Study on Two-level Supply Chain Inventory Management
Collaboration Strategies under Carbon Emission Caps

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Abstract. This paper builds the carbon-emission-based Economic Order Quantity (EOQ) model in the two level supply chain with a supplier and a retailer, and studies the effects of interests of enterprises and supply chain system caused by three different collaboration ways of supply chain considering the carbon emission caps. These three collaboration ways are listed as follow: ① the retailer or supplier is dominant and another is compliant; ② the retailer and supplier have equal status and collaborate with independent carbon emission caps; ③ the retailer and supplier have equal status and collaborate with shared carbon emission caps.

Introduction

Entering the 21st century, with the deterioration of global warming, detecting and controlling the emission of greenhouse gas which is composed mostly by carbon dioxide has been attached great importance all over the world.


Wahab et al [4] considered three conditions as retailers and suppliers in the same country, different countries or in combination with the cost of carbon emissions in a two-stage supply chain. Benjaafar et al [5] have introduced carbon emission factor into the supply chain system.

This passage studies the effects to the interest of enterprise and supply chain in two level supply chain with one supplier and one retailer, retailers and suppliers under three different modes in consideration of carbon emissions limits policy. The three cooperations follow as:

① Retailers (or suppliers) dominate, suppliers (or retailers) account for the status of the subject.

② Retailer and supplier status are equal, and the two sides cooperate with each other in the case of carbon quota independence.

③ Retailer and supplier status peer, the two sides in the case of carbon quota sharing mutual cooperation.
Model Hypothesis and the Symbol Definition

Assume that demand of the products is fixed, replenishment has no lead time. The total cost of the enterprise is consisted of ordering cost and holding cost. At the same time transportation and inventory operation produce carbon emissions.

Decision variables and parameters:
- D: The rate of market demand for the product;
- T: Retailer's ordering cycle (decision variables);
- a: Fixed cost of retailer for each order;
- A: Fixed cost of supplier for each order;
- h: Unit inventory storage costs of retailer;
- H: Unit inventory storage costs of supplier;
- $\hat{a}$: Retailer's carbon emissions per order;
- $\hat{A}$: Supplier’s carbon emissions per order;
- $\hat{h}$: Retailer’s carbon emissions per inventory holdings;
- $\bar{H}$: Supplier’s carbon emission per inventory holdings;
- m: The retailer’s carbon emissions quota;
- M: The supplier’s carbon emissions quota;

For the classical EOQ model (no considering carbon emissions), average total cost of retailers and suppliers is respectively $\frac{a}{T} + \frac{hDT}{2}$ and $\frac{A}{T} + \frac{HDT}{2}$, the retailer's optimal order cycle is $\sqrt{\frac{2a}{hD}}$, the optimal cost is $\sqrt{2aDh}$. The supplier's optimal order cycle is $\sqrt{\frac{2A}{H}}$, the optimal cost is $\sqrt{2AHD}$.

Considering the carbon limitation: Total cost of retailer $Z_b = \frac{a}{T} + \frac{hDT}{2}$, emissions of retailer $E_b = \frac{\hat{a}}{T} + \frac{\hat{h}DT}{2} \leq m$. Supplier’s Cost $Z_s = \frac{A}{T} + \frac{HDT}{2}$, supplier’s emissions $E_s = \frac{\hat{A}}{T} + \frac{\bar{H}DT}{2} \leq M$.

Total cost of supply chain $Z = Z_b + Z_s = \frac{a+A}{T} + \frac{(h+H)DT}{2}$. Overall carbon emissions of supply chain $E = E_b + E_s = \frac{\hat{a}+\hat{A}}{T} + \frac{(\hat{h}+\bar{H})DT}{2}$.

Impact Analysis with Carbon Emission Constraint

Cooperation 1: One Party Dominant (Retailers or Suppliers)

Assumeing the retailer in a dominant position and supplier a follower, retailers minimize their own costs to maximize their own interests and consider suppliers' emissions limits, otherwise suppliers might be regulated and punished.

