An Improved United Compass Ambiguity Decorrelation Algorithm

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ABSTRACT: An improved unity ambiguity decorrelation algorithm of Beidou was proposed due to the drawback of unity ambiguity algorithm in high-dimensional calculations. The algorithm will transform vector matrix K through rounding one by one, and have a descending order according to the size of the diagonal elements after the end of iteration. Comparing the new algorithm with the standard unity decorrelation algorithm through simulation, the results show that the new algorithm can achieve smaller condition number and larger decorrelation number no matter in low-dimensional or high-dimensional calculations, which can solve the problem of ability decreased of decorrelation in the original algorithm. At the same time, compared with LDLT algorithm based on sorting, the effect of decorrelation of the new algorithm is smaller and less time consuming.

Keywords: unity decorrelation; Compass; ambiguity; descending order; integral by vector

With gradually improved self-developed compass navigation system in China, and higher requirements on the baseline resolution, that is, precise positioning and less resolution time, to develop a new kind of algorithm to preferably balance both relations becomes the current research hotspot.

The ambiguity resolution is the most critical part in the process of high-precision baseline resolution of compass. If the ambiguity of whole cycles can be quickly fixed, we can correspondingly shorten the observation time, improve work efficiency and expand real-time high-precision positioning applications. Currently, LAMBDA method [1] is the most widely used in the ambiguity resolution, which was first proposed by Teunisse [2], in which the ambiguity decorrelation and ambiguity integer search are the most critical. Currently, the researched ambiguity decorrelation algorithm can be divided into three categories as a whole: integer Gauss decorrelation algorithm, algorithm based on Cholesky factorization [3] and orthogonal decomposition algorithm represented by LLL algorithm [4-5]. United decorrelation algorithm is actually an evolution of integer Gauss decorrelation algorithm, which has its superior in speed compared with several other algorithms. Therefore, the research of its algorithm in line with the requirements of high precision and high speed. A new method successively takes integral from the transformation matrix K, and obtains conclusion through the test and comparison with low-dimensional and high-dimensional calculation. The improved united decorrelation algorithm has been improved in terms of the decorrelation capacity compared with the original algorithm, especially in high-dimensional calculation, which makes up the shortcomings of poor decorrelation effect of the original algorithm in the high-dimensional calculation, and also enhances the decorrelation effect of the original algorithm in the high-dimensional calculation.

1 DECORRELATION ALGORITHM

1.1 United decorrelation algorithm

The difference between the united decorrelation algorithm and integer Gauss decorrelation algorithm is that, the former varies directly on the covariance matrix Qi, while the latter varies on the matrix L after Cholesky decomposition, and then inversion. Therefore, the calculation speed of united decorrelation algorithm is faster. There are mainly three steps [6]:

1. To rank the diagonal elements of the ambiguity covariance matrix by the use of the rotation transformation matrix L from small to large.

\[ Q_i = L Q_i L^T \]  

(1)
(2) Variation of Gaussian elimination. To successively vary the covariance matrix \( Q \) after descending order by the use of the transformation matrix \( K \) to the diagonal matrix \( D \) in the following ways, of which the superscript indicates the number of iterations.

\[
Q^t = K^t Q K^t = \begin{bmatrix} q_1 & 0 \\ 0 & X_t \end{bmatrix}
\]

(2)

Where, \( Q^t = Q^t \) can be obtained from n-1 transformation.

\[
Q^{t-1} = KQK^t = D = \text{diag}[q_1, q_2, \ldots, q_n]
\]

(3)

Where, the transformation matrix \( K \) meets the following conditions, of which \( m \) represents the number of rows to be transformed.

\[
K_m = \begin{bmatrix}
1 & & & \\
\cdot & k_{m+1,m} & 1 & \\
\cdot & k_{m+2,m} & 1 & \\
& & & \\
k_{m,m} & 1 & \cdots & \cdot
\end{bmatrix}_{m+1,m}
\]

(4)

