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Suspension System Dynamics Modeling and Simulation Analysis

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Abstract. A dynamic model of 7-DOF wheeled vehicles was established using the Lagrange's theorem, vibration response of vehicle body's vertical, pitching, lateral directions and unsprung mass can be carried on simultaneously based on the model. The random road spectrums of standard road surface and typical off-road terrain were deduced separately, pavement structure analysis in time and frequency domain was also completed. Systematical dynamic programming computation was conducted using random road spectrums as input. Optimal iteration algorithms were adopted to match and optimize the elastic-damping characteristics of suspension systems, and the variation laws of suspension damping ratio under different road conditions and vehicle speeds were obtained, which provides theoretical basis for determination of hydro-pneumatic suspension composite characteristic in prototype vehicle.

Introduction

In order to analysis dynamic characteristics of vehicle, it must establishing dynamics model. Vehicle vibration systems are very complicated, and dynamic response results are influenced by numerous factors. In this article, the dynamics model is built up through parameters off-road vehicle, which has seven freedom degrees. After that, simulation and optimization of suspension properties are completed, it is important to improve ride comfort and handling stability of the vehicle.

Dynamic Model of the Vehicle

Physical Model

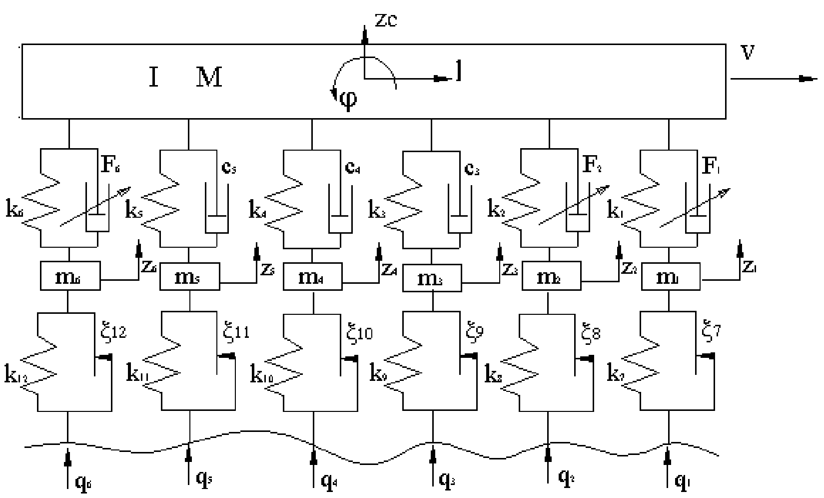

Figure 1. Physical model of the vehicle.

Off-road vehicle dynamic model shown in Figure1, which has seven freedom degrees, point O is mass center of body, and the vehicle travels along the X direction.

Model Deduction

Dynamic differential equation is:[1-2]

$$[M]\{\ddot{z}\}+[C]\{\dot{z}\}+[K]\{z\}=[L]\{q(t)\} \tag{1}$$

In which, $\{z\}=\{z_c \quad z_1 \quad z_2 \quad z_3 \quad z_4 \quad z_5 \quad z_6\}^T$

$$\{q(t)\}=\{q_1(t) \quad q_2(t) \quad q_3(t) \quad q_4(t) \quad q_5(t) \quad q_6(t)\}^T$$

The mass matrix is:

$$[M]=\begin{bmatrix} M & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & m_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & m_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & m_6 \end{bmatrix}_{8\times 8} \tag{2}$$

