Observability Analysis and Optimal Meter Placement in Active Distribution Networks

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Abstract. In this paper, the observability analysis method based on meter placement was employed. Each consumer is classified, and the Distributed Generation (DG) is considered as a special negative load with constant power, which contains two types: photovoltaic (PV) power and wind turbine generation (WTG). The load variation is represented using typical load patterns. An efficient graph partitioning method is adopted to partition the weighted tree in the performing meter placement to achieve the network observability. Then an algorithm of optimization meter placement based on observability degree is presented to improve the state estimation accuracy. Finally, the validity of the proposed method has been tested in the IEEE 33-bus distribution network.

Introduction

With the development of intelligent distribution automation and the monitoring of the distribution network (DN), intelligent control and balance of new energy connect are the focus of future progress. But the un-observability has seriously restricted the development of DN automation and intelligent power grid [1]. Therefore, it is necessary to study the overall observability of the DN. Observability analysis is an important function in state estimation. An electrical network is determined as observable if the available measurements are sufficient to estimate all the state variables uniquely within that network. However the distribution network lack in the observability research [2]. A novel probabilistic approach to DN observability is proposed in [2], it takes the uncertainty of the state estimation into account and assesses the network observability depending on the accuracy of the estimated network state. Furthermore, it is found that even if the system is observable, the SE will not converge or the estimation result is not ideal. The observability of the system can only reflect the possibility of the estimation state output of the system, but the estimation accuracy and speed can not be reflected. So, it is necessary to quantify the system observability and to judge the observability degree of the system [3].

More recently, DN observability may be achieved by incorporating the prior information (the so-called pseudo-measurements), the meter placement problem has been addressed targeting a given distribution system state estimation (DSSE) accuracy specification [4] or minimizing the number of instruments [5]. Singh et al. [4] have proposed a measurement placement algorithm which seeks to reduce the errors in both the voltage magnitude and angle estimates simultaneously. In [5], an optimization algorithm is proposed that is suitable for choosing the optimal number and position of the measurement devices in DSSE procedures used in modern electric distribution networks.

Based on the existing research results, this paper discusses the observability analysis method and the algorithm of meter placement in active distribution network (ADN). The network segmentation algorithm is adopted to partition the weighted tree in the performing meter placement. Then the system observability is implemented when the measurement matrix is full rank. In addition, an optimal meter placement algorithm based on observability degree is proposed for improving DSSE accuracy.
Observability Analysis of Active Distribution Network with DG

All the users in the DN are divided into \( n \) types, including the conventional load and the DG. The DG is considered as a special negative load, which is divided into two types: photovoltaic (PV) and wind turbine (WT). The change rule of each kind users is expressed by the typical load mode [6], and the importance level of the users is determined according to the average active power. In this paper, we define the area between several configuration meters on the branch as the measurement area, and the branch power measurement is preferred. By configuring the branch measurements under the desired network, ignoring network loss, the measurement equation can be obtained as follows:

\[
Z_t =HX_t + V_t
\]

(1)

where \( Z_t \) stands for a vector of dimension \( m \times 1 \), \( m \) is the number of measurement areas; the element \( Z_{(m)} \) stands for the measured power of the \( m \)-th measurement area at the time \( t \); \( H \) is a constant coefficient matrix of dimension \( m \times n \), \( n \) is the number of the categories of users, the element \( h_{m \times n} \) is the sum of average power load of \( n \)-type user in the \( m \)-th measurement area; \( X_t \) is a vector of dimension \( n \times 1 \), the element \( x_{(n)} \) stands for the normalized typical load mode of the \( n \)-th user at time \( t \); \( HX_t \) is the calculated value of \( Z_t \). So \( V_t \) is the residual phase of dimension \( m \times 1 \).

When the solution to formula (1) exists: \( m=n \) and the rank of matrix \( H \) is \( n \), or \( m>n \), in other words, the measurement system is observable. The classical weighted least square (WLS) estimation method is employed to get the normalized typical load modes as:

\[
X_t =W_t Z_t
\]

where \( W_t = (H^T R_t^{-1} H)^{-1} H^T R_t^{-1} \), \( R_t \) stands for the measurement error variance diagonal matrix of dimension \( m \times m \) at time \( t \); \( R_t^{-1} \) is the weighted matrix of measurement.