Finally, the following equation can be obtained:

\[
K = K_1K_2 \cdots K_mK
\]

(5)

(3) United decorrelation. Assuming that \( U = KL \), \( [U] = [K][L] \), \( Q = [U][Q][U]^t \). To repeat the steps 1-3, and strengthen the decorrelation effect, iteration is over until \([U]\) is unit matrix. Finally, the ambiguity transformation matrix \( Z \) satisfies:

\[
Z = \begin{bmatrix}[U_1][U_{n-1}] \vdots [U_1][U_l][U_i]
\end{bmatrix}
\]

(6)

1.2 \( \text{LDL}^T \) algorithm based on ordering

\( \text{LDL}^T \) algorithm can improve the degree of decorrelation through ordering. Compared with white-filtering decorrelation algorithm \([7]\), \( \text{LDL}^T \) algorithm has less calculated amount and rapid speed, with the following steps:

(1) To rank the diagonal elements of the ambiguity covariance matrix by the use of the rotation transformation matrix \( S \) from small to large.

\[
Q_s = S Q_s S^t
\]

Integral \( \text{LDL}^T \) resolves.

\[
Q_s = \text{LDL}^T
\]

(2) Inversion and iteration, of which the superscript of \( Q_l \) represents the number of iterations.

Finally, the ambiguity transformation matrix \( Z \) satisfies:

\[
Q = [L_1]^t S Q_s S^t [L_1]^t = [L_1]^t S Q_s [L_1]^t
\]

(9)

1.3 Improved united decorrelation algorithm

Compared with other algorithms, the united decorrelation algorithm has a simple calculation and significant speed, but also has some shortcomings. With the rotation transformation matrix \( L \), during diagonalization of the covariance matrix \( Q \) after ascending order, \( q_i \) refers to the \( i \)-th value on the diagonal, which is not necessarily the minimum value among the \( i \)-th value to the \( n \)-th value on \( Q \) diagonal. Therefore, the absolute value of \( k_m \) sometimes could not be the maximum. If each calculation is based on the minimum \( q_i \), the absolute value of \( k_m \) is always able to maintain the maximum, thus improving the degree of decorrelation. In addition, the reference \([8]\) proposes to use the method of descending order to make the diagonal elements after variation comply with the least square discrete search based on the sequential conditions. However, after simulation and verification, the covariance matrix finally obtained is just approximate descending order, rather than entire descending, so this paper proposes a strategy of first ascending and finally descending, in order to ensure that the covariance is eventually in a descending order. Meanwhile, in all of the united decorrelation algorithms, assuming that \( K = K_{n-1}K_{n-2} \cdots K_1 \), \( K \) can be obtained after taking integral of each element in \( K \). In fact, this way virtually increases errors. Therefore, this paper adopts the successive approximation method in subsequent calculation to take integral of each \( K \) matrix as shown in equation (11) and then multiply to obtain \( K^t = [K_{n-1}][K_{n-2}] \cdots [K_1][K_1] \). The simulation experiment proves that this way can improve the degree of decorrelation.

\[
K_n = \begin{bmatrix}
1 & & & \\
\cdot & \tilde{k}_{m+1,m} & 1 & \\
\cdot & \tilde{k}_{m+2,m} & 1 & \\
& & & \\
\tilde{k}_{m,m} & 1 & \cdots & \cdot
\end{bmatrix}_{m,m}
\]

(11)

The specific process of algorithm is as follows:

(1) To rank the ambiguity covariance matrix by the use of the rotation transformation matrix \( L \) based on the ascending order, \( Q_l \) can be obtained from the equation (1).
(2) Variation of Gaussian elimination. \( Q_l \) is converted into equation (2) by the use of matrix \( K_1 \). The experiment simulation indicates that, even if the order is made previously, but in \( X_1 \), \( q_i > q_m \), resulting that the absolute value of \( k_m \) during decorrelation is unable to be guaranteed at the maximum as much as possible. Therefore, the rotation transformation matrix \( S \) is introduced to shift the minimum value in the remaining diagonal elements to the diagonal location at the second line. To transform based on the above method until like equation (13).

