Research on the Location of Mobile Robot Based on an Improved Monte Carlo Algorithm

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Abstract. The particle filter algorithm solves the problem that the stochastic quantity must satisfy the Gaussian distribution for the non-Gaussian filtering. In recent years, it has been widely used in research of target tracking and positioning. The problem of particle scarcity and the choice of the proposal distribution function existed in the robot localization method. In allusion to the two problems, a method of robot localization based on particle filter of Markov Chain Monte Carlo (MCMC) is proposed. By calculating the probability of filtering particles and adaptively adjusting the boundary of the proposal distribution function, it not only maintains the diversity of particles, but also inhibits particle scarcity. The algorithm and the common Sampling Importance Resampling (SIR) algorithm are applied to the simulation experiment of target location. The simulation results showed the good performance of the improved particle filter algorithm, and the phenomenon of article degeneracy can be effectively curbed. Moreover, the positioning accuracy is higher, and the robustness is better than SIR algorithm.

Introduction

The localization problem is one of the three major problems on mobile robot research[1-2]. It is to determine the accurate position of the robot in unknown environment, and it is a prerequisite for the robot to complete a variety of tasks. It belongs to the problem of optimal state estimation of nonlinear stochastic dynamic systems[3]. Aiming at the nonlinear and non-gaussian, the experts proposed particle filter algorithm[4] in the research on mobile robot localization In recent years which effectively solves the problem that the random quantity must satisfy the gaussian distribution when the nonlinear system is filtering. It has been widely used because of its excellent characteristics. This kind of particle filter localization method is also called Monte Carlo Localization algorithm(MCL). The particle filter algorithm is to use a set of discrete weighted particle sets and approximates the posterior probability density function. It avoids a large number of integral computing in the bayesian recursive formula[5]. Due to the stochastic nature of the estimate in the algorithm, the problem of sampling degradation and poverty is common to this method. In order to solve the degradation, the researchers have proposed a kind of Sampling Importance Resampling (SIR) algorithm. However, the algorithm only replicates the high-weight particles and ignores the low-weight particles, so there will be an extreme phenomenon. That is, the concentration of particles in a particular particle and its replication particles, leading to serious problems of particle deficiency[6]. In this paper, an improved MCL algorithm is used to realize the localization of mobile robots. By setting a threshold in the algorithm, whether resampling is necessary to be judged. Resampling is to be done only when the degradation is exist. In this way, the number of resampling can be reduced, and the efficiency of the algorithm can be improved. At the same time, an extended resampling Monte Carlo algorithm is applied in the paper. It is an improved MCMC (Markov Chain Monte Carlo) method which can adaptively adjust the boundary of the proposed distribution function. It uses a uniform distribution as a function of probability density so that its distribution boundary can be effectively adjusted according to the actual situation.
of particle dispersion. The number of effective particles in sampling can be improved by this method, and the problem of particle shortage is solved effectively.

The Location Principle of Monte Carlo Algorithm

Monte Carlo localization algorithm[7] is proposed as a deformation algorithm based on the Bayesian probability algorithm. The principle is to simulate a posterior probability density function of the estimated state through a set of sample spaces with weighted particles. The observed value \( y_k \) is obtained by each sensor to estimate the probability density \( p(x_k | y_{1:k}) \) of the state value \( x_k \).

Solving \( x_k \) is generally recursive using formula (1), the formula is as follows:

\[
p(x_k | y_{1:k}) = \frac{p(y_k | x_k)p(x_k | y_{1:k-1})}{p(y_k | y_{1:k-1})}
\]

(1)

However, the solution to the nonlinear system does not exist or is not easy to solve, so the probability density \( p(x_k | y_{1:k}) \) of the state estimator \( x_k \) is approximated by finding a set of weighted samples \( S_k = \{x'_i, w'_i\}_{i=1,2,...,N} \), the formula is as follows:

\[
p(x_k | y_{1:k}) \approx \sum_{i=1}^{N} w'_i \delta(x_k - x'_i)
\]

(2)

In the formula (2), \( N \) is the total number of particles; \( w'_i \) is the weight (indicating the importance of the sample \( x'_i \)), its value is non-negative and the sum of the weights is equal to one. Since \( x_k \) is only related to \( x_{k-1} \) and \( y_k \), its weight is determined by significance principle.

\[
w'_k \propto w'_{k-1} \frac{p(y_k | x'_k)p(x'_k | x'_{k-1})}{q(x'_k | x'_{k-1}, y_k)}
\]

(3)

In the formula (3), \( q(x'_k | x'_{k-1}, y_k) \) is the particle proposal distribution function.

