A MILP Approach to Accommodate More Wind Generation in Distribution Networks

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Abstract. Distribution networks with renewable energy encounter two challenges. One is that during the peak load, generation is normally low or zero and it will cause the voltage drop. Meanwhile, during peak generation period, the generated power will exceed the load and be injected to the grid, which will cause the voltage rise. This paper proposes a MILP (Mixed Integer Linear Programming) approach to accommodate more wind generation in distribution networks. Optimal system contains battery energy storage system (BESS), wind power, external grid, critical load and controllable load. By coordinating the states of controllable load, battery and wind power direction, the objective to minimize the total cost from grid can be reached. Detailed case study is given to demonstrate the feasibility of this approach.

Nomenclature

\begin{align*}
C_{\text{net}}(t) & \quad \text{Net power cost at time } t; \\
C_{\text{buy}}(t) & \quad \text{Electricity purchase price at time } t; \\
C_{\text{sell}}(t) & \quad \text{Electricity selling price at time } t; \\
P_{\text{g\_sell}}(t) & \quad \text{Power supplied to the grid at time } t; \\
P_{\text{g\_buy}}(t) & \quad \text{Power got from grid at time } t; \\
P_{\text{load}}(t) & \quad \text{Uncontrollable load at time } t; \\
P_{\text{ac}} & \quad \text{Rated power of air conditioner}; \\
P_b(t) & \quad \text{Power flow in battery at time } t; \\
P_w(t) & \quad \text{Forecasted wind power generation at time } t; \\
P_{\text{ret}}(t) & \quad \text{Power being returned to grid from battery at time } t; \\
P_{\text{lev}}(t) & \quad \text{Battery charge level at time } t; \\
P_{\text{g\_max}} & \quad \text{Maximum power capacity of grid}; \\
\rho & \quad \text{Outdoor air density}; \\
R & \quad \text{Blade radius of wind turbine}; \\
\end{align*}
$v_w$ Wind speed;
$v_c$ Cut-in wind speed;
$v_r$ Rated wind speed;
$v_f$ Cut-off wind speed;
$c_p$ Efficiency coefficient of wind turbine;
$P_{\text{rated}}$ Rated power of wind turbine;
$T_{\text{air}}^{\text{min}}$ Indoor air lowest permissible temperature;
$T_{\text{air}}^{\text{max}}$ Indoor air highest permissible temperature;
$T_{\text{w}}^{\text{min}}$ Wall lowest permissible temperature;
$T_{\text{w}}^{\text{max}}$ Wall highest permissible temperature;
$T_{\text{amb}}$ Ambient temperature;
$C_{\text{p_a}}, C_{\text{p_w}}$ Heat capacity of the air and the wall;
$M_{\text{a}}, M_{\text{w}}$ Mass of air inside the house and wall inside the house;
$Q_{\text{ac}}$ Cooling energy delivered by air conditioner;
$Q_{\text{ex-w-a}}$ Heat exchange between the wall and indoor air;
$Q_{\text{gain-w}}$ Heat gain by the wall from the ambient;
$Q_{\text{gain-a}}$ Heat gain by the indoor air from the ambient;
$R_{eq}$ Equivalent thermal resistance of the house envelope;
$R_{w-a}$ Thermal resistance between the wall outer surface and the ambient;
$R_{w-i}$ Thermal resistance between the wall inner surface and the indoor air;
$T_{w}, T_{r}$ Wall temperature and indoor air temperature of the smart house

**Introduction**

In order to solve problems of global warming and depletion of energy resources, the world has been turning to alternatives of fossil fuels. Wind energy is one of the most rapidly growing renewable power source and wind power penetrations have been increasingly demanded [1]. The most significant characteristic of power generated from wind turbine is fluctuation due to variation of wind speed. The increasing penetration of wind power would impose direct impacts on network power quality, protection settings, and etc. Therefore, in order to avoid violating system constraints, some amount of wind power may need to be curtailed when the load is low but the wind is high [2].

