A Novel High-precision Single-frequency BeiDou High Kinematic Positioning Algorithm with Pseudo Range and Carrier Phase

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Abstract. High-precision Positioning is one of the key technologies of BeiDou receiver in high dynamic situations. Its implementation requires an optimized and dedicated hardware. The real-time processing places several constraints such as area occupied, power consumption, speed, etc. Using a Hatch filter in a single frequency receiver reduces multipath and receiver noise but introduces an induced bias that depends on the rate of ionospheric delay and a carrier smoothing time. However the cycle slips may easily occur during a long carrier smoothing time. Suppose carrier phase is directly used for positioning, there will be a lot of difficulty in solving the problem of integer ambiguity. A hybrid approach for pseudo range and carrier phase is given, and adopts a current statistic jerk tracking model, which makes positioning highly accurate under the condition of single point and single frequency in high dynamic situations. Besides, the delay of electric inhibition abscission layer, the troposphere delay and the transient change of the multi-path effect have little influence on signal.

Introduction

For the dynamic and real-time Navigation and positioning, using pseudo-range data can avoid calculating ambiguity and handling cycle slip [1]. Generally the pseudo-range precision of P code is ±0.2m, and the pseudo-range precision of C/A code is about ±2m [2]. The positioning precision can satisfy the demands of most navigation users.

Generally, the precision of carrier phase observation is 2-3 in magnitude higher than that of pseudo-code ranging observation value [2]. Therefore, the field of GPS data processing attracts the wide attention and research. The earliest and also most widely used algorithm is called Hatch Filtering [3-7]. Meanwhile, there is the possibility of divergence with this algorithm. To avoid the disadvantage for a single frequency user, various methods have been proposed to mitigate the divergence: a complementary Kalman filter with GPS velocity [8], and an optimal hatch filter using a multipath model parameterized by an elevation angle and Klobuchar model [6].

In this work, the effect of ionosphere and acceleration rate on pseudo-ranging and carrier phase pseudo range are seen as colored noise. The acceleration rate can not be regarded as a stochastic process with zero mean in many high dynamic situations. Therefore, the pseudo-range and carrier phase are combined and processed with a current statistic jerk tracking model. Under the single frequency, single point and highly dynamic state, positioning precision will reach rather a high level for BeiDou users.

The paper is organized as follows. Method of Phase Smoothing Pseudo-range are described in Section 1. The Current Statistic Jerk Tracking Model is given in Section 2. The simulation results and discussions are presented in Section 3. We state a brief conclusion in Section 4.

Method of Phase Smoothing Pseudo-range

If pseudo-range observation value is directly used to determine the ionosphere delay, the precision will be at a low level; if carrier phase observation value is used, there exists the problem of ambiguity although precision is high. Therefore, phase smoothing is often used for amendments. The
observation formula of BeiDou carrier phase pseudo-range and pseudo code ranging under the single frequency and single point can be expressed as:

\[
\lambda \Phi_i^j(t) = \rho_i^j(t) + \lambda N_i^j + c \Delta \delta_i^j(t) - \Delta^{\text{iono}}(t) + \Delta^{\text{trop}}(t) + \varepsilon_o(t) \\
- \Delta^{\text{iono}}(t) + \Delta^{\text{trop}}(t) + \varepsilon_o(t)
\]

(1)

\[
R_i^j(t) = \rho_i^j(t) + c \Delta \delta_i^j(t) + \Delta^{\text{iono}}(t) + \Delta^{\text{trop}}(t) + \varepsilon_o(t) + \varepsilon_s(t) \frac{\pi}{6}
\]

(2)

where \( \Phi_i^j(t) \) is the difference phase between receiver i and satellite j. The unit here is cycle; Carrier wavelength is \( \lambda \); \( \rho_i^j(t) \) is the geometric distance between receiver i and satellite j; \( N_i^j \) is the integer ambiguity; the gap between satellite clock error and receiver clock error is \( \Delta \delta_i^j(t) \), \( \Delta^{\text{iono}}(t) \) is the ionosphere delay of pseudo code ranging, positive data; the ionosphere retardation of carrier phase pseudo range is \( \Delta^{\text{iono}}(t) \), \( \Delta^{\text{trop}}(t) \) is the troposphere retardation of carrier pseudo range and pseudo code ranging; \( \varepsilon_o(t) \) and \( \varepsilon_s(t) \) correspond to the measurement noise of carrier pseudo range and pseudo code ranging.