The model:

\[
\min \ Z_1 = \frac{a}{T} + \frac{hDT}{2},
\]

s.t. \[
\begin{align*}
E_b &= \frac{\hat{a}}{T} + \frac{\hat{h}DT}{2} \leq m. \\
E_s &= \frac{\hat{A}}{T} + \frac{\bar{H}DT}{2} \leq M.
\end{align*}
\]
To minimize the retailer's emissions, the order cycle $T_1^* = \sqrt{\frac{2a}{hD'}}$ When retailer’s carbon emissions being maximal, $T_1 = \frac{m - \sqrt{m^2 - 2a \Delta D}}{hD}$, $T_2 = \frac{m + \sqrt{m^2 - 2a \Delta D}}{hD}$. Now $T_1$ and $T_2$ represent respectively the minimal and the maximal order cycle of retailer with carbon emissions constraint. Similarly we have supplier’s minimal order cycle $T_3$ and the maximal order cycle $T_4$ under carbon emissions constraints, $T_3 = \frac{M - \sqrt{M^2 - 2a \Delta D}}{\bar{h}D}$, $T_4 = \frac{M + \sqrt{M^2 - 2a \Delta D}}{\bar{h}D}$.

**Theorem 1.** With cooperation method 1 considering carbon emissions the optimal order lot $\tilde{T}_1^*$ satisfies $\tilde{T}_1^* \in \left[T_1', T_2'\right]$ , and

$$\tilde{T}_1^* = \begin{cases} T_1' & \text{if } T_1' \leq T_1^* \\ T_1^* & \text{if } T_1' < T_1^* < T_2' \\ T_2' & \text{if } T_1^* \geq T_2' \end{cases}$$

Theorem 1 shows the connections of the optimal ordering cycle with emission cap $\tilde{T}_1^*$, the optimal ordering cycle without emission cap $T_1^*$, and the ordering cycle interval with emission cap $\left[T_1', T_2'\right]$. If $T_1^*$ is between $T_1'$ and $T_2'$, then considering carbon emissions, the enterprise can still operate according to the original optimal ordering cycle, so the cost of leading enterprise remains minimal. Otherwise, the enterprise shall react accordingly. Similarly, in the case of suppliers dominating position, the order cycle becomes the supplier's delivery cycle. So we do not discuss this cooperation method to avoid liability.

**Cooperation 2: Equal Status and Independent Carbon Caps**

Condition: all members in the supply chain collaborating with each other, targeting at minimizing total cost of the supply chain, emissions cap still independent. This method is also common in industrial manufacturing, such as in the cooperation plan and replenishment plan [1].

The model:

$$\min Z_2 = \frac{a + A}{T} + \frac{(h + H)DT}{2}.$$  \hspace{1cm} (5)

$$\begin{cases} E_b = \frac{\hat{a}}{T} + \frac{\bar{h}DT}{2} \leq m. \\ E_s = \frac{\hat{A}}{T} + \frac{\bar{h}DT}{2} \leq M. \end{cases}$$ \hspace{1cm} (6)

So we can have

$$T_2^* = \sqrt{\frac{2(a + A)}{(h + H)D}}.$$ \hspace{1cm} (7)

**Theorem 2.** With shared total cost and independence emission cap, the optimal ordering cycle $\tilde{T}_2^*$

$$\tilde{T}_2^* = \begin{cases} T_1' & \text{if } T_1' \leq T_1^* \\ T_2' & \text{if } T_1' < T_2^* < T_2', \text{ and } \tilde{T}_2^* \in \left[T_1', T_2'\right], \\ T_2' & \text{if } T_1^* \geq T_2' \end{cases}$$

**Corollary 1:**
1. If \( \frac{a}{h} = \frac{A}{H} \), then \( Z_1 = Z_2 \), \( E_1 = E_2 \).

2. If \( \frac{a}{h} \neq \frac{A}{H} \), then \( T_1^* \leq T_1 \), \( T_2^* \leq T_1' \), or \( T_1' \geq T_2^* \), \( T_2^* \geq T_2' \), then \( Z_1 = Z_2 \), \( E_1 = E_2 \); Or \( Z_1 > Z_2 \), \( E_1 \neq E_2 \).

Corollary 1 shows that if \( \frac{a}{h} = \frac{A}{H} \), which is all the business conditions of the retailer and the supplier are the same, then \( T_1^* \) equals \( T_2^* \), and the supply chain is at the best state and costs do not change which situation is not common in real life. The following situations is more common: when \( \frac{a}{h} \neq \frac{A}{H} \), and \( T_1' \) or \( T_2' \) in the interval \([T_1', T_2']\), we can effectively reduce the total cost of the whole supply chain and optimize the supply chain using cooperation method 2. Because in this case enterprises work together instead of considering their own profit.

**Cooperation 3: to Share the Total Cost and Carbon Quotas**

The seller and the buyer of the supply chain further expand cooperation scope, sharing their own carbon quota, at the same time targeting at minimizing the total cost of supply chain. If the enterprise on the supply chain is not independent but subjects to a branch of the major corporation or has established a good relationship of cooperation, the cooperation method is common.