\[
Q = (S_m K_m) Q^T (S_m K_m)^T = \begin{bmatrix} q_1 & q_2 & \ldots & q_{m-1} & q_m \end{bmatrix}
\]

(13)

Where, the superscript of \( Q \) represents the number of iterations. The matrix \( K \) can be obtained from equation (4), and \( k \) satisfies the following relations like equation (14). \( m \) represents the number of rows of \( q_m \), \( l \) and \( q \) correspond to the elements in \( Q_i \).

\[
k_m + s_m = -l_m + s_m / q_m
\]

(14)

(3) Integral transformation. The following equation can define the united transformation matrix \( U \), assuming that \( U = S_m [K_m] S_m [K_m] \ldots [K_m] [S_m] [S_m] [K_m] \), \( U = U_L \), each element of \( U \) is an integer, and the diagonal matrix may not be obtained by calculation of the covariance matrix \( Q = U Q_U U \) with united transformation, so it is necessary to improve the degree of decorrelation via iteration. Assuming that \( Q = Q _o \), to repeat the steps 1-3, the iteration is over until \( U \) is unit matrix. Finally, the matrix is in descending order through the transformation matrix \( L_1 \).

\[
Z = L_1 U_n U_{n-1} U_{n-2} \ldots U_2 U_1
\]

(15)

2 ANALYSIS OF EXPERIMENTAL RESULTS

2.1 Analysis of simulation experiment

The simulation experiment of this paper uses MatlabR2014a as simulation software, and the method in the reference [9] is used to select the ambiguity covariance matrix to generate an analog matrix for experimental comparison. The spectral condition number and decorrelation number in the references [10-11] are used to measure the decorrelation effect of the algorithm. The spectral condition number \( e \) is shown in equation (16). If it is closer to 1, it indicates that it is closer to the diagonal matrix. Therefore, the smaller the spectral condition number is, the better the decorrelation effect is.

\[
e = \sqrt{\frac{\lambda_{\text{max}}}{\lambda_{\text{min}}}}
\]

(16)

The decorrelation number \( r \) is shown in equation (17) and (18), and its value is within the range of (0, 1). If it is closer to 1, it indicates that the decorrelation effect is better; if it is closer to 0, it indicates that the decorrelation effect is worse.

\[
R_u = [\text{diag}(Q_u)]^{1/2} Q_u [\text{diag}(Q_u)]^{1/2}
\]

(17)

\[
r = \sqrt{\det R_u} \quad 0 \leq r \leq 1
\]

(18)

This paper first selects 100 groups of 8-dimensional random covariance matrix to do test.

![Figure 1](image1.png)

![Figure 2](image2.png)
As shown in Figure 1 and Figure 2, the original algorithm and the new algorithm can play a good role of decorrelation in low-dimensional conditions, and reduce the spectral condition number of the original matrix, and improve its decorrelation number, proving that its improved algorithm is slightly better.

Then, to randomly select 100 groups of 30-dimensional random covariance matrix to do test, the test results are as follows:

![Figure 3](image3.png)

**Figure 3. Comparison of decorrelation number of old and new algorithm in high-dimensional calculation.**

![Figure 4](image4.png)

**Figure 4. Comparison of spectral condition number of old and new algorithm in high-dimensional calculation.**

As can be seen from Figure 3 and Figure 4, (Note: for ease of comparison, here, the vertical ordinate values are different) generally, in high-dimensional calculation of united decorrelation algorithm, the decorrelation effect is not only reduced, and sometimes the correlation is increased, so that the search space becomes worse. The method of this paper still keeps a lower spectral condition number, and correspondingly increases the decorrelation number, thus obtaining conclusion that the improved united decorrelation algorithm can maintain a good decorrelation effect in low-dimensional calculation, and also improve the decorrelation capacity of the original method in high-dimensional calculation.

