Subspace-based Localization Approach for Near-field Source in Impulse Noise Environment

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Abstract. In this paper, we propose a subspace-based matching (SBM) approach based on fractional lower order moment (FLOM) matrix to estimate the location of near-field acoustic signals in additive nonGaussian impulsive noise environment. By matching the subspace in the fingerprint database and the subspace computed from the current received signals, we associate the received signal with its transmission location with high accuracy. The proposed algorithm has two advantages: firstly, it can locate multiple sources without estimating bearing and range of sources. Secondly, it is more robust to multipath, environment changing and noise level. The simulation results show the efficacy of our proposed algorithm.

Introduction

Localization of passive source would enable a variety of diverse applications in radar, sonar, and navigation fields. The localization of near-field source is a hot topic in the last few decades [2, 7]. Various techniques applied for near field signal localization have been proposed, including linear prediction (LP) method [1], one-dimensional (1-D) MUSIC method [9], high-order ESPRIT method [8] etc. However, the above methods fail to estimate the location of near-field signal where the additive noise is impulsive.

In this paper, subspace-based matching (SBM) method is introduced using Fractional Lower-Order Moments (FLOM) of the array outputs. Hypothetically, direction-of-arrival (DOA) and range are estimated with a uniform linear array (ULA). Localization consists of an off-line phase and an on-line phase. In the off-line phase, some training points placed in the target area send signals, measurement is conducted to construct a database of subspace for these nodes. In the on-line phase the subspace of received signal is matched with the subspace in the database, which is constructed in the off-line phase, to estimate the signal source location. Furthermore, second-order statistics (SOS) based SBM and fourth-order cumulants (FOC) based SBM are added to compare with the FLOM based SBM.

Near-Field Signal Model in Impulsive Noise Environment

Assume that \( M \) near-field uncorrelated narrow band signals are observed by a ULA of \( 2N+1 \)-element with inter-element spacing \( d \). Let the array center be the phase reference point. Then the received signal at the \( n \)th sensor can be modeled as

\[
x_n(t) = \sum_{l=1}^{M} e^{j\tau_l} s_l(t) + w_n(t), -N \leq n \leq N
\]  

(1)

where \( s_l(t) \) denotes the \( l \)-th source signal, \( w_n(t) \) is a sequence of i.i.d isotropic complex Symmetric Alpha Stable (S\( \alpha \)S) random variables with \( 1 < \alpha \leq 2 \) [5], and \( \tau_l \) is the phase shift associated with
the propagation time delay between sensor 0 and sensor \( n \) of the \( l \)-th source signal, which can be regarded as a function of source signal parameters, including range \( r_l \), angle \( \theta_l \) and wavelength \( \lambda \), given by

\[
\tau_{nl} = \frac{2\pi}{\lambda} \left( \sqrt{(r_l^2 + (nd)^2) - 2r_l d \sin \theta_l} - r_l \right) \tag{2}
\]

As the source \( l \) is in the Fresnel region, which means that range \( r_l \) satisfies the condition \( 0.62 \left( \frac{D^2}{\lambda} \right)^{1/2} < r_l < \left( \frac{2D^2}{\lambda} \right) \), with \( D \) denoting the aperture of the array [4], \( \tau_{nl} \) can be approximated by using the second-order Taylor expansion [1]

\[
\tau_{nl} = \left( \frac{-2\pi d}{\lambda} \sin \theta_l \right)m + \left( \frac{-2\pi d^2}{\lambda r_l} \cos^2 \theta_l \right)m^2 + O\left( \frac{d^2}{r_l^2} \right) \tag{3}
\]

where \( O\left( \frac{d^2}{r_l^2} \right) \) represents terms of order greater than or equal to \( \frac{d^2}{r_l^2} \). The second-order Taylor series approximation is found in many references on near field source localization with ULA [4]. Using this approximation, equation (1) can be replaced by

\[
x_n(t) = \sum_{i=1}^{M} e^{\left( \frac{2\pi d}{\lambda} \sin \theta_l \right)x_i t} e^{\left( \frac{\pi d^2}{\lambda r_l} \cos^2 \theta_l \right)x_i^2} s_i(t) + w_n(t) \tag{4}
\]

