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Research on the Calculation Method of the Collision Probability of Satellite Formation Flying

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Abstract. For some reasons (such as the failure of a satellite or the formation of a satellite formation mission), it is necessary to reconstruct the formation. In this process, the research on satellite collision avoidance has received extensive attention. In this paper, by deriving the initial state covariance matrix of the satellite, the collision probability density function was integrated in a specific region of the satellite, then the collision probability of formation satellites was obtained. The simulation experiments show that the method has great significance for the collision between satellites.

Introduction

Since the late 1990s, the research work of satellite formation flying is not interrupted [1]. Especially in recent years, with the development of micro satellites, has become a hot research topic. At present, the spacecraft formation flying is used to accomplish the task of space science experiment in the world. Such as A-Train plans and MMS plans from NASA [2,3].

Satellite formation flying technology is one of the major characteristics of a small satellite in space to form a specific configuration collaborative work, close contact, in order to form a large distribution of virtual spacecraft (also called "distributed satellite system"). Thus, the phenomenon of "Emergence" in the system theory is produced, and the performance of the system is beyond the single satellite system.

In formation flying, due to the influence of various perturbations, the formation of the formation will drift, and due to the various hardware and software problems, it will increase the probability of collision in the formation process. How to avoid the collision between formation satellites is an important problem to be considered in the design of satellite formation flying. In this paper, the integral of collision probability density function in the specific area of the satellite by the initial state covariance matrix of the recursive satellite. And the collision probability of formation satellites is calculated. Finally, numerical simulation is carried out.

Collision Probability Calculation of Satellite Formation Flying

Hill Equation of Satellite Formation Flying

When consideration is given to the reference satellite in the near circular orbit (the orbital eccentricity "e=0") [5], the relative motion equations can be simplified as:

$$\begin{aligned} &\ddot{x} - 2\omega\dot{z} = 0 \\ &\ddot{y} + \omega^2 y = 0 \\ &\ddot{z} + 2\omega\dot{x} - 3\omega^2 = 0 \end{aligned} \tag{1}$$

Eq. 1 is also known as C-W equation or Hill equation[5,6]. The analytical solution of this equation is:

$$x = 2\left(2x_0 + \frac{\dot{y}_0}{\omega}\right) - \left(3x_0 + 2\frac{\dot{y}_0}{\omega}\right)\cos(\omega t) + \left(\frac{\dot{x}_0}{\omega}\right)\sin(\omega t)$$

$$y = \left(y_0 - 2\frac{\dot{x}_0}{\omega}\right) + 2\left(\frac{2\dot{y}_0}{\omega}\right)\sin(\omega t) + 2\left(\frac{\dot{x}_0}{\omega}\right)\cos(\omega t) - 3(\dot{y}_0 + 2\omega x_0)$$

$$z = \frac{\dot{z}_0}{\omega}\sin(\omega t) + z_0\cos(\omega t) \tag{2}$$

We can also transform the C-W equation into a standardized form:

$$\dot{X} = AX + Dw + \Gamma u \tag{3}$$

Relative state quantity X is:

$$X = \begin{bmatrix} x & y & z & \dot{x} & \dot{y} & \dot{z} \end{bmatrix}^T \tag{4}$$

Where: w — interference vector in three directions:

u — control vector in three directions.

In the equation,

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2\omega & 0 & 0 \\ 0 & 3\omega^2 & -2\omega & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -\omega^2 & 0 \end{bmatrix}, \quad D = \Gamma = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{5}$$

Where: ω as reference satellite orbit angular velocity.

The solution Eq. 3 can be obtained:

$$X(t) = \phi(t, t_0)X(t_0) + \int_0^t \phi(t)[Dw(\tau) + \Gamma u(\tau)]d\tau \tag{6}$$

Where: $\phi(t)$ as state transition matrix.

$$\phi(t) = \begin{bmatrix} 1 & 0 & 6\omega t - 6\sin(\omega t) & (4\sin(\omega t) - 3\omega t)/\omega & 0 & (2 - 2\cos(\omega t))/\omega \\ 0 & \cos(\omega t) & 0 & 0 & \sin(\omega t)/\omega & 0 \\ 0 & 0 & 4 - 3\cos(\omega t) & (2\cos(\omega t) - 2)/\omega & 0 & \sin(\omega t)/\omega \\ 0 & 0 & 6\omega t - 6\cos(\omega t) & 4\cos(\omega t) - 3 & 0 & 2\sin(\omega t) \\ 0 & -\omega\sin(\omega t) & 0 & 0 & \cos(\omega t) & 0 \\ 0 & 0 & 3\omega\sin(\omega t) & -2\sin(\omega t) & 0 & \cos(\omega t) \end{bmatrix} \tag{7}$$

