The Optimal Strategy of the Goodgrant Foundation

Mei XIONG\textsuperscript{a}, Long-Wei CHEN\textsuperscript{b,*,}\textsuperscript{,} Yu CAO\textsuperscript{c}, Xing-Yu Li\textsuperscript{d}, Yu-Qi ZHANG\textsuperscript{e}

Statistics and Mathematics College, Information School, Yunnan University of Finance and Economics, Kunming, China, 650221

\textsuperscript{a}clwxmff@163.com, \textsuperscript{b}532086724@qq.com, \textsuperscript{c}807382231@qq.com, \textsuperscript{d}814897422@qq.com, \textsuperscript{e}503045014@qq.com

*Corresponding author

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Abstract. The Goodgrant Foundation want to help improve educational performance of undergraduates attending colleges and universities in the United States, and it maintains for five years, starting July 2016, and they wonder how to invest. To begin with, we primarily screen 7804 school samples and elect 2977 relatively excellent schools. Investors are divided into three categories according to their attitude towards the risk (risk preference, risk neutral and risk aversion). The optimal investment plan of risk preference, risk neutral, and the risk aversion investments are 54 schools, 93 schools, and 95 schools respectively by the means of technique of Markowitz mean-variance theory.

Introduction

Without many American foundations, the growth rate of educational performance of undergraduates attending colleges and universities in the United States cannot be so rapidly. To do this, Goodgrant Foundation intends to donate a total of $100,000,000 (US100 million) to an appropriate group of schools per year, for five years, starting July 2016. In doing so, they do not want to duplicate the investments and focus of other large grant organizations such as the Gates Foundation and Lumina Foundation.

Goodgrant foundation has the following four requirements:

\begin{enumerate}
\item Screening relatively excellent schools, using the data provided in the attachment, references and network resources;
\item Take the school’s demonstrated potential for effective use of private funding and investors’ attitude towards the risk into consideration, using the appropriate method to filter the alternative investment schools;
\item Develop a model to determine an optimal investment strategy that identifies the schools, the investment amount per school, the return on that investment, and the time duration that the organization’s money should be provided to have the highest likelihood of producing a strong positive effect on student performance. This strategy should contain a 1 to N optimized and prioritized candidate list of schools you are recommending for investment based on each candidate school’s demonstrated potential for effective use of private funding, and an estimated return on investment (ROI) defined in a manner appropriate for a charitable organization such as the Goodgrant Foundation;
\end{enumerate}

\textsuperscript{*}This problem is from “2016 Mathematical Contest in Modeling (MCM) Problem C”.

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Markowitz illustrates the mean-variance theory in 1952[1], which is the start of the financial investment research, it is the important tool in financial investment theory. This theory use the return on assets as random variables, probability theory and optimization techniques to model the investment behaviour under uncertainty[2]. Many scholars have to vote carried out on the basis of this theory Research portfolio theory later[2,3,4].

The Model Design

Screen of Data

In order to determine three optimal investment strategies and 1-N ranked list of schools, the process of developing models is shown the following steps:

In the first step, all schools were screened in accordance with possible investment potential schools given in the table. We primarily screen 7804 school samples from Table Problem C Most Recent Cohorts Data (Scorecard Elements) and elect 2977 relatively excellent schools from Table Problem C Most Recent Cohorts Data (Scorecard Elements).

In the second step, we use Markowitz mean-variance theory to set up screening and arraying model. In such a case, selected schools can be sorted by different attitudes towards risk (risk preference, risk neutral and risk aversion).

In the third step, with the help of Markowitz portfolio theory, we establish multi-goal Programming model of Investment Portfolio (MLP) to select optimal allocation of different schools and return-on-investments (ROI) in three cases, respectively.

The efficient frontier can be determined according to the portfolio theory of Markowitz mean-variance method theory, and investors can be divided into three categories according to their attitude towards the risk. It follows the optimal range given in Table 1:

<table>
<thead>
<tr>
<th>Attitude towards the risk</th>
<th>Optimal range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Risk preference</td>
<td>$(\bar{z}_j + 2.5\sigma_j, +\infty)$</td>
</tr>
<tr>
<td>Risk neutral</td>
<td>$(\bar{z}_j + 2\sigma_j, +\infty)$</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>$(\bar{z}_j + 1.5\sigma_j, +\infty)$</td>
</tr>
</tbody>
</table>

$\bar{z}_j$ is the mean of variable in each column while $\sigma_j$ is variance of variable in each column.

