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Double-Objective Optimization of Laminated Composite Plates for Natural Frequency Separation and Radiation Acoustic Power

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Abstract. In this study, frequency separation maximization and radiation acoustic power minimization of the laminated plate are simultaneously considered based on genetic algorithm. A double-objective genetic algorithm is used to obtain Pareto-optimal designs for frequency separation maximization and radiation acoustic power minimization. A four-layer laminated plate is used as an example, and the numerical simulations show that different weight coefficient ratios are chosen, Pareto optimal solution of double objective of the laminated plate are obtained.

Introduction

Because of their lightweight and many superior properties, laminated composite plates or panels have been widely used in the construction of vehicle, aircraft and spacecraft structures. In order to decrease the likelihood of resonance, some of researchers focused on maximizing the natural frequency or maximizing frequency separation between natural frequencies. For example, Y. Narita et al [1] studied the optimal design of thin laminated plates and obtained results for maximum fundamental frequency using finite element method to determine the frequency response. Abdalla et al [2] considered maximization of the natural frequency of composite panels. The variation of the stiffness properties was parameterized using lamination parameters, and the fundamental frequency was predicted based on classical lamination theory. In addition, there has been much research on acoustic power optimization of laminated composite plates. Wodtke et al [3] studied the acoustic radiation of damping sandwich ring structure, and the optimal objective was the acoustic radiation power minimum.

The objective of the present study is to find the optimum stacking sequence that gives the maximum frequency separation and minimum sound power for the fiber composite laminated plates.

Sound Power Analysis of Laminated Plates

Consider a planar structure vibrating with an angular frequency ω and radiating sound into the upper half space V exterior to the panel surface S for $z>0$. The density of the medium is ρ, and the sound speed is c. The surface area of the vibrating plate is S. The plate is divided into J elements with equal areas. The vector of the normal velocities of each element is denoted as $\mathbf{U}(\omega)$. The acoustic power can be expressed as [4]

$$W(\omega)=\frac{\rho c S}{2}\mathbf{U}(\omega)^{H}\cdot\mathbf{R}\cdot\mathbf{U}(\omega) \tag{1}$$

where the superscript H denotes the complex conjugate transpose. The matrix $\boldsymbol{R}$ is a $J\times J$ matrix of the real part of the acoustic transfer impedance between each pair of elements.

Natural Frequency Analysis of Laminated Plates

Based on the layer-wise theory, each laminate in the direction of the thickness needs to be interpolated, and the laminated plate geometry and coordinate system are shown in Figure 1. The displacement of the laminated plate can be written as [5]:

$$U(x,y,z,t)=\sum_{i=1}^{2n+1}\mu_i(x,y,t)\Psi_i(z);$$

$$V(x,y,z,t)=\sum_{i=1}^{2n+1}v_i(x,y,t)\Psi_i(z);$$

$$W(x,y,z,t)=\sum_{i=1}^{2n+1}w_i(x,y,t)\Psi_i(z). \tag{2}$$

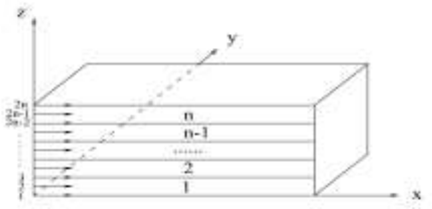

Figure 1. The laminated plate geometry and coordinate system.

According to finite element theory, $u_i(x,y,t)$, $v_i(x,y,t)$ and $w_i(x,y,t)$ in Eqs. (2) can be rewritten as:

$$u_i(x,y,t)=\sum_{k=1}^{m}N_k(x,y)u_kT(t);$$

$$v_i(x,y,t)=\sum_{k=1}^{m}N_k(x,y)v_kT(t);$$

$$w_i(x,y,t)=\sum_{k=1}^{m}N_k(x,y)w_kT(t). \tag{3}$$

where $\mathbf{N}$ is the unit shape matrix.

According to finite element theory, the total power equation can be obtained.

$$\mathbf{M}\ddot{\mathbf{X}}+\mathbf{C}\dot{\mathbf{X}}+\mathbf{K}\mathbf{X}=\mathbf{F} \tag{4}$$

where $\mathbf{M}$ is the unit mass matrix, $\mathbf{C}$ is the damping matrix, $\mathbf{K}$ is the unit stiffness matrix, and $\mathbf{F}$ is the element nodal force matrix.

From Eq. (6), the surface normal displacement vector X can be determined, and the normal velocity vector $U_X(x,y,\omega)$ can be obtained. $U_X(x,y,\omega)$. And natural frequency ω can be obtained from $\left|\boldsymbol{K}-\omega^2\boldsymbol{M}\right|=0$.