The conventional MCL algorithm is described as follows:

a) Forecast update: No normalized samples \( \bar{x}' \) are sampled from the motion probability model \( p(x_k | x_{k-1}) \) and the weights are evenly distributed to each particle. That is, \( \bar{w}'_k = 1/N \) is assigned to sample \( S'_k = \{(\bar{x}'_i, \bar{w}'_i) | i = 1,...,N\} \).

b) Perception update: the observation model is applied to update each particle’s weight

\[
\hat{w}'_k = \bar{w}'_k \cdot p(y_k | \bar{x}'_k), \quad \hat{w}'_k = \hat{w}'_k / \sum_{j=1}^{N} \hat{w}'_j
\]

and the weight is normalized. Finally, a new sample set \( S''_k = \{\tilde{x}'_i, \tilde{w}'_i\} | i = 1,...,N\} \) is obtained.

c) Sampling Importance Resampling: By the Sampling Importance Resampling, the particles with little effect on location and small weight will be eliminated in the sample set. At the same time, the high-weight particles are replicated and resampled. However, when the observation error occurs, it will reflect the real position and weight of the smaller particles eliminated and produce sampling degradation, reducing the location accuracy.

The Extend Monte Carlo Localization Algorithm

The mobile robot wants to know its own position, so it is necessary to carry out repeated iterations and updates. However, after a number of iterations, there will be sampling degradation problems. It
makes the weight of most particles smaller and smaller, only the very large weight of individual particles. This results in a large amount of computational resources being wasted on those particles which contribute less to the posterior distribution. Therefore, in order to solve this problem, a threshold $N_{\text{thres}}$ is set to $3\times N/4$. When the value of $N_{\text{eff}} = 1/\sum_{i=1}^{N} \left( \hat{w}_i \right)^2$ is smaller than the threshold $N_{\text{thres}}$, resampling is performed. By judging the degradation of the sample, it is possible to perform resampling when the sample is degraded.

In this way, it reduces the number of resampling and improves the performance of the algorithm. In the process of resampling, there is still a problem, which is particle deprivation. This phenomenon exist due to the fact that the ordinary resampling only replicates the particles with larger weights and discards the particles with smaller weights. It reduces the posterior probability density of the particles in the sample space. The traditional Markov Chain monte Carlo (MCMC) algorithm is adopted by the resampling method of Metropolis Hastings. it solves the problem of particle scarcity to a certain extent, but the sampling range of its specific proposed distribution $q(x_i^* | \tilde{x}_i^j)$ is fixed and cannot be appropriately adjusted according to the specific situation of its own sampling. Even if it affects the number of sampled effective particles. In this paper, the dispersion of the current particles can be adjusted. When the number of effective particles is large, the dispersing range of the particles should be narrowed and dispersed in a small range near the real value; When the effective number of particles is small, the dispersion range of the particles should be expanded. So that it has more chance to spread around the real value. In the algorithm, we search the nearest particle $\tilde{x}_i^j$ for particle $\tilde{x}_i^k$ and the proposed distribution $q(x_i^* | \tilde{x}_i^j)$ becomes a uniform distribution whose boundary size is proportional to the distance between $\tilde{x}_i^j$ and $\tilde{x}_i^k$. The range of sampling can be adjusted in a certain range, and make the number of sampled effective particles be improved. The distribution range of particles is adjusted in different degree of convergence, and particles with good convergence more concentrated around the true value. But the particles with poor convergence become more dispersed. By such a method can better prevent the occurrence of scarcity phenomenon. It can better prevent emergence of particle deprivation. The Extended Monte Carlo resampling method is as follows:

At k-1 moment, the weight of the particle set is $S_{k-1} = \{ (x_{k-1}^{i}, w_{k-1}^{i}) | i = 1, \ldots, N \}$.

a) Forecast update: the $\tilde{x}_i^j$ is sampled from the movement probability model $P(x_i^k | x_{k-1}^{i})$. And the weights of $\tilde{w}_i^j = 1/N$ are evenly distributed to each particle. Getting a particle set $S_t = \{ (\tilde{x}_i^j, \tilde{w}_i^j) | i = 1, \ldots, N \}$.

b) Perception update: Using the observation model to update each particle weight $\tilde{w}_i^j = \tilde{w}_i^j \cdot p(y_k | \tilde{x}_i^j)$, and the weight $\tilde{w}_i^j = \tilde{w}_i^j / \sum_{j=1}^{N} \tilde{w}_i^j$ is normalized. Finally, a new sample set $S_t = \{ (\tilde{x}_i^j, \tilde{w}_i^j) | i = 1, \ldots, N \}$ is obtained ( $\tilde{x}_i^j$ is equal to $\tilde{x}_i^j$).