To address this issue and improve more wind power penetration in distribution network, many efforts have been done. Using controllable loads to accommodate the variability of wind power production was proposed in recent years. In [3], the author presented an approach to identify loads having appropriate characteristics and adjust those loads in real time using control approaches similar to those used in controlling generation; in [4], the author contrast the effect of demand side versus
supply side policies by analyzing the role that load following costs can have in counteracting the impact of un-predictable renewable energy sources on system operation, and proposed a load-following ramping reserve example. The author in [5] tried to match demand with wind generation considering the wind forecast errors in the power system scale. Literature [6] proposed a genetic algorithm to optimize the operational planning of wind generator, photovoltaic facility, diesel generator and battery energy storage system (BESS) based on price sensitive. In [7], the author presented a methodology to minimize the interconnection point power flow fluctuation by using distributed controllable loads such as battery and heat pump; In [8], the authors model the load and generation of micro grids with wind farms to implement optimal power flow using particle swarm optimization; In [9], the authors developed an optimal operation method with the aim of power supply from power system within the acceptable range and reduction of max-min interconnection point power flow error as low as possible to smooth supply power from distribution system; and in [10], the paper provided an optimization algorithm, which was based on direct load control(DLC), to determine the optimal control schedules. By applying an aggregator to the controllable devices of the virtual power plant, optimization of load reduction over a specified control period can be achieved.

By reviewing the literature, we can find that most authors’ optimal objectives concentrate on the minimization of the interconnection point power flow fluctuation and the optimal management strategy in power supply side. In this paper, we propose a distribution system with wind power unit, battery storage system and controllable load. We choose air conditioner as controllable load in the proposed network because the air-conditioning equipments are significant electricity energy consumers in the summer days. Therefore, designing the suitable scheme to schedule the cycling of the air conditioner loads (ACLs) can effectively curtail the peak loads in summer days. Since the primary usage of the air-conditioning equipments is to provide thermal comfort for the occupants, the most important constraint of the ACL scheduling is obviously minimizing the customers’ thermal discomfort.

Our optimal objective, based on real-time electricity price, is to minimize the cost customers spend on electricity. It can be achieved through depleting power generated by wind unit to larger content and cooperating all devices in this proposed system. Here, we use the day-ahead forecast data and ignore the forecast errors in this study. As for the optimization method, a Mixed Integer Linear Programming (MILP) approach is used. The simulation is developed by Matlab with Mosek Optimization tool.

This paper is organized as follows. After introduction section, the main optimal objectives are introduced, followed by the concept of proposed MILP approach. After that, the thermal inertia model is introduced. Then a distribution system is used as a case to verify the proposed method. Finally, conclusions are drawn in last section.

**MILP Model Formulation**

In this paper, the proposed distribution network consists of battery storage system, wind power, critical load and controllable load. The main optimal objectives of this system contain three parts listed below.

1) To determine the switch status of air conditioner within the comfortable temperature range, which acts as the controllable load, to further consume and adjust wind generation.

2) To alter the pattern of energy use so that the battery can be charged during the high generation and low load period, on the other hand, the battery will supply power in the peak load time when electricity from grid is expensive.

3) To accommodate more wind power generation through determining the wind generation direction. It can ensure power will not be abandoned when wind generation exceed the load demand.
Objective Function

The objective is to minimize the total power cost from external grid in this system. This means to minimize the cost that the proposed system gets from grid, meanwhile, maximize benefit that the distribution system supplies to grid. The objective function is described as below.

\[
\text{Minimize } C_{\text{net}}(t) = P_{g\_buy}(t) \times C_{buy}(t) - P_{g\_sell}(t) \times C_{sell}(t)
\]  

(1)

The net cost from grid is decided by the real-time power flow between grid and system, and the real-time electricity retail and purchase price. From the objective function, it can be seen that given the safe and stable operation of loop system, shrinking the power provided by grid and enlarging the amount power supplied to grid will be the solution for this system.

Constraints

Due to different devices involved in the proposed system, following constraints should be met.

1. Energy balance and supply-demand relationship

\[
P_{\text{load}}(t) + P_{ac} \times S_{ac}(t) = P_b(t) + P_{g\_buy}(t)
\]

(2)

\[
P_{g\_sell}(t) = -S_{\text{wind}}(t) \times P_{ac}(t) - P_{\text{breed}}(t)
\]

(3)

\[
P_{g\_buy}(t) \leq D_b(t) \times P_{g\_max}
\]

(4)

\[
P_{g\_sell}(t) \geq -(1 - D_b(t)) \times P_{g\_max}
\]

(5)

Power from system and grid must match the total load (including critical load and controllable load) in any time. More-over, the amount of power interacting between grid and system should not exceed the permissible ranges of grid.