Numerical Examples

Formulation of the Optimization Problem

The problem of the multi-objective optimization is defined in the following mathematical form:

$$\begin{cases} V-\min: \; f(x)=[f_1(x), f_2(x), \cdots, f_p(x)]^T \\ s.t. \quad : x \in X \\ \quad\quad X \subseteq R^m \end{cases} \tag{5}$$

where: V-min is denoted vector minimization, i.e., each sub-objective function of vector objective function $f(x)=[f_1(x), f_2(x),..., f_p(x)]^{\mathrm{T}}$ is expected to tend to be minimized as possible. $f(x)$ is multi-objective optimization vector function. If there are $\bar{x} \in X$ and $\bar{x}$ is Pareto optimal solution or non-inferior solution of multi-objective optimization.

The genetic algorithm based on objective weight is used in this paper. Each sub-objective function has own weight coefficient, these the sum of each sub-objective function multiply by own weight coefficient is new one objective function. The multi-objective function weight problem is defined in the following form,

$$F(x)=\sum_{i=1}^{m} \lambda_i \cdot f_i(x) \tag{6}$$

where $\lambda_i (i=1,2,...,m)$ is each sub-objective function non-negative weight coefficient, and satisfy $\sum_{i=1}^{m} \lambda_i = 1$. So according to the problem of the frequency parameter Ω_{12} and acoustic radiation power $W(\omega_p)$ optimization is defined in the following mathematical form:

$$\begin{aligned} & Find: X=([\theta_1/\theta_2/\cdots/\theta_n]) \\ & f(x)=\lambda_1 f_1(x)+\lambda_2 f(x) \\ & Min \;\; f_1(x)=-(\Omega_2([\theta_1/\theta_2/\cdots/\theta_n])-\Omega_1([\theta_1/\theta_2/\cdots/\theta_n])) \\ & Min \;\; f_2(x)=W(\omega_p) \\ & Subject\ to: \omega_p \geq F_1; -90 \leq \theta_k \leq 90, k=1,2,\cdots,n \end{aligned} \tag{7}$$

where the appropriate λ_1、λ_2 is chosen, and then $\lambda_1+\lambda_2$=1. The different λ_i is chosen, the different Pareto optimal solution can be obtained.

Results and Discussion

The physical properties of the four-layer laminate plate are presented in Table 1. Consider fully clamped boundary conditions of the laminate plate. Assuming the locations of one point force (force amplitude of 1 N) is center of the plate. The exciting frequencies respectively are 200Hz and 500Hz. The initial orientation angle is $[0^{\circ}]_{4}$.

Double objective functions which are the maximum natural frequency separation and minimum acoustic power are simultaneously optimized by used genetic algorithm. In order to select a suitable λ_1、λ_2, in this paper the twelve different weight coefficients λ_1、λ_2 are selected and the double objective functions' Pareto optimal solution of the four-layer laminated plate are shown in Table 2. The four discrete orientation angles are -45º,0º,45º, and 90º.

Table 1. Material properties of the laminated plate.

Length (m)	0.3480
Width(m)	0.3048
Thickness(m)	0.002
Young's modulus (GPa)	E_1=181,E_2=E_3=10.3
Poisson's ratio	$\upsilon_{21}=\upsilon_{31}=\upsilon_{23}=0.28$
Shear modulus (GPa)	G21=G23=G31=7.17
Damping ratio	0.05
Density(kg/m^3)	1560

Table 2. Pareto optimal solution of the four-layer laminated plate with different weight coefficient ratio.

(a) Exciting force frequency is equal to 200Hz

λ_1:λ_2	Ω_{12}	$W(\omega_p)$	Optimum angle
10:1	81.66	82.30	[90º/0º/90º/0º]
5:1	81.66	82.30	[90º/0º/90º/0º]
1:1	81.66	82.30	[90º/0º/90º/0º]
1:5	81.66	82.30	[90º/0º/90º/0º]
1:10	74.78	81.84	[90º/0º/-45º/0º]
1:15	74.78	81.84	[90º/0º/-45º/0º]
1:20	64.95	81.70	[90º/45º/0º/0º]
1:30	64.95	81.70	[90º/45º/0º/0º]

(b) Exciting force frequency is equal to 500z

λ_1:λ_2	Ω_{12}	$W(\omega_p)$	Optimum angle
10:1	81.66	84.52	[90º/0º/90º/0º]
5:1	81.66	84.52	[90º/0º/90º/0º]
1:1	81.66	84.52	[90º/0º/90º/0º]
1:5	81.66	84.52	[90º/0º/90º/0º]
1:10	81.66	84.52	[90º/0º/90º/0º]
1:15	67.80	77.93	[45º/45º/45º/45]
1:20	43.60	76.20	[45º/45º/90º/90]
1:30	43.60	76.20	[45º/45º/90º/90]

From Table 2, according to Ω_{12} and $W(\omega_p)$, there are three critical weight coefficients $\lambda_1:\lambda_2$, which are respectively selected $1:1$, $1:10$, and $1:20$. So the three weight coefficients λ_1、λ_2 are respectively selected in this paper.

