Study on Underdetermined Inverse Problems for Grounding Grid

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Abstract. This paper proposes a solution of inverse problems for grounding grids. The grounding grid determines the safe and reliable running of substation. The study on inverse problem of grounding grid will provide an effective means for its state evaluation. The grounding grids model is half homogeneous space model, these problems are usually underdetermined due to the lack of enough data to contain a unique solution. In this paper, we present a method for the solution of underdetermined inverse problems. We present a general adjoint state formulation which can improve the calculation speed of sensitivity matrices.

Introduction

Galvanized steel is the commonly used conductor material of grounding grid, the annual corrosion rate of galvanized steel is between 2mm~8mm¹¹. The grounding grid plays an important role in the safe and reliable running of substation²². Poor welding and earthing short circuit current during construction cause poor electrical connection point of failure, that will inevitably lead to the safety performance suffered severe damage³³. The accidents of power system because of corrosion and fracture often happen in our country. On the other hand, the distribution state needs to be determined by a lack of grounding grid quantitative data.

The commonly used traditional methods such as magnetic fault diagnosis⁴⁴-⁶⁶, electric network theory⁷⁷-⁹⁹ and electrochemical detection¹⁰¹ are difficult to realize quantitative evaluation of the fault and accurate positioning, these methods can only achieve the qualitative assessment through the magnetic fields distribution or resistance, then high resolution image that reflect the conduction performance of the ground grids is very meaningful. The assessment of grounding grid condition through rebuilding the image will be a new trend of grounding grid detection. Grounding grids diagnosis system based on electrical impedance tomography measurement principle¹¹¹ and transient electromagnetic apparent resistivity imaging for break point diagnosis of grounding grids¹²¹ are put forward in recent years. To overcome the shortcomings of existing traditional methods¹³¹, this paper proposes a new analyzes method for half homogeneous space model.

The Basic Theories

The half homogeneous space model of grounding grids is shown in Figure1, the upper half space is air and the subsurface space is soil, the exciting source was situated on a ground level, grounding grids were located in the soil.

The image reconstruction method for grounding grid is described, emission source sending electromagnetic signals to underground medium through transmitting antennas, through electromagnetic field propagation between grounding grid and the surrounding medium, the receiver system receive the signal which contains the electrical characteristics information. Finally the image reconstruction can be realized through inverse problem analysis.
A linear, overdetermined inverse problem is the simplest case during the solutions, the problem in this paper is the complications of the underdetermined case, and we are often present when solving real-world inverse problem. We begin with a discussion of the simplest case, we assume a set of n field measurements, a physical theory or “forward model” governing the behavior of the system and a set of m unknown physical parameter value. Assuming for the time being that our forward model is a system of linear equations, we write the model as an n×m matrix G, and Gs is a calculation of expected measurements. The data in y contain measurement error, it is unrealistic to assume that the equation y=Gs, one possible objective function that can be minimized is the 2-norm of the residual, \[ \min(y - y_p)^T (y - Gs), \]
where \(y_o\) and \(y_p\) are observed measurements and predicted measurements.

In practice, solving inverse problems about grounding grid is typically much more difficult, it is overdetermined inverse problem. In many case, the distributed unknown field might not be measured such as the data points located in soil or grounding grid, we can only obtain the data which are on the ground. Our data about the system are often sparse which is n<m, then there is not enough data to constrain our solution to a unique parameter field estimate. In order to solving inverse problem the sensitivity matrix is indispensable.

**Adjoint State**

When the inverse problem is undermined, calculation of the sensitivity matrix is time-consuming. Adjoint operators are an often used method in order to reduce the computational burden of inverse problems\cite{14}. Conventional solving methods for sensitivity matrix H is finite difference approximation, the matrix can be obtained through altering each element of s slightly and approximated as

\[
H_{ij} = \frac{\partial h_i}{\partial s_j} \approx \frac{h_i(s + \Delta s_j) - h_i(s)}{\Delta s_j}
\]

where s is unknown solution parameters. In order to solve sensitivity matrix we need to run the forward model for each perturbation \(\Delta s_j\), this require m+1 forward model runs, but m is very large in underdetermined problems, the calculation speed will be too slow. Choosing a suitable \(\Delta s_j\) for each unknown parameter is important, that determines the approximation of the derivative and numerical noise in the estimate.