The damping matrix is:

$$[C]=\begin{bmatrix} \sum_{i=1}^{6} c_i & \sum_{i=1}^{6} l_i c_i & -c_1 & -c_2 & -c_3 & -c_4 & -c_5 & -c_6 \\ \sum_{i=1}^{6} l_i c_i & \sum_{i=1}^{6} l_i^2 c_i & -l_1 c_1 & -l_2 c_2 & -l_3 c_3 & -l_4 c_4 & -l_5 c_5 & -l_6 c_6 \\ -c_1 & -l_1 c_1 & c_1 & 0 & 0 & 0 & 0 & 0 \\ -c_2 & -l_2 c_2 & 0 & c_2 & 0 & 0 & 0 & 0 \\ -c_3 & -l_3 c_3 & 0 & 0 & c_3 & 0 & 0 & 0 \\ -c_4 & -l_4 c_4 & 0 & 0 & 0 & c_4 & 0 & 0 \\ -c_5 & -l_5 c_5 & 0 & 0 & 0 & 0 & c_5 & 0 \\ -c_6 & -l_6 c_6 & 0 & 0 & 0 & 0 & 0 & c_6 \end{bmatrix}_{8\times 8} \tag{3}$$

The stiffness matrix is:

$$[K]=\begin{bmatrix} \sum_{i=1}^{6} k_i & \sum_{i=1}^{6} l_i k_i & -k_1 & -k_2 & -k_3 & -k_4 & -k_5 & -k_6 \\ \sum_{i=1}^{6} l_i k_i & \sum_{i=1}^{6} l_i^2 k_i & -l_1 k_1 & -l_2 k_2 & -l_3 k_3 & -l_4 k_4 & -l_5 k_5 & -l_6 k_6 \\ -k_1 & -l_1 k_1 & k_1+k_7 & 0 & 0 & 0 & 0 & 0 \\ -k_2 & -l_2 k_2 & 0 & k_2+k_8 & 0 & 0 & 0 & 0 \\ -k_3 & -l_3 k_3 & 0 & 0 & k_3+k_9 & 0 & 0 & 0 \\ -k_4 & -l_4 k_4 & 0 & 0 & 0 & k_4+k_{10} & 0 & 0 \\ -k_5 & -l_5 k_5 & 0 & 0 & 0 & 0 & k_5+k_{11} & 0 \\ -k_6 & -l_6 k_6 & 0 & 0 & 0 & 0 & 0 & k_6+k_{12} \end{bmatrix}_{8\times 8} \tag{4}$$

Vibration equation of the system is:

$$\begin{cases} M\ddot{z}_c + \sum_{i=1}^{6} c_i\left(\dot{z}_c + l_i\dot{\varphi} - \dot{z}_i\right) + \sum_{i=1}^{6} k_i(z_c + l_i\varphi - z_i) = 0 \\ I\ddot{\varphi} + \sum_{i=1}^{6} c_i\left(\dot{z}_c + l_i\dot{\varphi} - \dot{z}_i\right)l_i + \sum_{i=1}^{6} k_i(z_c + l_i\varphi - z_i)l_i = 0 \\ m_1\ddot{z}_1 - c_1\left(\dot{z}_c + l_1\dot{\varphi} - \dot{z}_1\right) - k_1\left(z_c + l_1\varphi\right) + \left(k_1 + k_w\right)z_1 = k_w q_1(t) \\ m_2\ddot{z}_2 - c_2\left(\dot{z}_c + l_2\dot{\varphi} - \dot{z}_2\right) - k_2\left(z_c + l_2\varphi\right) + \left(k_2 + k_w\right)z_2 = k_w q_2(t) \\ m_3\ddot{z}_3 - c_3\left(\dot{z}_c + l_3\dot{\varphi} - \dot{z}_3\right) - k_3\left(z_c + l_3\varphi\right) + \left(k_3 + k_w\right)z_3 = k_w q_3(t) \\ m_4\ddot{z}_4 - c_4\left(\dot{z}_c + l_4\dot{\varphi} - \dot{z}_4\right) - k_4\left(z_c + l_4\varphi\right) + (k_4 + k_w)z_4 = k_w q_4(t) \\ m_5\ddot{z}_5 - c_5\left(\dot{z}_c + l_5\dot{\varphi} - \dot{z}_5\right) - k_5\left(z_c + l_5\varphi\right) + \left(k_5 + k_w\right)z_5 = k_w q_5(t) \\ m_6\ddot{z}_6 - c_6\left(\dot{z}_c + l_6\dot{\varphi} - \dot{z}_6\right) - k_6\left(z_c + l_6\varphi\right) + \left(k_6 + k_w\right)z_6 = k_w q_6(t) \end{cases} \quad (5)$$