The modal:

\[
\begin{align*}
\min Z_3 &= \frac{a+A}{T} + \frac{(h+H)T}{2}. \\
\text{s.t.} E_3 &= \frac{\hat{a}+\hat{A}}{T} + \frac{(\hat{h}+\hat{H})T}{2} \leq m + M.
\end{align*}
\]

No considering carbon emissions, the optimal ordering cycle \( T_3^* = T_2^* = \frac{\sqrt{2(a+A)}}{(h+H)D} \) When carbon emissions \( E_3 \) reaches its peak \( m + M \), then:

\[
T_5 = \frac{m+M-\sqrt{(m+M)^2-2(\hat{a}+\hat{A})(\hat{h}+\hat{H})D}}{(\hat{h}+\hat{H})D}, \quad T_6 = \frac{m+M+\sqrt{(m+M)^2-2(\hat{a}+\hat{A})(\hat{h}+\hat{H})D}}{(\hat{h}+\hat{H})D}.
\]

Interval \([T_5, T_6]\) is the feasible region of the cooperation method.

**Theorem 1.** With shared total cost and considering emission caps, the optimal ordering cycle \( T_3^* \) satisfies \( T_3^* \in [T_5, T_6] \), and

\[
T_3^* = \begin{cases} 
T_5 & \text{if } T_3^* \leq T_5 \\
T_1' & \text{if } T_5 < T_3^* < T_6, \\
T_6 & \text{if } T_3^* \geq T_6.
\end{cases}
\]

At the same time we can have \( \hat{T}_2^* \in [T_5, T_6] \).

We can see under cooperation 2 the optimal ordering cycle \( \hat{T}_2^* \) is always in the cycle interval \([T_5, T_6]\) of cooperation 3 with emission cap. Compared with independent carbon caps, shared emission quota makes a wider choice of enterprise ordering cycle and more flexible operation, reducing the carbon quota restrictions on enterprise operation, and much fully used the this resource of the interesting parties.

**Corollary 2.**

1. If \( \frac{a}{h} = \frac{A}{H} \), if \( T_2^* \in [T_1', T_2'] \), then \( Z_2 = Z_3 \); if \( T_2^* \notin [T_1', T_2'] \), then \( Z_2 > Z_3 \).

2. If \( \frac{a}{h} \neq \frac{A}{H} \), (a) if \( T_2^* \notin [T_1', T_2'] \), then \( Z_2 > Z_3 \); (b) if \( T_2^* \in [T_1', T_2'] \), then \( Z_2 = Z_3 \).
By the corollary 2 we can see that using the cooperation 3 under certain conditions can further reduce supply chain cost, and supply chain cost being minimal in the three ways although using this method will cause the originally low-cost smaller firms to increase their cost slightly. This kind of method of cooperation is suitable for two branch firms which belong to the same stem company.

**Corollary 3.**

1. If \( \frac{a}{h} = \frac{A}{H} \), then \( Z_1 = Z_2 = Z_3 \); or \( Z_1 = Z_2 > Z_3 \).
2. If \( \frac{a}{h} \neq \frac{A}{H} \), then (a) if \( T^*_1 \leq T^*_1 \), \( T^*_2 \leq T^*_1 \), then \( Z_1 = Z_2 > Z_3 \). (b) if \( T^*_1 \in [T^*_1, T^*_2] \), then \( Z_1 > Z_2 = Z_3 \). (c) if \( T^*_2 < T^*_1 < T^*_1 < T^*_2 \), then \( Z_1 > Z_2 > Z_3 \).

We can see from corollary 3, if \( \frac{a}{h} = \frac{A}{H} \), \( T^*_2 \notin [T^*_1, T^*_2] \), sharing carbon quota can kelp to reduce the total cost of supply chain, which is not achievable using carbon quota independent cooperation method. If \( \frac{a}{h} \neq \frac{A}{H} \), under certain conditions, carbon quota sharing way can help to further reduce the total cost of supply chain.

Integrating the above conclusions, the cooperation method of sharing carbon emissions quota is the optimal way for the interests of the whole supply chain.

**Summary**

In 21 century, the competition between enterprises is not only the competition between enterprises and enterprises, but the competition between supply chain and supply chain. The results of this article can be used to determine the optimal ordering strategy for enterprises under the carbon emission environment in the supply chain, so as to provide some reference for the effective competitive strategy of the supply chain.

**References**