c) Judgment: $N_{\text{eff}}$ represents the number of particles that are efficient for state estimation in a particle set, and can be obtained from the formula below.

$$N_{\text{eff}} = 1/\sum_{i=1}^{N} \left( \tilde{w}_i \right)^2$$

(4)

If the $N_{\text{eff}}$ is less than the threshold $N_{\text{thres}}$, it is necessary to resample the particle set; otherwise, the $x_i^k$ is equal to $\tilde{x}_i^j$ and the particle set $S_t = \{ (\tilde{x}_i^j, \tilde{w}_i^j) | i = 1, \ldots, N \}$ is output.
d) Improved MCMC algorithm: Search the nearest particle \( \tilde{x}_k^i \) for particle \( x_k^i \) and calculate the proposal distribution function. The formula is as follows:

\[
q(\tilde{x}_k^i | x_k^i) = U\left( -\frac{|\tilde{x}_k^i - \tilde{x}_k^{i'}|}{2}, \frac{|\tilde{x}_k^i - \tilde{x}_k^{i'}|}{2} \right)
\]  

(5)

In the formula (5), \( \tilde{x}_k^i \) is sampled from the proposal distribution. Finally, the acceptance ratio is calculated by the following formula.

\[
\alpha(\tilde{x}_k^i, x_k^i) = \frac{p(\tilde{x}_k^{i'} | y_k)}{p(x_k^i | y_k)}
\]  

(6)

If \( \alpha \geq 1 \), the \( x_k^i \) is equal to \( \tilde{x}_k^i \); otherwise, the \( x_k^i \) is equal to \( \tilde{x}_k^{i'} \). At the same time, the weights of \( w_k^i = 1/N \) are evenly distributed to each particle and a new particle set \( S_k = \{(x_k^i, w_k^i) | i = 1, ..., N\} \) is obtained.

**Simulation Results and Analysis**

A order to verify the effectiveness of the algorithm, the algorithm is compared with the SIR algorithm through simulation experiments and focus on the tracking effect of two samples, the posterior density, and standard deviation. The simulation platform is the computer with CPU frequency of 1.86GHz, the memory of 4G and simulation development tools is Matlab 7.1. The following scalar system was used for analysis.

State equation of the system:

\[
X(k) = 0.5X(k-1) + \frac{2.5X(k-1)}{1+X^2(k-1)} + 8\cos(2k) + W(k)
\]  

(7)

Observation equation:

\[
Y(k) = \frac{X^2(k)}{20} + V(k)
\]  

(8)

In the formula (7), \( W(k) \) is the process noise, which is a White Gaussian Noise with zero mean and 10 variance; In the formula (8), \( V(k) \) is the process noise, which is a White Gaussian Noise with zero mean and unit variance. White Gaussian Noise. According to the formula above, this paper conducted 50 experiments for 100 particles with two algorithms respectively. The diagram of Simulation results are shown below.

Figure 1. Simulation and Tracking Effect of SIR Algorithm.
Figure 2. Simulation and tracking effect of extended Monte Carlo Algorithm.

Figure 1 is the tracking effect diagram of SIR algorithm and Figure 2 is the tracking effect diagram of extended Monte Carlo algorithm. Through the comparison of the two diagrams, it can be seen that the extended Monte Carlo algorithm estimates the position closer to the actual position than the SIR algorithm. It can be explained that the algorithm used in this paper has better positioning effect and higher precision.

Figure 3. The distribution of the posterior density of the SIR Algorithm.

Figure 4. The distribution of the posterior density of the Expanded Monte Carlo Algorithm.

Figure 3 shows that the distribution of the posterior density of the particles sampled using the SIR algorithm. Figure 4 shows that the distribution of the posterior density of the particles sampled using the extended Monte Carlo algorithm. In the above two diagrams we can see the extended Monte Carlo algorithm has greatly improved the posterior density in the sample space after sampling, and the number of effective sampling particles has also increased obviously. The ability to adjust the dispersion of particles has been effectively proved, and the problem of particle scarcity has been effectively solved.
Conclusion

The algorithm determines whether the sampling is degraded by setting a threshold value. It can reduce the number of resampling and improve the performance of the algorithm. At the same time, an improved MCMC method which can adaptively adjust the boundary of the proposed distribution function is presented in this paper. By comparing the simulation with important resampling algorithm, the algorithm can adjust the boundary according to the actual situation, improve the number of effective particles, and solve the problem of particle shortage caused by ordinary important sampling. It proved that the localization algorithm of extended Monte Carlo has higher positioning accuracy and better robustness.

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