predicate of the facts. As shown in the left part of Fig. 3, the information tree contains 12 nodes, corresponding to 12 lists of facts. By clicking each node, the user can expand and collapse the associated list of facts.

![Figure 3. Hierarchical structure about reasoning results.](image)

In fact, the deduction route for each fact has been obtained during the deduction process, and they have been included in the tree structure as a part of the information. For example, after clicking the node 6 (i.e., parallelogram node) in the tree of Fig. 3, all the parallelograms in the list, i.e., parallelogram ABCD and AECF are shown up as its sub nodes, based on which the fact could be deduced.

**RESULTS**

By integrating the reasoning engine into the DGS, a CAGR system is proposed. To evaluate the reasoning ability of the proposed system, some tests are performed.
We collected 132 geometry proof exercises from the geometry textbook and supplementary materials of the secondary school, covering various knowledge points and different level of difficulty.

Test results shows that the CAGR could successfully prove 107 of them, and the maximum execution time is not more than ten seconds. The remaining 25 exercises are rather difficult problem, mainly related to the knowledge points of similarity and circle, and most of them could not be solved without adding auxiliary lines. In addition, for the exercises that CAGR failed to prove, lots of geometric facts still have been reasoned out. Moreover, as it is based on the forward chaining approach, for these difficult problems that the CAGR could not prove out, construction method by adding auxiliary point or line can be easily applied based on the CAGR system, which is also an important knowledge point in geometry proving teaching.

CONCLUSION

As a key part of geometry education, geometry proving is one of the most complex activities and one which secondary students experience the most difficulty with. To counter this, AGR is incorporated into some DGS to assist the learning and teaching of reasoning ability in geometry. Teaching and learning of geometry proving needs human-readable and traditional proofs, which means that the proofs can be understood by the students, each step has clear geometric meanings, and the proof are short enough for students to repeat with pencil and paper, which is so called CAGR in this paper.

A CAGR system is designed and realized in this paper, with a user-friendly DGS interface. To make the CAGR system more adapted to the requirement of teaching and learning of reasoning ability, a tool of automatic generation of geometry reasoning exercise and checking the students’ answer is incorporated to the CAGR system. Several types of test are performed to evaluate the reasoning ability and the efficiency of the exercise-generation of the resulted CAGR system. Out of 132 exercises, CAGR could successfully solve 107 of them, which could meet the requirement of the geometry reasoning education.

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