Fourier transform to the formula (1): [3-4]

$$-w^2 MX(w) + jwCX(w) + KX(w) = F(w) = K_q Q(w) \quad (6)$$

Random Excitation of Road

Pavement power spectral density is expressed:[5-6]

$$G_q(n) = G_q(n_0)\left(\frac{n}{n_0}\right)^{-W} \qquad n_l < n < n_u \quad (7)$$

In which, n is time frequency, n_u and n_l are upper limit, lower limit of spatial frequency of road spectrum, n_0=0. 1 m^{-1} is reference space frequency, W is frequency index.

The road spatial power spectrum density could be change to time power spectrum density:

$$G_q(w) = G_q(f) = \frac{1}{u}G_q(n) = \left(2\pi\right)^2 n_0{}^2 G_q(n_0)\frac{u}{w^2} \quad (8)$$

And power spectrum matrix of 4 wheels is:

$$\left[G_q(n)\right] = \begin{bmatrix} 1 & coh(n) & e^{-j2\pi nl} & coh(n)e^{-j2\pi nl} \\ coh(n) & 1 & coh(n)e^{-j2\pi nl} & e^{-j2\pi nl} \\ e^{j2\pi nl} & coh(n)e^{j2\pi nl} & 1 & coh(n) \\ coh(n)e^{j2\pi nl} & e^{j2\pi nl} & coh(n) & 1 \end{bmatrix} G_q(n) \quad (9)$$

In which, $coh_{xy}(n)$ is the coherence coefficient for the wheels, it can be expressed as:

$$coh_{xy}^2(n) = \begin{cases} (a - bn)^r & 0 \le |n| \le 0.1 \\ (a - 0.1b)^r & 0.1 < |n| \le 2 \\ 0 & 2 < |n| \end{cases} \quad (10)$$

And $a = 1$; $b = (1 - 0.1^{1/r})/0.1$; $r = B/0.25$, B is the tread.

The Evaluation Index System of Suspension Performance

The suspension determined the vehicle ride comfort and stability, which performance can be evaluated through the following three basic parameters:

(1) The vibration acceleration response

It is expressed through root mean square of vertical body acceleration:

$$\sigma_{\ddot{x}_1} = \sqrt{\frac{1}{2\pi}\int_0^{+\infty} 2S_{\ddot{x}_1}(w)dw} = \sqrt{\frac{1}{2\pi}\int_0^{+\infty} 2w^4 G_{11} dw} \quad (11)$$

(2) Suspension stroke

Suspension stroke parameter is defined as the wheels and the body displacement difference RMS value, it can be used to describe the degree of relative displacement of the static state, have great influence on handling stability of vehicle.

The four suspension stroke is defined as:

$$
\begin{aligned}
f_{d1} &= x_1 + L_{s1}x_2 - L_1x_3 - x_4 \\
f_{d2} &= x_1 - L_{s1}x_2 - L_1x_3 - x_5 \\
f_{d3} &= x_1 + L_{s2}x_2 + L_2x_3 - x_6 \\
f_{d4} &= x_1 - L_{s2}x_2 + L_2x_3 - x_7
\end{aligned} \tag{12}
$$