The measurement noise is ignored here since the noise is too low. If formula (1) is substituted into formula (2), the relationship between pseudo code ranging and carrier phase pseudo range can be shown as follows:

\[
R_i^j(t) = \lambda \Phi_i^j(t) - \lambda N_i^j + 2 \Delta^{\text{iono}}(t)
\]

(3)

During the continuous tracking measurement of BeiDou satellite j, the carrier phase \( \Phi_i^j(t) \) automatically counts. Without cycle slip, the integer ambiguity \( N_i^j \) remains unchanged. Therefore, for the continuous observation of n epoch, there are:

\[
\begin{align*}
R_i^j(t_1) &= \lambda \Phi_i^j(t_1) - \lambda N_i^j + 2 \Delta^{\text{iono}}(t_1) \\
R_i^j(t_2) &= \lambda \Phi_i^j(t_2) - \lambda N_i^j + 2 \Delta^{\text{iono}}(t_2) \\
&\quad \vdots \\
R_i^j(t_n) &= \lambda \Phi_i^j(t_n) - \lambda N_i^j + 2 \Delta^{\text{iono}}(t_n)
\end{align*}
\]

(4)

From the above formula, the estimated value of carrier integer ambiguity \( N_i^j \) can be shown:

\[
\lambda N_i^j = \frac{1}{n} \sum_{i=1}^{n} (\lambda \Phi_i^j(t_i) - R_i^j(t_i)) + \frac{2}{n} \sum_{i=1}^{n} \Delta^{\text{iono}}(t_i)
\]

(5)

If formula (5) is substituted into formula (3), the pseudo range that carrier phase smoothes can be shown:

\[
\overline{R}_i^j(t_n) = \lambda \Phi_i^j(t_n) \left( 1 - \frac{1}{n} \sum_{i=1}^{n} (\lambda \Phi_i^j(t_i) - R_i^j(t_i)) \right) + \frac{2}{n} \sum_{i=1}^{n} \Delta^{\text{iono}}(t_i) + 2 \Delta^{\text{iono}}(t_n)
\]

(6)

The last two items after the equality of formula (6) are caused by the opposite effect that ionosphere delay has on carrier and code phase. For the ionosphere is relatively steady, provided that the receiver is less dynamic, then

\[
\frac{2}{n} \sum_{i=1}^{n} \Delta^{\text{iono}}(t_i) = \frac{2}{n} \sum_{i=1}^{n} \Delta^{\text{iono}}(t_i) = 2 \Delta^{\text{iono}}(t_n)
\]

(7)

The last two items in formula (6) nearly offsets each other. Thus, when carrier phase smoothes pseudo range, even though the time window in use is quite long, pseudo range can be smoothed in a good way at the high precision of positioning. If the receiver is quite dynamic, even if time is very limited, the ionosphere delay may still change significantly. In this case, the appropriate length
should be chosen for smoothing time window. Otherwise, the pseudo-range data of the single-frequency carrier phase smoothing may be divergent. That is, the pseudo-range data of carrier phase smoothing gradually deviates from the original pseudo code ranging data. The test shows, on the highly dynamic BeiDou signal source, when the receiver is at the dynamic level of 600m/s, the pseudo ranging data that has been smoothed for 500s may deviate from the original pseudo code ranging pseudo ranging data as long as 5m.

The Current Statistic Jerk Tracking Model

For the above method of carrier phase smoothing pseudo range, when the dynamic level is low, rather a long time window is preferred for smoothing, thus improving the precision of pseudo range to a higher level. However, when the dynamic level is high, there is the possible divergence for the smoothed pseudo range.