The abscissa refers to the number that each index lies within the optimum range, the ordinate refers to the number of schools lies within the optimum range. For example, when the horizontal axis is 7, the vertical axis is 14. It represents 7 indexes about 14 schools lies within the optimum range among 2977 schools while investors who prefer risk.

The more fall the number of indexes into the optimal, the higher is the ranking of schools. A list of schools can be obtained according to the number of index lies within the optimum range, which can be classified into three attitudes towards risk of investors.

On the one hand, the class standardization adjusted each variable of schools as percentages, eliminating the extreme impact that large magnitude variable data have on other data. On the other hand, after screening 7804 school samples from Table Problem C Most Recent Cohorts Data (Scorecard Elements) and elect 2977 relatively excellent schools from Table Problem C
Most Recent Cohorts Data (Scorecard Elements). Under the optimal number of variables in each layer level descending sort, we pick out three excellent schools as feasible set. This program can not only sort large amounts of raw data to filter and to provide data support for subsequent optimum model, but also come to the preliminary sorting scheme of schools.

**The Optimal Investment Models (MSLP)**

Assume that Goodgrant Foundation chose \( n \) different school investments, planning to invest for \( m \) years; the proportion of investment in the investment portfolio accounted for

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ij} = 1, \quad x_{ij} \geq 0 (i = 1, 2, \ldots, m, j = 1, 2, \ldots, n) \tag{1}
\]

where \( i \) stands for certain years, \( j \) stands for certain school.

Then we set the actual rate of return on the investments for schools in certain years as \( r_{ij} \), and set average annual yield of the certain \( j \) school as

\[
r_{ij} = \frac{1}{m} \sum_{i=1}^{m} r_{ij} \quad (j = 1, 2, \ldots, n) \tag{2}
\]

So we get the total educational performance of investment portfolio:

\[
R = \sum \sum x_{ij} r_{ij} \tag{3}
\]

In order to reduce the errors, we introduce the bias symbol:

\[
e_{ij} = r_{ij} - r_{ij} = r_{ij} - \frac{1}{m} \sum_{i=1}^{m} r_{ij} \quad (i = 1, 2, \ldots, m, j = 1, 2, \ldots, n) \tag{4}
\]

Take the sum of absolute value of deviation and measuring indicators of Markowitz profitable theory into consideration, we get the total risk of investment portfolio:

\[
\sigma = \sum \sum |e_{ij}| x_{ij} \tag{5}
\]

**Establish and Solve Multi-goal Programming Model (MLP)**

If we only consider to maximize the total educational performance and minimize the total investment risk, from the analysis and assumptions above, we can get an optimization model (MLP), shown as follows[5,6]:

\[
\max R = \sum \sum r_{ij} x_{ij}, \quad \min \sigma = \sum \sum e_{ij} x_{ij} \tag{6}
\]

s.t.

\[
\begin{align*}
\sum_{j=1}^{n} x_{ij} &= 1 \\
0 &\leq x_{ij} \leq b
\end{align*} \tag{7}
\]

where the upper limit of investment amount is \( b \).

For ease of notations and calculation, we simplifying MLP give SLP. Seeing the reciprocal of risk makes it minimum, we define return-on-investment (ROI):
We set ROI as a goal function.

From the analysis above, we can get single-goal Programming model (SLP), shown as follows:

$$\max R = \sum_{i=1}^{n} \sum_{j=1}^{m} e_{ij} x_{ij} + \frac{1}{\sum_{j=1}^{m} \sum_{i=1}^{n} e_{ij} x_{ij}}$$

$$s.t. \sum_{j=1}^{m} x_{ij} = 1$$

$$0 \leq x_{ij} \leq b$$

In SLP, real earnings in the next five years can be forecast by a known previous five years.