And the four suspension stroke RMS for:

$$
\begin{aligned}
\sigma_{f_{d1}} &= \sqrt{\sigma_{x1}^2 + b_1^2\sigma_{x2}^2 + L_1^2\sigma_{x3}^2 + \sigma_{x4}^2} = \sqrt{\frac{1}{2\pi}\int_0^{+\infty} 2(G_{11} + b_1^2G_{22} + L_1^2G_{33} + G_{44})dw} \\
\sigma_{f_{d2}} &= \sqrt{\sigma_{x1}^2 + b_1^2\sigma_{x2}^2 + L_1^2\sigma_{x3}^2 + \sigma_{x5}^2} = \sqrt{\frac{1}{2\pi}\int_0^{+\infty} 2(G_{11} + b_1^2G_{22} + L_1^2G_{33} + G_{55})dw} \\
\sigma_{f_{d3}} &= \sqrt{\sigma_{x1}^2 + b_2^2\sigma_{x2}^2 + L_2^2\sigma_{x3}^2 + \sigma_{x6}^2 + b_2^2\sigma_{x7}^2} = \sqrt{\frac{1}{2\pi}\int_0^{+\infty} 2(G_{11} + b_2^2G_{22} + L_2^2G_{33} + G_{66} + b_2^2G_{77})dw} \\
\sigma_{f_{d4}} &= \sqrt{\sigma_{x1}^2 + b_2^2\sigma_{x2}^2 + L_2^2\sigma_{x3}^2 + \sigma_{x6}^2 + b_2^2\sigma_{x7}^2} = \sqrt{\frac{1}{2\pi}\int_0^{+\infty} 2(G_{11} + b_2^2G_{22} + L_2^2G_{33} + G_{66} + b_2^2G_{77})dw}
\end{aligned} \tag{13}
$$

(3) Relative dynamic load of wheels

The relative dynamic load parameters are defined as the variation root mean square relative to the static load of the tires

The four wheels dynamic load are:

$$
\begin{aligned}
\frac{F_{d1}}{G_1} &= \frac{k_{t1}(x_4 - q_1)}{G_1} = c_1(x_4 - q_1) \\
\frac{F_{d2}}{G_2} &= \frac{k_{t2}(x_5 - q_2)}{G_2} = c_2(x_5 - q_2) \\
\frac{F_{d3}}{G_3} &= \frac{k_{t3}(x_6 - q_3)}{G_3} = c_3(x_6 - q_3) \\
\frac{F_{d4}}{G_4} &= \frac{k_{t3}(x_7 - q_4)}{G_4} = c_4(x_7 - q_4)
\end{aligned} \tag{14}
$$

The frequency response function of relative dynamic load on input excitation is:

$$
H_{\frac{F_d}{G}-q} = \begin{bmatrix} c_1(H_{41}-1) & c_1H_{42} & c_1H_{43} & c_1H_{44} \\ c_2H_{51} & c_2(H_{52}-1) & c_2H_{53} & c_2H_{54} \\ c_3H_{61} & c_3H_{62} & c_3(H_{63}-1) & c_3H_{64} \\ c_4H_{71} & c_4H_{72} & c_4H_{73} & c_4(H_{74}-1) \end{bmatrix} \tag{15}
$$

Power spectral density of dynamic load response and input excitation have the following relationship:

$$
\left[G_{\frac{F_d}{G}}(w)\right] = \left[H_{\frac{F_d}{G}}{}^*\right]\left[G_q(w)\right]\left[H_{\frac{F_d}{G}}\right]^T \tag{16}
$$

After that, the root mean square value of dynamic tire load is:

$$
\sigma_{\frac{F_{di}}{G}} = \sqrt{\frac{1}{2\pi}\int_0^{+\infty} 2G_{\frac{F_d}{G}}(i,i)dw} \tag{17}
$$

Vehicle Dynamics Simulation

The vehicle dynamics model is calculated through matlab programm. Road and vibration acceleration simulation results are as follows. The vertical vibration acceleration responses of the vehicle body changing with different suspension damping characteristics lay the foundation for establishing control strategy of semi-active suspension.