2. Battery constraints

\[-P_{\text{max}} \leq P_{\text{blev}}(t) \leq P_{\text{max}}\]

(6)

Battery charge level magnitude must lie within the permissible capacity ranges to ensure safe operation of the battery.

3. Wind power constraints

\[
\begin{align*}
P_{\text{u}} &= \frac{1}{2} \rho \pi R^2 V_w^3 C_p & v_c \leq v_w \leq v_r \\
P_w &= P_{\text{rated}} & v_r \leq v_w \leq v_f \\
P_w &= 0 & v_w \leq v_c \cup v_r > v_f
\end{align*}
\]

(7)

The output of wind turbine is largely depending on wind speed. In general, generated power increases as the cube of wind speed until it reaches rated speed. Then the wind turbine will generate the fix amount of power once wind speed exceeds the rated value. If wind speed is smaller than cut-in speed or larger than fatigue speed, there will be no power generated from wind turbine.

4. Thermal comfort constraints

The thermal comfort of residents should be satisfied, thus the constraints are formulated as:

\[
T_{r\_min} \leq T_r \leq T_{r\_max}
\]

(8)

\[
T_{w\_min} \leq T_w \leq T_{w\_max}
\]

(9)
**MILP Optimization Solver**

The MILP is a recently developed approach to efficiently solve global optimization problems. Standard MILP is described as follows.

\[
\min f(x) = c^T x
\]

\[
b_l \leq Ax \leq b_u
\]

\[
x_l \leq x \leq x_u
\]

Where \( A \in \mathbb{R}^{m \times n} \), \( c, x, x_l, x_u \in \mathbb{R}^n \), \( b_l, b_u \in \mathbb{R}^m \) and \( x \) should be an integer. The rows corresponding to the portion of \( x \) are integer constraints, while the rest of the rows are continuous constraints. An optimal solution \( x \), is a vector of variables for which under the objective function is smallest; in the meanwhile \( x \) can satisfy the constraints [11]. MILP approach can be a suitable solution to optimize the operation of distribution network, where the continuous constraints can represent the output of wind power and the integer constraints can be the status of controllable loads, battery charging state and direction of power flow.

**Thermal Inertia Modeling**

The key to ensure the occupants’ thermal comfort is to fully understand and model the thermal transition process of the buildings. In the smart house study, the one parameter thermal model has been widely used in many literatures, which is shown in Figure 1 (a). The one-parameter model takes into account of parameters like internal and external temperatures, but only considers the thermal resistance of the wall and neglects the wall’s thermal capacitance. In this paper, a more complex two-parameter is represented in Figure 1 (b). The house is divided into two components, where one is the inside of the house and the other is the additional thermal mass such as walls wall with a notably different thermal capacity.

![Figure 1. Schematic of thermal models: (a). One parameter model (b). Two parameter model.](image)

The change of indoor air temperature of a house could be considerably different from that without considering thermal capacity of walls. This is due to the heat gain of a house through walls consists of the relatively steady-state transmission that occurs because of the temperatures differences between the indoor air and outdoor ambient, and the unsteady-state gain resulting from the varying intensity of solar radiation on the outer surface of the wall. The thermal capacity within a wall brings about the complicated unsteady-state heat flow across the wall, and a certain amount of heat through the wall could be captured and later released to the indoor air or outdoor ambient. Therefore, the thermal dynamic model of a two-parameter model could be expressed by Eqs. (13)-(14).

\[
\frac{dT_i(t)}{dt} = \frac{1}{M_u \times C_p_u} \left( \frac{dQ_m}{dt} - \frac{dQ_{m,\text{in}}}{dt} - \frac{dQ_{m,\text{out}}}{dt} \right) \forall t \in T
\]  

(13)
\[ \frac{dT_r(t)}{dt} = \frac{1}{M_p \times C_p_r} \left( \frac{dQ_{gen,r}(t)}{dt} + \frac{dQ_{ex,r}(t)}{dt} \right), \quad \forall t \in T \]  

(14)