The received signal vector \( \mathbf{x}(t) = [x_{-N}(t), \cdots, x_N(t)]^T \), with the superscript \([\cdot]^T\) denoting matrix transposition, can be written in

\[
\mathbf{x}(t) = \mathbf{A} \mathbf{s}(t) + \mathbf{w}(t), \tag{5}
\]

where \( \mathbf{s}(t) = [s_1(t), \cdots, s_M(t)]^T \) is the signal vector, \( \mathbf{w}(t) = [w_{-N}(t), \cdots, w_N(t)]^T \) is the noise vector, and \( \mathbf{A} \) is the array manifold matrix given by \( \mathbf{A} = \left[ \mathbf{a}(r_1, \theta_1), \cdots, \mathbf{a}(r_M, \theta_M) \right] \) with the steering vector \( \mathbf{a}(r_l, \theta_l) \) being expressed as

\[
\mathbf{a}(r_l, \theta_l) = \left[ e^{\left( \frac{2\pi d}{\lambda} \sin \theta_l \right)x_{-N}}, \cdots, e^{\left( \frac{\pi d^2}{\lambda r_l} \cos^2 \theta_l \right)x_{-N}^2}, \cdots, e^{\left( \frac{\pi d^2}{\lambda r_l} \cos^2 \theta_l \right)x_N^2} \right]^T. \tag{6}
\]

**SBM Approach Based on FLOM**

We present the main idea of our proposed SBM approach based on FLOM. FLOM is an efficient statistic for dealing with impulse noise. We define the FLOM matrix as follow

Definition1[5]: \( \mathbf{C} \) is a FLOM matrix, whose \((i, k)\)-th entry is defined as

\[
C_{ik} = E\left[ x_i(t) x_k(t) \right]^{p-2} x_i^*(t), 1 < p < \alpha \leq 2 \tag{7}
\]

where \( x_i(t) \) and \( x_k(t) \), \( \forall i \) and \( k \) \( (1 \leq i, k \leq L) \), are given by (1), and \( C_{ik} \) is bounded [6]. The matrix form \( \mathbf{C} \) can be written as

\[
\mathbf{C}_{\text{FLOM}} = \mathbf{A} \mathbf{A}^H + \gamma \mathbf{I} \tag{8}
\]

where \( \mathbf{A} \) is given by (6), \( \mathbf{I} \) denotes identity matrix, \( \mathbf{A} \) and \( \gamma \) are statistics related to signal and
noise [6]. For the \((m,n)\)-th entry is

\[
\Lambda_{mn} = \delta_{mn} E \left\{ s_m(t) \sum_{i=1}^{M} s_i(t) + w_m(t) \left[ \sum_{i=1}^{M} s_i(t) + w_m(t) \right]^* \right\}
\]

Corresponding to the \(k\)-th row of \(C\), \(\gamma\) is expressed as

\[
\gamma = E \left\{ w_k(t) \sum_{i=1}^{M} s_i(t) + w_k(t) \left[ \sum_{i=1}^{M} s_i(t) + w_k(t) \right]^* \right\}
\]

Let \(T\) be the number of snapshots, then the sample Spatial Sign Covariance Matrix is [9]

\[
\hat{C}_{SCM} = \frac{1}{T} \sum_{t=1}^{T} S(x_i(t))S^H(x_j(t))
\]

where \(x_i(t)\) is the \(i\)-th snapshot of the received signal vector. It has been shown in [6, 9] that \(C_{FLOM}\) and \(C_{SCM}\) have a similar subspace structure which enables the use of subspace methods. Obviously, calculating the SCM matrix, instead of FLOM matrix, is much simpler and can obtain the subspace expected.

**Subspace-Based Matching (SBM) Technique**

Subspace-based matching (SBM) technique is adopted in several studies. And this technique determine the distance between two subspaces using some definition as follows:

**Definition 2** [6]: Let \(S_1\) and \(S_2\) be the subspace in \(L\)-dimensional complex vector space with the same dimension \(q\) (Note that \(L\) denotes the number of array elements, in this paper, \(L = 2N + 1\), same as below). The angles \(\alpha_1, \cdots, \alpha_q \in [0, \pi/2]\) between \(S_1\) and \(S_2\) are defined recursively as follows. For \(\|u\| = \|v\| = 1\), we define

\[
\cos \alpha_i = \min_{u \in S_1, v \in S_2} u^\dagger v
\]

where \((\cdot)^\dagger\) denotes the conjugate transpose. \(\alpha_i\) represents the largest deviation of \(S_1\) from \(S_2\) and is called the principal angle between \(S_1\) and \(S_2\), and \(\sin \alpha_i\) is a distance between the two subspaces [6]. Therefore, we use the sine of principal angle between two signal subspace \(S_1\) and \(S_2\) to judge the distance between \(S_1\) and \(S_2\), the smaller the values is, the closer the distance is, and vice versa.