Calculation of Collision Probability

If the relative state error can be considered as random error, Gauss distribution is unbiased distribution. Assume that ω follows as the N (0, P) distribution. Then, $X(0) = N(0, P_0)$; $X(t) = N(0, P(t))$. The covariance matrix of the relative state at any time can be expressed as:

$$\dot{P}(t) = AP_0 + P_0A^T + DWD^T \tag{8}$$

Where: W as covariance matrix of perturbation acceleration; P as covariance matrix of initial state. For circular reference orbits, analytic solution of P can be obtained.

$$P(t) = \phi(t,t_0)P_0\phi^T(t,t_0) + (\omega+1)\int_0^t \phi(t)DWD^T\phi^T(t)d\tau \tag{9}$$

Assuming that the covariance matrix P of the relative state is calculated at a certain time, the probability density function of X is:

$$pdf(X) = \frac{1}{\sqrt{(2\pi)^n |P|}} \exp\left(-\frac{1}{2}X^T P^{-1} X\right) \tag{10}$$

In order to simplify the calculation, assume that the satellite's danger zone is a circle with a radius of 5m. If two satellites form a formation flying, the integral region is transformed into a circular region with a radius of 10m. Figure 1 is a schematic diagram of collision probability calculation method in the encounter plane.

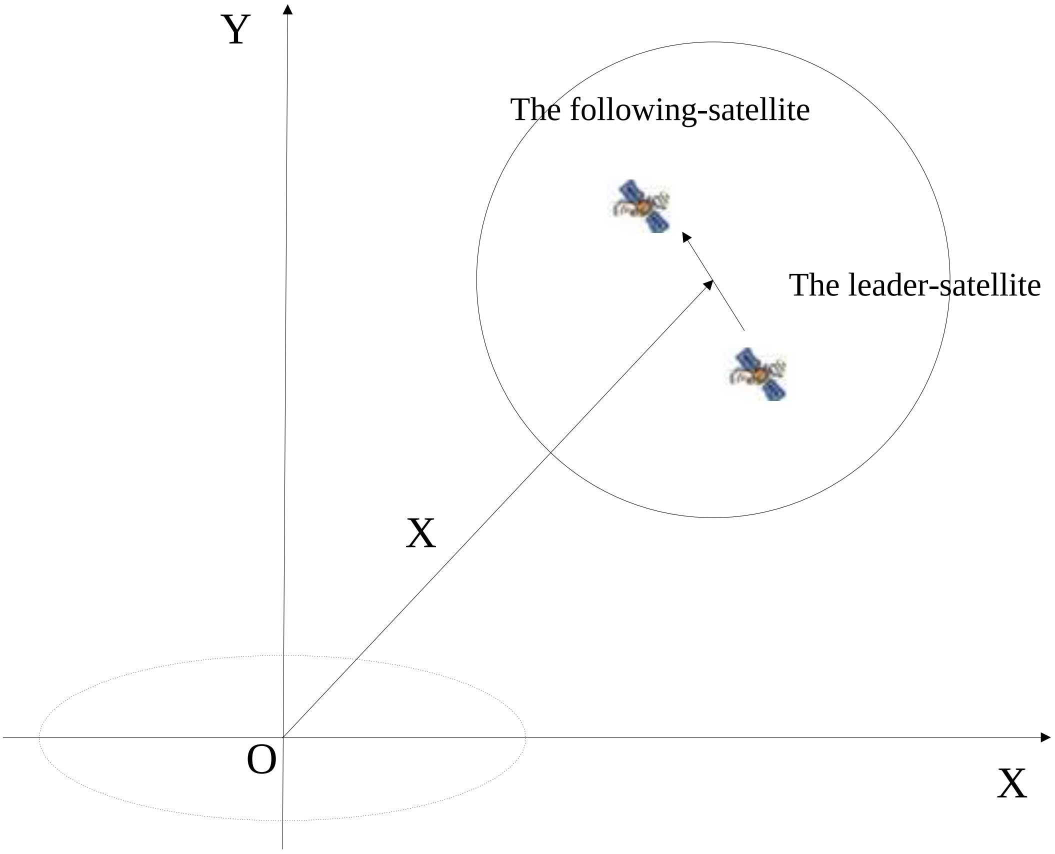

Figure 1. Schematic diagram of collision probability calculation method in the encounter plane.