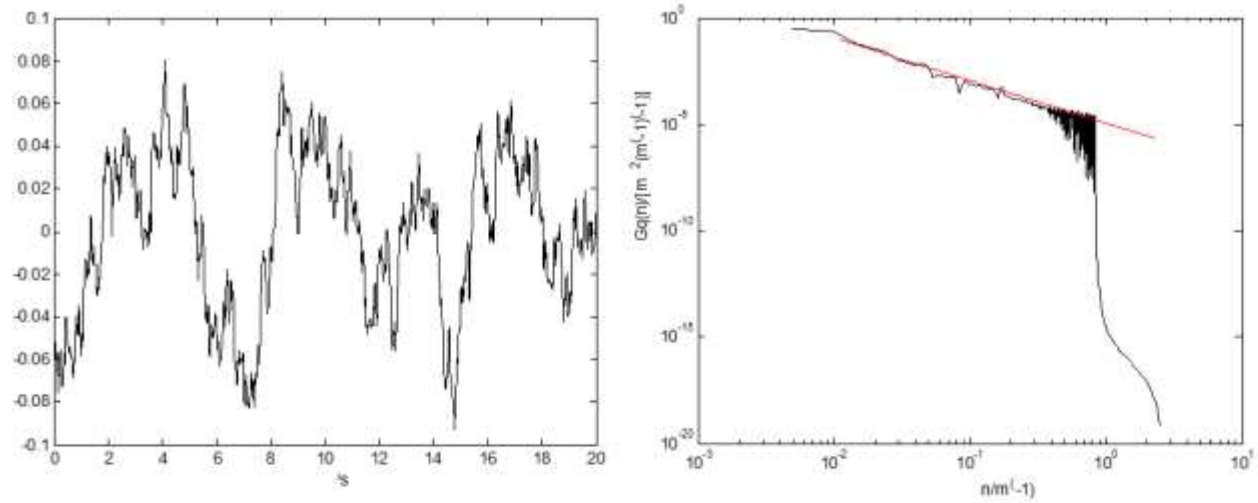

Figure 2. Random excitation of the road.

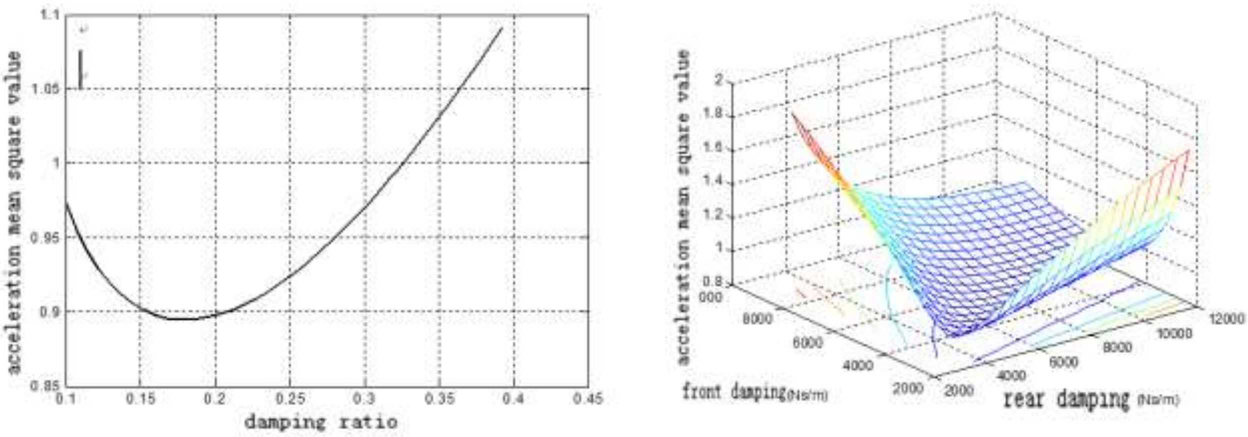

Figure 3. Vertical vibration acceleration response of different damping.

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