Table 3 are respectively the double objective optimization results with weight coefficient $\lambda_1:\lambda_2=1:1$, $1:10$, and $1:20$ at the exciting force frequency is equal to 200Hz. Table 2 and Table 3 show that with $\lambda_1:\lambda_2$ is increased, the change of Ω_{12} and $W(\omega_p)$ is increased, especially at $\lambda_1:\lambda_2=1:20$, the change of Ω_{12} and $W(\omega_p)$ are the largest.

Table 3. The double objective optimization with exciting force frequency is 200Hz.

(a) $\lambda_1:\lambda_2=1:1$

Angle increment	Optimum angle	Ω_{12}	$W(\omega_p)$
initial angle	$[0º]_4$	32.38	83.43
90	[90º/0º/90º/0º]	81.66	82.30
45	[90º/0º/90º/0º]	81.66	82.30
30	[0º/-60º/0º/90º]	84.54	81.91
15	[45º/-30º/60º/30º]	87.02	83.48

(b) $\lambda_1:\lambda_2=1:10$

Angle increment	Optimum angle	Ω_{12}	$W(\omega_p)$
90	[90º/0º/-45º/0º]	74.78	81.84
45	[0º/60º/0º/90º]	84.52	81.91
30	[0º/60º/0º/90º]	84.52	81.91
15	[90º/0º/90º/0º]	81.66	82.30

Table 4. the double objective optimization with exciting force frequency is 500Hz.

(a) $\lambda_1:\lambda_2=1:1$

Angle increment	Optimum angle	Ω_{12}	$W(\omega_p)$
initial angle	$[0º]_4$	32.38	76.79
90	[90º/0º/90º/0º]	81.66	84.52
45	[90º/0º/90º/0º]	81.66	84.52
30	[0º/-60º/0º/90º]	84.54	85.96
15	[45º/-30º/60º/30º]	87.02	86.72

(b) $\lambda_1:\lambda_2=1:10$

Angle increment	Optimum angle	Ω_{12}	$W(\omega_p)$
90	$[0º]_4$	32.38	76.79
45	[90º/0º/90º/0º]	81.66	84.52
30	[-30º/90º/90º/30º]	68.71	80.28
15	[-30º/90º/90º/30º]	68.71	80.28

(c) $\lambda_1:\lambda_2=1:20$

Angle increment	Optimum angle	Ω_{12}	$W(\omega_p)$
90	[90º/90º/0º/0º]	57.98	81.76
45	[90º/0º/-45º/0º]	74.78	81.84
30	[90º/0º/-60º/0º]	84.54	81.91
15	[90º/15º/-45º/0º]	75.21	81.63

(c) $\lambda_1:\lambda_2=1:20$

Angle increment	Optimum angle	Ω_{12}	$W(\omega_p)$
0	$[0º]_4$	32.38	76.79
45	[-45º/45º/90º/90º]	43.62	76.20
30	[-60º/30º/60º/60º]	43.87	78.53
15	[45º/-45º/75º/75º]	46.28	76.76

When the exciting frequency is $\omega_p=500\text{Hz}$ and the weight coefficients λ_1:λ_2=1:1，1:10，1:20, the double objective optimization results are shown in Table 4.

In addition, from Table 3 to Table 4, when the weight coefficient ratio $\lambda_1:\lambda_2=1:1$， the contribution of objective function Ω_{12} is obviously much larger than the role of acoustic radiation power. And the effect of optimization of double objective function is almost just optimization of acoustic power. It is particularly evident when the exciting frequency is 500Hz. But with the weight coefficient ratio increased, for example $\lambda_1:\lambda_2=1:10$ and even $\lambda_1:\lambda_2=1:20$, the two optimization objectives contributions to the final result tend to be equal. When the exciting force frequency is 200Hz, the weight coefficient ratio λ_1:λ_2=1:20, Pareto optimum solution can satisfy the double optimum objective. But when the exciting force frequency is 500Hz, the weight coefficient ratio $\lambda_1:\lambda_2=1:10$ could be suitable.

Conclusions

In this paper, the maximum the natural frequency separation and minimum acoustic power for a four-layer laminated plate is optimization using genetic algorithms. Pareto-optimal solutions can be obtained by the weight coefficients. The different weight coefficient ratios are selected and the optimum orientation angle design can be acquired by using different angle increment interval of laminated plate. The optimum results can be improved the vibration and acoustic character of the laminated plate.

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