Because we only care about the relative position error covariance matrix of the satellite collision probability, the upper left corner matrix 3*3 of the relative state covariance matrix $P(t)$ is obtained [7]. Marked as: P_X . Then, the probability density function is:

$$pdf(X) = \frac{1}{\sqrt{(2\pi)^3 |P_X|}} \exp\left(-\frac{1}{2}X^T {P_X}^{-1} X\right) \tag{11}$$

The probability density function is integrated in the region where possible collision is possible. Then, collision probability can be calculated.

$$P_c = \frac{1}{\sqrt{(2\pi)^n |P_X|}} \iiint \exp\left(-\frac{1}{2} X^T P_X^{-1} X\right) dxdydz \tag{12}$$

Numerical Simulation

Formation flying of two satellites is used as an example, and its orbital parameters [8] are shown in Table 1.

Table 1. The leader-satellite and the following-satellite orbit parameters.

orbit parameter	the leader-satellite	the following-satellite
a	6892937.0018	6892937.0018
e	0.001168	0.001162
i	97.4433830	97.443830
Ω	90	89.99687
ω	0	0.002516
M	0	0.21226

The initial relative position error is 0.1m; the initial relative velocity error is 0.0005m/s; the simulation time is 6 days. Figure 2 is Collision probability of "xoy".

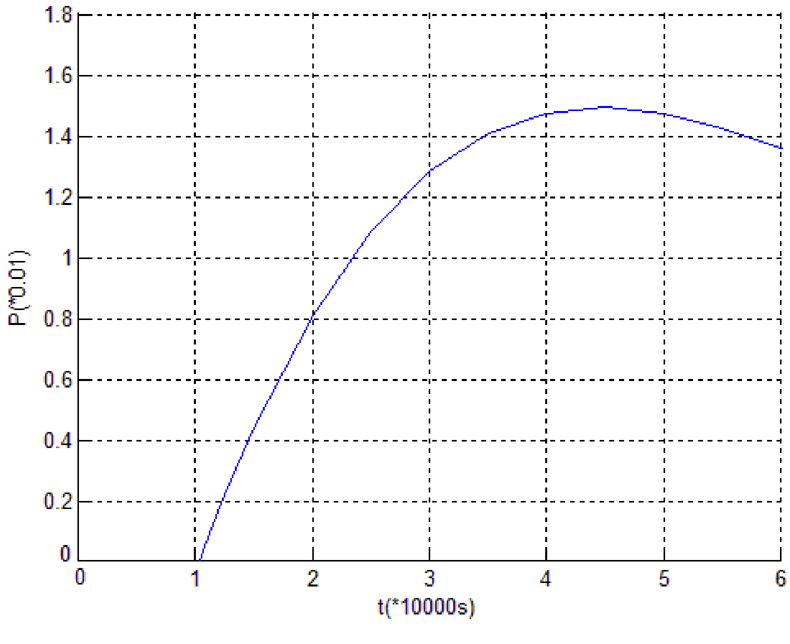

Figure 2. Collision probability of xoy.

From Figure 2, we can see that the collision probability increases gradually, and then reaches a peak, and then gradually decreases. At the beginning of the prediction, position covariance matrix values are very small, at this time, the distance between the center of the probability ellipse and the collision region is relatively large, so the collision probability is very small. As time goes on, the position covariance matrix increases rapidly, which lead to increase the value of collision probability. When the collision probability reaches the maximum value, two satellites are not necessarily collision. Two satellites may still be in a safe distance in the other direction. Collision avoidance is not required at this time. A common practice is: At first, the collision probability of the "xoz" projection surface is calculated. Then the probability value and the security threshold are compared. If the value of the collision probability is greater than the threshold value, the probability of the "xoy" projection surface is calculated. Only these two probability values are greater than the threshold value to avoid operation.

Summary

In this paper, the initial state covariance matrix of the formation satellite is derived. The collision risk between formation satellites is analyzed based on the method of collision probability. When the collision probability of the two satellites is larger than the threshold value, the appropriate avoidance strategy should be adopted. The research of evasion strategy will be the direction of future efforts.

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