In addition, we use principal component analysis to analyze Net price for the lowest income/highest income students, Median debt for completes, Completion rate, Median rate, Median earnings, share of Fmr.students Earning more than HS Graduates and Share of Pell Recipients. We calculated the variables of each school’s first principal component scores in different years as the estimated value $r_{ij}$, which reflected the potential for effective use of private funding.

The solution methods of SMLP are similar to that of screening and arraying model. By the means of Markowitz mean-variance theory, we divided the attitude towards risk into three cases.

**Risk Preference**

It is straightforward to show the maximum return-on investment (ROI). Below we explain the results in most detail: According to the order of school, we take the former $1 \sim N(N = 24, 33, 48, 54, 68, 91, 118, 132, 159, 218)$ schools into consideration by the means of technique of the model SLP, with LINGO software to solve, follows the optimal investment allocation.

As figure 1 illustrates:

![Figure 1. The Correlations between Maximum ROI and the Number of Schools in Risk Preference.](image)
N stands for the number of school to be invested.
The figure shows the trend of maximum ROI with the number of schools grows.
The return-on-investment (ROI) raises its highest when the number of schools become largest and then decreases.
When N is 54, the amount of ROI reaches the highest at 156203.8. It means that the Foundation should invest 54 schools, and the value of goal function is 156203.8.
The optimal investment strategy: Bowdoin College occupies the largest percentage of investments, and the amount of investment is $0.6 million, and the time duration is three years.
In general, the less number of schools be invested, the more centralized amount of investment in certain minority schools, the gap of amount of investment between school are bigger. The more number of schools be invested, the less amount of investments assigned to each school, relatively evenly with difference.

Risk Neutral
The solution of Risk neutral is similar to that of Risk preference. Below we explain the results in most detail: According to the order of school, we take the former N ∈ \{24, 49, 54, 60, 68, 80, 93, 134, 151, 219, 331\} schools into consideration by the means of technique of the model SLP, with LINGO software to solve, follows the optimal investment allocation.
As figure 2 illustrates:

![Figure 2](image)

Figure 2. The Correlations between Maximum ROI and the Number of Schools in Risk Neutral.

1. The figure shows the trend of ROI with the number of schools grows.
2. The return-on-investment (ROI) raises its highest when the number of schools become largest and then decreases.
3. When N is 100, the amount of ROI reaches the highest at 222092. The Foundation should invest 100 schools, and the value of goal function is 222092.
4. The optimal investment of allocation shows in table 1 at appendix.
The allocation of investments in Risk neutral is similar to Risk preference. We offer different strategies for charity Foundation in Risk preference.

Risk Aversion
The solution of Risk aversion is similar to that of Risk preference. Below we explain the results in most detail: According to the order of school, we take the former N ∈ \{20, 31, 48, 63, 80, 95, 119, 136, 163, 190, 252\} schools into consideration by the means of
technique of the model SLP, with LINGO software to solve, follows the optimal investment allocation.

(1) The figure shows the maximum ROI as the number of schools grows.
(2) The trend of return-on-investment (ROI) raise its height when the number of schools become maximum and then decreases.
   When N is 100, the amount of ROI reaches the highest.
(3) The allocation of investments in Risk neutral is similar to the above two cases.

As figure 3 illustrates:

![Figure 3. The Correlations between Maximum ROI and the Number of Schools in Risk Aversion.](image)

**Conclusion**

By comparing different degree of risk preferences to select the optimal number of schools, it can be found that the most optimal number of schools increase with the reduction of degree of risk preference, which conform the reality. In practical situation, the more schools be invested, the more diversified of investment, the lower of the risk.

Meanwhile, according to the box plots in three cases, we can see that the more schools be invested, the much evenly that funds administered to each school; the fewer schools be invested, the greater the gap of capital investment in each school.

The optimal strategy we get from multi-goal Programming model is consistent with real-life situation. Most of America’s top universities are included in our proposed investment school, which has a strong practical significance and reference value. Investors can tend to choose different investment strategies according to their preferences of risk and the degree of dispersion of the funds. Investment programs under three different preferences of risk exactly give investment per school, return-on-investment (ROI) and the time duration. All these parameters can be visually query from optimal investment allocation table.

**References**