**SBM Algorithm Based on FLOM**

With the near-field signal model in impulsive noise environment. In the offline phase, through Eigen-decomposition of the FLOM matrix of each grid, we can acquire and store the signal subspace into fingerprint database, which is spanned by the eigenvectors corresponding to large eigenvalues. In the online phase, the same steps like above are adopted, hence a signal subspace of FLOM in the on-line phase can be obtained. SBM technique can be used to determine the distance of two signal subspace.

The steps below are the process of SBM approach based on FLOM. Assume that there are \(Q\) training grids in the target area in the off-line phase, we calculate SCM matrix instead of FLOM matrix.
One must determine the dimension of signal subspace of the sample spatial sign covariance matrix, $L_q$ and $L_l$ in both off-line phase and on-line phase. Here, we present a novel method which is not sensitive to noise level to determine the number of signal subspace of the sample spatial sign covariance matrix:

Assume that $[\lambda_1, \cdots, \lambda_L] = \text{diag}(\Xi)$ are the eigenvalues of SCM. And we define the ratio of eigenvalue $\kappa$ as $\kappa_i = \frac{\lambda_i}{\lambda_{i+1}}, i=1,2,\ldots,L-1$. The dimension of signal subspace can be given by choosing the index of the largest value of $\kappa$.

To compare the performance of our proposed SBM approach based on FLOM, second-order (SOS) statistic matrix [1] and fourth-order cumulants (FOC) matrix [8] can be applied to the above procedures, respectively. We call the SBM using FLOM matrix as SBM-FLOM, SBM-SOS and SBM-FOC are similarly defined.

**Simulation results**

A ULA with $L=11$ and $d=0.3\lambda$ is employed to receive the near-field signals. 100 training points are evenly distributed in the grid of the target area whose size is $10m \times 10m$. The computer randomly generates 100 test points, which are in the Fresnel region of the array ($3.22\lambda < r < 18\lambda$). The complex isotropic noise is used as the additive noise which is characterized by the characteristics exponent $\alpha=1.3$, and the dispersion $\gamma=1$. To compare the performance of our proposed SBM-FLOM algorithm, the performance of SBM-SOS and SBM-FOC are shown in the same condition. All the simulation results are performed by 100 Monte Carlo trials at different signal-to-noise ratios (SNRs) (from 0 to 30dB) and with the same snapshots (100). Figs. 1 and 2 display the average deviation of the estimated bearing and range with the change of SNRs, respectively. From the two figures, we find that our proposed SBM-FLOM algorithm has better performance in both bearing estimation and range estimation with different noise levels. Based on the results of bearing and range estimates, Fig. 3 illustrates the average deviation of distance between the estimated points and test points with the same change of SNRs. As can be seen from Fig. 3, the localization accuracy of SBM-FLOM outperforms that of SBM-SOC and SBM-FOC methods whenever the noise level is high or low.

![Figure 1. Average deviation versus SNR, bearing estimation.](image1)

![Figure 2. Average deviation versus SNR, range estimation.](image2)
To further compare the performance of our proposed algorithm to the other algorithms, we also show the localization error of cumulative distribution function (CDF) of these algorithms under different SNRs. Figs. 4, 5, and 6 demonstrate the CDFs of localization error when SNR is 5dB, 15dB, and 25dB, respectively. From the three figures, one can find that SBM-FLOM is the most reliable of all, with a high accuracy.

**Conclusion**

With regard to the DOA estimation of near-field signals in impulsive noise environment, this paper introduces the SBM method, which is based on the subspace spanned by the eigenvectors corresponding to the large eigenvalues through the Eigen-decomposition of three different matrix computed by the received signals. The simulation results show that the SBM-FLOM, yielding a better performance compared to the other two methods, can be considered an efficient method.

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**References**