We solve H through coefficient form PDE definition

\[
f = \nabla \cdot (-c\nabla u - bu + \gamma) + au + \beta \cdot \nabla u
\]

\[
j u = r
\]

For half homogeneous space model of grounding grids, coefficients can be \(c=1, b=0, \gamma=0, \alpha=io\mu s, \beta = 0, f = \mu J, j = 1, r = 0\), then coefficient form PDE definition can be calculated as Eq.(3).
We are able to define our observations as integrals of an arbitrary function $k_i$ over the whole domain.

$$h_i = \int k_i(u, s_1, s_2, \ldots, s_m) dA$$

where $u$ is the dependent variable of our PDE. To maintain full generality $k_i$ may be a function of the dependent variable of the PDE(u) and unknowns $(s_1 \cdot \cdot \cdot s_m)$, as represented as Eq.(5)

$$k_i = -i\omega u(r)\delta(r-r_i)$$

Then we can obtain the expression of $h_i$, the sensitivity matrix can be calculated as Eq.(7).

$$h_i = \int -i\omega u(r)\delta(r-r_i) dA = -i\omega u(r_i) = E(r_i)$$

$$\frac{\partial h_i}{\partial s_j} = \int \left[ \frac{\partial k_i}{\partial s_j} + \frac{\partial k_i}{\partial u} \frac{\partial u}{\partial s_j} \right] dA$$

Obviously, $u$ is intrinsically dependent on our unknown parameters $(s_1 \cdot \cdot \cdot s_m)$. Since we have PDE and boundary conditions that govern the behavior of $u$, both the PDE and boundary conditions with respect to $s_j$ can be differentiated, that govern $\frac{\partial u}{\partial s_j}$ can be obtained. This method would obtain more accurate estimate, but solving processes is still complicated in underdetermined problems.

Using mathematical rules such as Green’s Theorems and the Divergence Theorem[15], we can avoid solving for $\frac{\partial u}{\partial s_j}$ from Eq.(8).

$$\frac{\partial h_i}{\partial s_j} = \int \frac{\partial k_i}{\partial s_j} dA + \int \psi (i\omega \mu u \frac{\partial s}{\partial s_j} - \frac{\mu \partial J}{\partial s_j}) dA$$

Since $J_s$ has no relevant to $s$, $\frac{\partial J}{\partial s_j} = 0$, then Eq.(8) can be represented as Eq.(9).

$$\frac{\partial h_i}{\partial s_j} = \int \frac{\partial k_i}{\partial s_j} dA + \int \psi i\omega \mu u \frac{\partial s}{\partial s_j} dA$$

Basing on the expression of $k_i$, and Eq.(10), the adjoint state of our problem can be acquired from Eq.(11).

$$-\nabla^2 \psi + i\omega \mu u \psi = \alpha \psi + \nabla \cdot (-c \nabla \psi - \beta \psi) + b \cdot \nabla \psi \quad (c=1,b=0,\alpha=i\omega \mu s, \beta = 0)$$

$$\left[ i\omega \mu u \psi + (-\nabla^2 \psi) = i\omega \delta(r-r_i) \right]$$

$$\left[ \psi \right]_c = 0$$

where $\psi$ is adjoint variable, $\psi$ is found via solution of the PED from Eq.(11). $\frac{\partial k_i}{\partial s_j} = 0$, then we can obtain the expression from Eq.(9).

$$\frac{\partial h_i}{\partial s_j} = \psi_i i\omega \mu u \Delta A_j$$
The PDE for $\psi$ is dependent only on $i$, the observation number, finding $H$ with adjoint states can be executed with only $n+1$ forward model runs. The efficiency for underdetermined problems where $m\gg n$ can be improved drastically.

**Solving Process**

Basing on Eq.(12) a row of sensitivity matrix can be obtained through one measurement point. In order to validate the accuracy and rational of the method, mathematical-physical model of grounding grids can be built, $u$ can be acquired through the solving of forward problem, the data of forward problem solution are assumed the measurement data.

In the process of inverse problem solution, exciting source is a known quantity, it is wideband width pulse excitation, we suppose the initial value of $s$ is $s^{(0)}$, $u$ of the whole area can be solved through the forward problem. $\psi^{(0)}$ of the whole area can be solved from adjoint states. The number of measurement is $n$, a row of sensitivity matrix can be solved in every measure, the whole element of sensitivity matrix can be solved through $n+1$ times.

![Figure 2. Setup of example inverse problem.](image)

**Reverse Problem**

The reconstructed image was reconstructed using the observed data which are obtained from the analysis of forward problems, and the electrical characteristic parameters were reconstructed preliminarily using the nonlinear damped least square method. Through setting the objective function and solving the sensitivity matrix, two different conductivity samples of different shapes are reconstructed, the size of square sample is 0.03×0.03 and the radius of round is 0.08. The result of reconstruction is shown in Figure 3.

![Figure 3. Image reconstruction by the least square method.](image)

Electrical characteristic parameters were reconstructed preliminarily using the adjoint states method. Two different conductivity samples of different shapes are reconstructed, the size of inner square sample is 0.02×0.02 and the size of outside square sample is 0.05×0.05. The result of reconstruction is shown in Figure 4.
Summary
This paper proposes underdetermined inverse problems for grounding grid aiming at accurate positioning and quantitative evaluation of corrosion and breakpoint for power system grounding grid. We present a method for the solution of underdetermined inverse problems. We present a general adjoint state formulation which can improve the calculation speed of sensitivity matrices. The result of two methods is put forward in this paper to verify the theoretical derivation and the accuracy of propagation characteristics. The distribution of the field under different sample conditions is analyzed. The reconstruction of electrical characteristic parameters are solved by the damped least square method and adjoint state theory.

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References