From formula (4), we can see

$$ R_j(t_i) = R_j(t_{i-1}) + \lambda \Phi_j(t_i) - \lambda \Phi_j(t_{i-1}) + 2\Delta^{iono}(t_i) - 2\Delta^{iono}(t_{i-1}) $$

(8)

Let $\lambda \Phi_j(t_i) = \lambda \Phi(t_i) - \lambda \Phi_j(t_{i-1})$, then

$$ R_j(t_i) = R_j(t_{i-1}) + \lambda \Delta \Phi_j(t_i) + 2\Delta^{iono}(t_i) - 2\Delta^{iono}(t_{i-1}) $$

(9)

Due to the relatively steady ionosphere, and relatively short sampling interval (100ms), even when the dynamic level is high, the time ionosphere of an epoch changes very little. Therefore, we see $2\Delta^{iono}(t_i) - 2\Delta^{iono}(t_{i-1})$ as noise, and use the method of a current statistic jerk tracking model filtering to filter both carrier phase pseudo range and pseudo code ranging.

Let $\lambda \Phi_j(t_i)$ be pseudo range that time $t$ carrier phase corresponds to. Shortly following time $t_{n-1}$, time $t$ corresponds to the following pseudo range of carrier phase:

$$ \lambda \Phi_j(t_i) = \lambda \Phi_j(t_{n-1}) + \frac{\lambda d \Phi_j(t_i)}{dt} \bigg|_{t_{n-1}} (t - t_{n-1}) + $$

$$ \frac{\lambda d \Phi_j(t_i)}{2!} \bigg|_{t_{n-1}} (t - t_{n-1})^2 + $$

$$ \frac{\lambda d \Phi_j(t_i)}{3!} \bigg|_{t_{n-1}} (t - t_{n-1})^3 ... $$

(10)

Then

$$ \lambda \Delta \Phi_j(t_i) = \lambda \Phi_j(t_i) - \lambda \Phi_j(t_{i-1}) = \frac{\lambda d \Phi_j(t_i)}{dt} \bigg|_{t_{n-1}} (t_i - t_{i-1}) + $$

$$ \frac{\lambda d \Phi_j(t_i)}{2!} \bigg|_{t_{n-1}} (t_i - t_{i-1})^2 + $$

$$ \frac{\lambda d \Phi_j(t_i)}{3!} \bigg|_{t_{n-1}} (t_i - t_{i-1})^3 ... $$

(11)

Let $\Delta t = t_i - t_{i-1}$ be time interval of an epoch, then
\[ \frac{\lambda \Delta \Phi^j(t_e)}{\Delta t} = \frac{\lambda d \Phi^j(t)}{dt} \bigg|_{t_e} + \frac{\lambda}{2!} \frac{d^2 \Phi^{j+1}(t)}{dt^2} \bigg|_{t_e} \Delta t \]
\[ + \frac{\lambda}{3!} \frac{d^3 \Phi^{j+2}(t)}{dt^3} \bigg|_{t_e} (\Delta t)^2 \ldots \]
\[ \text{With formula (9), the following can be shown:} \]
\[ R^j(t_e) = R^j(t_{e-1}) + \frac{\lambda \Delta \Phi^j(t_e)}{\Delta t} \Delta t + 2 \Delta \Phi^j(t_e) - 2 \Delta \Phi^j(t_{e-1}) \]
\[ \text{Let} \]
\[ x_e = R^j(t_e), \quad \dot{x}_e = \frac{\lambda \Delta \Phi^j(t_e)}{\Delta t}, \]
\[ \ddot{x}_e = \frac{\lambda d \Phi^{j+1}(t_e)}{dt}, \quad \dddot{x}_e = \frac{\lambda d \Phi^{j+2}(t_e)}{dt} \]
\[ x_e \text{ correspond to pseudo code ranging;} \]
\[ \dot{x}_e \text{ correspond to frequency, that is, to speed;} \]
\[ \ddot{x}_e \text{ correspond to frequency rate, that is, to acceleration;} \]
\[ \dddot{x}_e \text{ correspond to acceleration rate, that is the jerk of the receiver.} \]

To achieve the high precision of positioning under the single frequency, the single point and the highly dynamic state, especially when the acceleration rate cannot be regarded as a stochastic process with zero mean, the current statistic jerk tracking model is chosen here to track maneuvering target [9]. With formula (12) and (13), the equation of discrete state and the equation of observation by the current statistic jerk tracking model can be shown as follows:

\[ X(n + 1) = F(n)X(n) + U(n)J + V(n) \]
\[ Z(n + 1) = H(n + 1)X(n + 1) + W(n) \]

Among which, \( X(n) = [x_e, \dot{x}_e, \ddot{x}_e, \dddot{x}_e]^T \), \( \overline{J} \) is the expectation of \( \dddot{x}_e \), \( V(n) \) is white system noise vector, and \( W(n) \) is white measurement noise vector.