Optimal Meter Placement in Active Distribution Networks

Meter Placement Principals

According to the analysis of distribution network observability, the minimum number of branch measurements is \( m=n \), and the matrix \( H \) in equation (1) is full rank. In a radial distribution network with \( L \) nodes, placing \( n \) branch meters in the \( L-1 \) branch to satisfy the above objective is a combinatorial optimization problem. After configuring the meters, if the measurement area which is formed by the result of the placement is regarded as a sub network, then the problem can be further transformed into the network partition problem. The network is divided into \( n \) sub network, and the power measurements are placed between different sub networks. The final solution is to select a set of sub network which measurement matrix \( H \) is full rank. In this paper, the network segmentation algorithm in the literature [7] is improved. Firstly, an initial weighted tree is formed for performing segmentation algorithm, and the weight of each node is expressed by the average active power of the user.

For weighted tree, several concepts are defined. 1) Parent/sub node: for \( i=p(j) \), node \( i \) is the adjacent contact point of \( j \) in the path of the node \( j \) to the root node, then the node \( i \) is the parent node of \( j \), node \( j \) is the sub node of \( i \); 2) Leaf node: node without sub node is leaf node; 3) Node length: the number of nodes in the path from the \( i \) to the root node (not including the root node); 4) Node length: the number of nodes in the path from the \( i \) to the root node (not including the root node); 4) Let \( T[i] \) represent the subtree that node \( i \) is regarded as the root node, and the weight of the subtree is given by:

\[
W(T[i]) = \sum_{j \in T[i]} W(j)
\]

(2)

Network Segmentation Algorithm

The goal of this algorithm is to divide the weighted tree \( G \), which contains \( L \) nodes, into \( n \) connected subsets and make the weight of each subset as close as possible to the \( W(T(L))/n \). In this paper, the starting point of the algorithm is determined by the type of load in the network. Assume that the types of users are \( n \), and each type includes a number of elements, and take a node from each type of users
to constitute a collection of search starting points of the algorithm, then choose a group starting points as the real starting point for the search. If a subtree, the root node of which is some search starting point, does not include other search starting point, the weighted tree will be "cut" to form a number of virtual leaf nodes and then continues to be split. The network segmentation algorithm must satisfy the following constraints: 1) In the process of network segmentation, each search starting point specified in this article can only be located in different measurement area. 2) When the network has been segmented into n−1 sub networks, then the segmentation termination is terminated. Because the head branches of the feeder are usually configured measuring meters.

**Optimal Meter Placement Based on the Observability Degree**

Observability judgment has binary nature (observable or unobservable), and the observability of the system can only reflect the possibility of the estimation state output of the system. The estimation accuracy and speed of the state variables can not be reflected. Therefore, it is necessary to quantify the system observability and to judge the observability degree of the system.

(1) Observability index $\eta_1$

The standard WLS method is adopted for the DSSE. The estimation error can be calculated in the following:

$$\Delta x = [H^T R^{-1} H]^{-1} H^T R^{-1} [Z - h]$$ (3)

where, $A=H^T R^{-1} H$ is the information matrix (symmetric positive definite), $B=A^{-1}$ is estimation error variance matrix, $C=H^T R^{-1} [Z-h]$ is free vector. Then the formula (3) can be converted to

$$AX = C$$ (4)

The accuracy of state estimation is generally reflected in the estimation error variance matrix, whose diagonal elements indicate the accuracy of the measurement system. Let $\eta_1$ represent the precision index of observability degree, it can be expressed: $\eta_1=tr(B)=\sum_{i=1}^{n}(1/\lambda_i)$, where, $\lambda_i$ is the real eigenvalue of matrix $A$, $\lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_n$; $n$ is the number of measurement meters.