### 2.2 Analysis of actual data

The actual data are the actual observed values of short baselines collected from CORS station and the erected moving station in No. 12 Building of Electronics and Information School of Nantong University, with sampling rate of 1 second, and time of 1h. The experimental board card is K501 dual-system quad-band high-precision RTK GNSS board card of Shanghai Sinan Navigation. With a reference satellite of Beidou satellite C03, double-difference observed value can be obtained. 2 to 100 epochs of ambiguity float solution and covariance can be obtained by the use of data processing software of Compass Solution. To randomly select five groups from front to back for decorrelation processing according to the improved united decorrelation algorithm and Cholesky algorithm based on ordering, the results are shown in the following tables.

Compared with the results in Table 1 and Table 2, the improved united decorrelation algorithm has a smaller spectral condition number compared with $LDL^T$ decorrelation algorithm based on ordering, but its decorrelation coefficient is lower than the latter one. Compared with their average number of iterations, the average number of iterations of the improved united decorrelation algorithm is 10 times, while the latter

<table>
<thead>
<tr>
<th>No.</th>
<th>Spectral condition number before decorrelation</th>
<th>Spectral condition number after ordering $LDL^T$ transformation</th>
<th>Decorrelation number before decorrelation</th>
<th>Decorrelation number after ordering $LDL^T$ transformation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0152x10^4</td>
<td>4.0944</td>
<td>1.73x10^-17</td>
<td>0.24533</td>
</tr>
<tr>
<td>2</td>
<td>0.7331x10^4</td>
<td>3.4060</td>
<td>1.81x10^-16</td>
<td>0.34164</td>
</tr>
<tr>
<td>3</td>
<td>0.5636x10^4</td>
<td>3.1247</td>
<td>9.05x10^-16</td>
<td>0.3215</td>
</tr>
<tr>
<td>4</td>
<td>0.4303x10^4</td>
<td>3.0757</td>
<td>2.43x10^-15</td>
<td>0.37359</td>
</tr>
<tr>
<td>5</td>
<td>0.3224x10^4</td>
<td>2.9543</td>
<td>8.91x10^-15</td>
<td>0.41083</td>
</tr>
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</table>

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<th>Spectral condition number after ordering $LDL^T$ transformation</th>
<th>Decorrelation number before decorrelation</th>
<th>Decorrelation number after ordering $LDL^T$ transformation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0152x10^4</td>
<td>4.5677</td>
<td>1.73x10^-17</td>
<td>0.28315</td>
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<tr>
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<td>1.81x10^-16</td>
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<td>3</td>
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<td>3.5148</td>
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</tr>
<tr>
<td>5</td>
<td>0.3224x10^4</td>
<td>3.2006</td>
<td>8.91x10^-15</td>
<td>0.42331</td>
</tr>
</tbody>
</table>
one is 13 times, indicating a higher efficiency of the new method.

3 CONCLUSION

With the development of science and technology, the timeliness and accuracy of positioning is increasingly emphasized. Currently, the use of carrier phase differential positioning technology can obtain centimeter-level positioning accuracy, thus improving accuracy and correspondingly improving the timeliness.

(1) For the shortcomings of united decorrelation algorithm and its improved algorithm in high-dimensional calculation, this paper proposes an improved united decorrelation algorithm, which takes integer from vector successively and makes descending order.

(2) Through the numerical simulation calculation, the new improved algorithm can achieve a desired effect in low-dimensional calculation and high-dimensional calculation. Meanwhile, its decorrelation effect in low-dimensional algorithm is better than that of the algorithm before improvement.

(3) Compared with the improved united decorrelation algorithm and Cholesky decorrelation algorithm based on ordering by the use of real observation data, their decorrelation effect is considerable. The new method can get a smaller spectral condition number, and also has a certain superior in computing efficiency, which is in line with rapid positioning research objectives, and has a certain practicality.

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REFERENCES