The input matrix \( U(n) \) can be derived as following:

\[ U(n) = \begin{bmatrix}
\frac{1}{2a^2} (2T - aT^2 + \frac{a^3}{3}T^3 - \frac{1}{a} (1 - e^{-aT})) \\
\frac{1}{a} (-T + \frac{aT^2}{2} + \frac{1 - e^{-aT}}{a}) \\
\frac{T - 1 - e^{-aT}}{a} \\
1 - e^{-aT}
\end{bmatrix} \]

where \( T = \Delta t \) is time interval of an epoch and \( \alpha \) is the characteristic parameter of maneuvering target. Thus, filtering algorithm based on the current statistic jerk tracking model is defined as following:

\[ \hat{X}(n + 1) = \hat{X}(n + 1 / n) + K(n + 1)[Z(n + 1) - H \hat{X}(n + 1 / n)] \]

\[ (16) \]
\[
\begin{align*}
\hat{X}_{n+1} & = F(n) \hat{X}_n + U(n) \\
P_{n+1} & = F(n)P_{n/n}F^T(n) + Q(n) \\
K(n+1) & = P(n+1/n)H^T[H P(n+1/n)H^T + R(n+1)]^{-1} \\
P(n+1) & = [I - K(n+1)H]P(n+1/n)
\end{align*}
\]

where \( \hat{X}_{n+1} \) is filter state prediction vector, \( P_{n+1/n} \) represents covariance of state prediction and \( P_{n/n} \) denotes covariance of state estimation. \( Q(n) \) is the variance of the process noise \( V(n) \) and \( R(n) \) is the variance of measurement noise. \( \ddot{j} \) is usually set as prediction of target jerk \( \dddot{x}_{n+1/n} \).

**Simulation Result**

For the simulation scene with the highly dynamic BeiDou signal source, the speed fluctuates between 900m/s~8000m/s with acceleration up to 20G and acceleration jerks up to 2G. Time interval of an epoch is 100ms, and 5000 points for the collected data of the continuous test. The variance of acceleration rate is 0.2 m/s³ and the reciprocal of jerk time is 1/120.

By comparing the calculated position with the simulated position, the error curve of the single-epoch coordinate (X, Y, Z) can be drawn for the single-point positioning (see Figure 1, 2, 3).

According to Figure 1, 2 and 3, after the smoothing of carrier phase, the error on the single-epoch coordinate is significantly decreased for the single-point positioning. The bulging position in the error curve corresponds to the time of acceleration jerk. Table 1 shows the mean value and variance of error in three directions of carrier phase smoothing position and direct pseudo code positioning.

<table>
<thead>
<tr>
<th>Directions</th>
<th>Mean</th>
<th>Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N-S</td>
<td>S</td>
</tr>
<tr>
<td>X</td>
<td>0.005</td>
<td>0.001</td>
</tr>
<tr>
<td>Y</td>
<td>0.002</td>
<td>0.005</td>
</tr>
<tr>
<td>Z</td>
<td>0.029</td>
<td>0.012</td>
</tr>
</tbody>
</table>

Where N-S denotes the non-smoothing result, S denotes the smoothing result.

![Figure 1. Single epoch error in X direction.](image_url)
Conclusions

By using the highly precise carrier phase observation data to smooth pseudo code ranging, the error effects on receiver measurement noise and multi-path can be better eliminated, and precision of pseudo code ranging observation data can be improved. However, due to the change and sudden jitter of the error in ionosphere refraction, there is the possible divergence for the single-frequency pseudo range phase smoothing. In this paper, the maneuvering target Singer tracking model is used to filter both pseudo code ranging and carrier phase, which better eliminates the negative effect that ionosphere delay has on carrier smoothing pseudo range. Thus, the high precision is achieved under single frequency and single point at the highly dynamic level.

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References