(2) Observability index $\eta_2$

The spectral condition number of the information matrix $A$ is given by

$$\text{cond}_2(A) = \frac{\lambda_n}{\lambda_1}$$ (5)

From the analysis of formula (4), assuming there is disturbance $\delta A$, corresponding to the disturbance of the vector $C$ is $\delta C$, then perturbation of solution $X$ is $\delta C$, then it can be obtained:

$$\frac{||\delta x||}{||x||} \leq \frac{||A^{-1}|| ||\delta A|| + ||\delta C||}{1 - ||A^{-1}|| ||\delta A|| / ||A||}$$ (6)

It can be seen that when the condition number is relatively small, the relative error of the estimated value ($||\delta x||/||x||$) is small, the reliability of the estimated value is high. So, the $\text{cond}_2(A)$ reflects the sensitivity of the system estimated value to the variation of the disturbance. The smaller the condition number, the better the numerical stability. In addition, the calculation speed can also be found from the condition number in some extent, the condition number is small, the calculation speed is relatively fast [3]. So, the index reflecting the numerical stability and the velocity of observability degree is described by: $\eta_2=\text{cond}_2(A)=\lambda_n/\lambda_1$.

Considering the different contribution of two indices to the system overall observability, analytic hierarchy process (APH) is employed to determine the weight of the two indices. Then the comprehensive observability index is given, $\eta=\eta_1 w_1 + \eta_2 w_2$, where $w_1$, $w_2 \in (0,1)$ and $w_1 + w_2 = 1$.

In summary, the flow chart of the optimal algorithm is shown in Figure 1:
Case Study

Test cases of IEEE 33-bus distribution network in [7] are used, a DG node that draw from the original node system are added to verify rationality of the algorithm. The expected network structure weight tree of IEEE 33-bus distribution network with DG is shown in Figure 2. WT(100), WT(200), PV(100), PV(200) means wind driven generator/ PV power generation with rated capacity of 100kW or 200kW. The active power of conventional load was obtained from the power consumption for a period of time: \( P = W(D_m \times 24) \), where \( W \) is the power consumption, \( D_m \) is number of days for this period. For DGs which were equipped with real measurement devices, the average of measurement data is used as the weight of the DG node. Then meter placement schemes will be got which can meet the requirement of network system’s observability. After optimizing and selection, the most optimal scheme can be got. The class information of node loads is shown in Table 1.

![Figure 2. The weighted tree of IEEE 33 node system with DG.](image-url)
Table 1. The class information of node loads.

<table>
<thead>
<tr>
<th>systems</th>
<th>Load types</th>
<th>Node number</th>
</tr>
</thead>
<tbody>
<tr>
<td>IEEE 33-bus distribution network</td>
<td>1</td>
<td>23,27,6,25,26,21,22</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>31,28,5,11,9,10</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>24,29,4,32,20,12,13</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>7,30,3,14,15,16,19</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>1,17,2,8,18</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>34,35,36</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>37,38,39</td>
</tr>
</tbody>
</table>

The optimal algorithm is employed in the performing optimal measurement placement. Firstly, APH is adopted to determine the weight of $\eta_1$ and $\eta_2$, $w_1=0.6$, $w_2=0.4$. The meter optimization results is shown in Table 2. In the optimal placement, the minimum value of index $\eta$ is 100.31, which achieve the largest observability degree under the premise of satisfying the network observability.

Table 2. The results of optimal meter placement.

<table>
<thead>
<tr>
<th>Area number</th>
<th>Nodes contained in area</th>
<th>Meter location</th>
<th>$\eta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>27~31,37</td>
<td>26-27</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>39</td>
<td>24-39</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>7,8,14~17,32,35,38</td>
<td>7-20</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>24</td>
<td>23-24</td>
<td>100.31</td>
</tr>
<tr>
<td>5</td>
<td>3~6,25,26,36</td>
<td>2-3</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>2,22,23</td>
<td>1-2</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1,9<del>13,18</del>21,33,34</td>
<td>1-31</td>
<td></td>
</tr>
</tbody>
</table>

Summary

In this paper, combined with the characteristics of the DN with DGs, the observability analysis method based on meter placement was employed to achieve the observability of the ADN. In addition, an optimal meter placement algorithm based on observability degree was proposed for improving DSSE accuracy. Practical cases based on the IEEE 33-bus distribution network are adopted to verify the efficiency and feasibility of the proposal. Finally, the optimal placement is obtained which achieve the largest observability degree under the premise of satisfying the network observability.

Acknowledgement

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References