**Linearization of the Thermal Dynamic Model**

The thermal dynamic model in (13)-(14) can be linearized for convenient calculating the indoor temperature variation. For each dispatch time interval, \( \Delta t \) is divided into K steps. We can suppose that the temperatures of the ambient, the wall, and the indoor air within any time step are constant as long as each step is large enough. Therefore, the change in temperatures can be presented by the temperature difference between two adjacent time steps, which can be linearized as Eqs. (15)-(18).

\[
T_r(n) = \left( 1 - \frac{1}{M_a \times C_p_a \times R_{eq}} \right) \times T_{r\_init} + \frac{1}{M_a \times C_p_a \times R_{eq}} \times T_{amb\_init} \\
+ \frac{T_{w\_init} - T_{r\_init}}{M_{air} \times C_p_a \times R_{wr}} - S_{ac\_init} \times \frac{Q_{ac}}{M_a \times C_p_a}, \quad n = 1
\]

(15)

\[
T_r(n) = \left( 1 - \frac{1}{M_a \times C_p_a \times R_{eq}} \right) \times T_r(n-1) + \frac{1}{M_a \times C_p_a \times R_{eq}} \times T_{amb}(n-1) \\
+ \frac{T_w(n-1) - T_{r\_init}(n-1)}{M_a \times C_p_a \times R_{wr}} - S_{ac}(n) \times \frac{Q_{ac}(n-1)}{M_a \times C_p_a}, \quad \forall n \in [2, N]
\]

(16)

\[
T_w(n) = T_{w\_init} + \frac{T_{amb\_init} - T_{w\_init}}{M_w \times C_p_w \times R_{wa}} + \frac{T_{r\_init} - T_{w\_init}}{M_w \times C_p_w \times R_{wr}}, \quad n = 1
\]

(17)

\[
T_w(n) = T_w(n-1) + \frac{T_{amb}(n-1) - T_{w}(n-1)}{M_w \times C_p_w \times R_{wa}} + \frac{T_{r}(n-1) - T_{w}(n-1)}{M_w \times C_p_w \times R_{wr}}, \quad \forall n \in [2, N]
\]

(18)

**Simulation Study**

The studied case in this research is composed of a wind unit, a BESS and 1000 households, which act as the total loads and are divided into 5 groups according to different types of house design. The simulation is conducted on Matlab/Mosek software.

In this radical distribution system, loads (including general uncontrollable loads, also known as critical loads, and controllable loads) directly consume the power stored in battery and the power from grid. In this case, air-conditioner cooling loads act as controllable loads that can be unified adjusted as long as room temperature being within the comfortable temperature range. Wind power can supply power to charge battery or be straightly sold to grid. The power interaction of this system is shown in Figure 2.

Given the various devices’ constraints, the optimal goal for this system is to get a strategy to accommodate more wind power and to minimize the energy cost the households spent through managing all the device operation statuses. In the meanwhile, the power utilization efficiency of the proposed network is also maximized.

Considering the optimization goal and relevant constraints, the operational status of air-conditioners in one group households is shown in Figure 3. 1 means air conditioners are toggled on, while 0 means air conditioners are toggled off. Performances of battery, wind power unit and grid...
are shown in Figure 4, Figure 5, and Figure 6 respectively. Direction of power between system and external grid is shown in Figure 6 and the amount of power plow is illustrated in Figure 7.

Figure 2. Power interaction between proposed radial system and grid.

Figure 3. Operational status of air-conditioners (0=off, 1=on).

Figure 4. Battery charge level.
From the results above, we can get a set of strategies of each component in this distribution system. Wind power will supply to grid when battery charge level is adequate enough to support the households load; battery can be charged from grid when wind speed is low but load is high. In this way, we can get a relatively efficient strategy to accommodate more wind power.
Conclusion

The fluctuation nature of wind power limits the utilization of wind energy in different scales. Wind power should be curtailed when power output exceeds the load demand, and it cannot cater the peak demand if wind generation is limited. So the strategy that wind-power-integrated distribution system with bi-direction power flow should be an ideal solution to utilize intermittent characteristics of wind power. This paper determines an optimal operation of proposed distribution network which consists of air conditioners, a wind power unit, BESS and external grid. The MILP approach is utilized to get a set of operational strategies for each participating unit in proposed system. In this way, the aim of increasing accommodation of wind power and minimizing the electricity cost households paid to grid can be achieved.

References


